INTRODUCTION AND RATIONALE
In my experience as a teacher the most popular question asked by learners is “Where am I going to use this mathematics?” This question emanates from seeing mathematics as unrelated abstract pockets of knowledge rather than a set of related and useful topics. My experience has been that when learners understand and relate a particular topic to their existing knowledge this question seldom crops up. I believe that learners’ inability to see mathematics as a worthwhile human activity is in part due to the low level of mathematical reasoning in classrooms. Learners who learn mathematics through mathematical reasoning may find the mathematics more meaningful. Mathematical reasoning allows learners to form connections between new and existing knowledge. This integration of knowledge facilitates sense making on the part of the learners. They are therefore in a better position to see mathematical activity as worthwhile activity. Learners who engage in mathematical reasoning may be in a better position to connect school mathematical activity to everyday mathematical activity. Mathematical reasoning enables the development of conceptual understanding which will allow learners to draw on their concepts in other situations. They will therefore experience mathematics as something they can understand and relate to.

The draft National Curriculum Statement (NCS) (2002) expresses the vision of a learner that can:

- Identify and solve problems and make decisions using critical and creative thinking
- Work effectively with others as members of a team, group and community

Herein are contained two of the focus points for my research. The first of the outcomes highlights the need for teaching mathematical reasoning. The second outcome embraces the notion of collaborative learning. Similar sentiments are echoed throughout the entire document. In response to the NCS (2002) this research was aimed at understanding the ways in which learners reason mathematically in a whole-class discussion.

Motivated by a need to teach in a way that will make mathematics more meaningful to my learners and guided by curriculum change, I formulated my research question as follows:

How do learners reason mathematically while working collaboratively during whole class discussions?

THEORETICAL FRAMEWORK AND LITERATURE REVIEW
Mathematical reasoning is made up of a number of processes. The learner makes observations and then connects these with existing knowledge and in so doing restructures this knowledge (Hatano 1996). Proficiency in “procedural fluency” and “conceptual understanding” (Kilpatrick, Swafford and Findell 2001) is needed for such restructuring. Key to enabling restructuring is to explain, communicate and justify assertions made, which are features of “adaptive reasoning” as argued by Kilpatrick et al. During this communicative process we see the learner evaluating the new knowledge and refining it to a point where the validity of other assertions can be checked.

A broader perspective regarding mathematical reasoning is the development of mathematical proficiency. Kilpatrick et al. (2001) identify five stands of mathematical reasoning namely Conceptual understanding, Procedural fluency, Strategic competence, Adaptive reasoning and Productive disposition. In this research I focused on adaptive reasoning. However as Kilpatrick et al. argue, the strands are interrelated and so the other strands were key in enabling adaptive reasoning to occur. Each strand complements the others, for example...
learners with better procedural proficiency can focus more readily on the underlying concepts at hand and will be better able to reason. Learners with a lack of procedural fluency will be bogged down by algorithms, leading to these algorithmic procedures becoming the focus point instead of the mathematical concepts and reasoning.

Collaborative learning is a communicative process whereby two or more parties gain new knowledge as a result of their interaction. Collaborative learning not only refers to an exchange of knowledge between the parties, but the interaction itself serves as a catalyst for the formation of new knowledge by the parties concerned. In my class I think of collaborative learning as a joint venture between learner/s and teacher and among learners themselves. This collaboration is governed by the pursuit of knowledge for the development of learner and teacher. How we reason mathematically or allow our learners to reason mathematically is in part dependent on the nature of collaboration between the parties. The nature of the learning which occurs is a complex interplay between individual and social construction (Hatano 1996; Wood, Cobb and Yackel 1992).

In the classroom the teacher tries to establish an educational discourse (Mercer 1995) that s/he would like to see learners emulate in mathematical discussions within the classroom and beyond. The classroom situation consists of learners from different backgrounds, with different knowledge, and many more differences. These differences have more potential to facilitate learning as opposed to one-sided teacher instruction. Direct teacher instruction addresses learner needs from the perspective of the teacher’s assessment of learner knowledge. Collaborative learning allow for a greater variety of issues to be addressed simultaneously within the learning environment. Learners are able to raise content and contexts of learning which relate to their understanding. My fear was always that collaborative learning would advantage weaker learners and disadvantage stronger learners. Reading, “Learning to see the invisible” (Schiffer 2001) provided me with insight into the ability for stronger learners to engage with weaker learners and develop deeper conceptual understandings. Deeper understandings unfold as stronger learners have to explain and justify their understandings.

METHODOLOGY
This was an action research project conducted in one of my grade 11 classes comprising 34 learners. During action research, collaboration with members in my research group was key. The school I teach at is a former Model C school situated in the south of Johannesburg. Data were collected by video taping learners while they were engaged in activities geared to elicit mathematical reasoning. The activities were selected and slightly modified from a text, which is currently being developed for the new FET curriculum (MEP, in preparation). Our research group worked together to select the activities. Three reasons were primarily responsible for the decision to focus on the material dealing with the quadratic function. One was that during our first year in the Honours programme we dealt with the teaching of functions in an investigative way as suggested by the activities. We were therefore on familiar territory. Also none of us had dealt with this section in our respective classes. More importantly these activities lend themselves to mathematical reasoning, and require learners to formulate ideas and justify them.

The focus of the analysis was on the classroom discussions that took place during these activities. In particular, I looked at how whole-class discussion and collaboration enabled the development of mathematical reasoning by one of the learners. The learner was selected based on her visible participation throughout the lesson. Analysis took the form of a chronological observation table (see Appendix). The analysis focussed on the development of her contributions as the class discussion progressed. Every visible contribution she made was recorded in a table describing the time, researcher observation and nature of learning that occurred. Once her learning had been described, the contributions the class discussion made in enabling key learning moves were analysed.

ANALYSIS
The class discussion that is the focus of the analysis was preceded by the learners working individually and then in groups, on the same task. The content under discussion was to relate the changes affected by the
horizontal translation of the graph of $y = x^2$ to the equation $y = (x-p)^2$ where $p$ was 3 and $-4$ respectively. The analysis follows Winnie’s learning through:

- observational assertions;
- making conceptual and representational links, firstly incorrect and then correct;
- explaining and justifying assertions made
- testing and evaluating new conceptual understanding
- using her new conceptual frame to test and point out the weaknesses in another learner’s assertions.

The analysis also deals with how this development was enabled through collaboration with other learners.

**Winnie’s Learning as Collaborative learning**

Winnie’s initial assertions were merely observational and she did not see a need for justification. This is suggested by her lack of explanations even though she was prompted by the teacher to clarify her assertions. The type of observations Winnie made can be seen in the observational table (Appendix, at 13:21:00). The teacher allowed another member in Winnie’s group, Gary, to give their findings on the next question in the activity. The class did not understand Gary’s contribution. A little more confidently Winnie then tried to simplify Gary’s assertions (Appendix, at 13:24:35), which forced her to begin connecting her observations to the equation. Winnie began to acknowledge a need to explain assertions, realising that Gary’s assertions needed explanation to her and probably the rest of the class. In so doing, her own reasoning developed. This suggests one way in which learning collaboratively feeds into the process of mathematical reasoning. While she did not see the need to clarify or explain her own assertions hearing another learner’s assertions, of which she was party to, prompted her to explain.

As learners in the class sought more clarity from Winnie with regard to Gary’s assertions, Winnie realised that she needed to switch modes. She needed to change from a verbal explanation to a written one (Appendix, at 13:25:44) and she requested permission to explain using the OHP. The content Whitney used here suggests that she moved to a higher level of reasoning. Not only did Winnie explain observations made, but began to make connections between her graphical observations and their representations in equations. The need to use a written representation to illustrate the translation of the graph of $y = x^2$ as $y = x^2 - 3$ (translation 3 units to the left) and $y = x^2 + 3$ (translation of 3 units to the right) served as a catalyst for making these connections. Although these connections are mathematically incorrect it does show that she was starting to make conjectures about certain patterns she observed in the written representations. The insistence from the class for clarity serves as a catalyst for explaining and justifying the assertions made.

A key turning point in the class discussion and in Winnie’s learning occurred when Mary-Anne posed a question directly to Winnie. Mary-Anne asked why the graph for $y = (x + 4)^2$ has a turning point of $-4$. She related the 4 inside the bracket as contradicting a turning point of $-4$ and asked Winnie to explain this. Her question served as a catalyst for a fervent discussion, which resulted ultimately in Winnie formulating her new conceptual frame. Winnie was confronted by cognitive conflict (Chazan & Ball 1999) resulting from Mary-Anne’s question. The question forced Winnie to reflect on the equation $y = (x + 4)^2$ instead of her example of $y = x^2 + 3$. Winnie was forced to re-evaluate her previous assertions.

During the class discussion that followed, Winnie did not return to her seat but occupied a seat in front of the class. She listened attentively throughout the discussion among 4 other learners, which followed Mary-Anne’s question. When she re-entered the discussion as a verbal participant she addressed her assertions to Mary-Anne. This suggests to me that Mary-Anne’s question served as a catalyst for Winnie’s reasoning that followed. It shows in a very powerful way how collaborative learning through a sense of responsibility for one’s own and others’ learning allows for mathematical reasoning to enter the classroom.

Winnie emerged from being a silent participant in the discussion with new knowledge. Winnie used adaptive reasoning as she explained the misconception she was party to. She correctly explains that the turning point of $-4$ for the graph $y = (x + 4)^2$ is an $x$-value and did not necessarily represent the 4 inside
the bracket (Appendix, at 13:34:40). She went further by asserting that the equation is a representation of the relational value of the variable quantities x and y. Winnie’s assertions initially took the form of a verbal contribution, which the other learners found difficult to understand. When they requested clarity, Winnie used visual representations once again to explain her new-found understanding. In explaining the second time using the OHP her explanation is not just clearer to the listener but her explanation has progressed to become more focussed and connected. Once again the quality of collaborative learning was present in Winnie’s reasoning. She modified her initial mathematically incorrect assertion to a mathematically correct assertion. Her explanation to the class facilitated their understanding but also assisted in refining her own understanding of the issue at hand. With this understanding she confidently explained Mary-Ann’s dilemma.

In Appendix at 13:36:30 – 13:14:18 we see Winnie again being confronted by Dion, who disagrees with her assertions. Armed with the knowledge of an equation as a representation of the relational value between the two variable quantities, she was in a position not only to affirm her assertions but also through preemptive questioning to convince Dion of the argument. Once again Winnie was struck by the reality that even though her reasoning was clear to herself, in a collaborative set-up it is only acceptable when she is able to convince others of her argument. This allowed Winnie to move to another level in mathematical reasoning. She started to justify her own conceptual frame to Dion. It can also be argued that Winnie took mathematical reasoning to an even higher level as she tested her own conceptual frame against that of Dion’s and used it to extract the weakness in his argument.

CONCLUSION

Informed by social-constructivist learning theories and through a process of qualitative action research, I have shown how a learner developed new knowledge through mathematical reasoning. I have shown that collaborative learning is an important contributor for learners to construct knowledge during classroom discussions. Collaborative classroom discussion is a catalyst for meaningful dialogue and it allows for a forum wherein the collective authority of the mathematics is established. I make no claim that my research findings can be generalised, as the research was conducted with only one of my own grade 11 classes. My research does however open a door into the possibilities in teaching mathematical reasoning in a learner-centred way. It is these possibilities that I wish to share with fellow teachers and researchers as a means of initiating more discussion and research about such teaching.

REFERENCES


APPENDIX

What follows is a chronological account of Winnie’s learning in table format. I hope that it will assist the reader in understanding the development with regards to learning that Winnie had undergone during the class discussion of task 3.

<table>
<thead>
<tr>
<th>Time</th>
<th>Researcher Observation</th>
<th>Winnie’s Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>13:21:00</td>
<td>Winnie hesitantly and unsure make the following assertions based on their group discussion: the turning point of the graphs differ; the y-values stay the same; the x-values changes; the size of the graphs stay the same; the equations of the graphs differ.</td>
<td>Winnie’s learning is at an observational level. She does not see the need for any explanations or justification.</td>
</tr>
<tr>
<td>13:24:35</td>
<td>A little more confidently Winnie responds to simplifying Gary’s assertion on the change in the equation by saying: To make it simpler we can just substitute x and because the y-value stays the same the equation must change. This assertion made by Winnie is too vague and calls for more clarity by the class.</td>
<td>Winnie starts to make connections between her observations and the equation. In a sense she is justifying why the equation must change. She enters a higher level of reasoning brought on by seeing a need to revoice Gary’s assertion for clarity.</td>
</tr>
<tr>
<td>13:25:44</td>
<td>Winnie goes on to explain what she means by using her own examples. She indicates that for the equation $y = x^2 + 3$ the graph will move three units to the right and for $y = x^2 - 3$ the graph will move three units to the left.</td>
<td>Winnie’s reasoning is extended to expressing the changes she observes in an alternate representation. She’s now not only connecting various aspects of the mathematics but she produces mathematical representations to express these connections in. Although this expression is wrong mathematically it demonstrates her reasoning in relation to the task.</td>
</tr>
</tbody>
</table>

At this point Winnie mainly become a silent participant in the discussion, which follows Mary-Anne’s question.

<table>
<thead>
<tr>
<th>Time</th>
<th>Researcher Observation</th>
<th>Winnie’s Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>13:33:38</td>
<td>From Winnie’s Oh! I got the idea that she suddenly understood the problem, which the discussion is hovering above. This assertion comes immediately after I have pointed out the concept of coordinate points through revision questioning and a number of comments, answers and other assertions from a number of learners.</td>
<td></td>
</tr>
</tbody>
</table>

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138
<table>
<thead>
<tr>
<th>Time</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>13:34:40</td>
<td>Winnie explains that the +4 and the −3 in the equations are not the x-values of the turning point but as she puts it “some other” values. She brings to the fore the equation as representative of the relationship between the x and y variables. She also indicates that the −4 and +3 on the two graphs are representative of the x-values of the turning points of the two graphs and not necessary the +4 and −3 in the equations. During these assertions it is evident that Winnie is more confident and self-assured that she is on the right track.</td>
<td>Winnie has moved to a level of reasoning where she is in a position to evaluate previous assertions and adapt it to her new found understanding. She is now in a position to make the proper connections between the value of the turning point and the representational equation. This learning comes after a relatively long period of silence from Winnie where I can only assume that she was quietly reasoning and adjusting her own understanding as the class discussion allowed for other learners to make their assertions.</td>
</tr>
<tr>
<td>13:36:30</td>
<td>Winnie reaffirms her stance on her reasoning when confronted by Dion indicating that he disagrees with her assertions. She emphasizes the fact that we use the equation to get the y-value by substituting the x-value into the equation.</td>
<td>Winnie realizes that her assertions need to be argued to convince her fellow learners of the truth of her statements in order to be accepted by the collective authority. She needed to justify her assertions.</td>
</tr>
<tr>
<td>13:43:04</td>
<td>Winnie at this point does not wait to be invited to give her contribution but is confidently and openly engaging Dion’s assertions. She indicates that Dion is merely telling us where the turning point should be. In her opinion he is not telling us where what the y value is. His only telling us what the x-value of the turning point is.</td>
<td>Winnie is using her conceptual understanding to test and spot the failures in Dion’s argument. This places her in a position to challenge Dion’s assertions.</td>
</tr>
<tr>
<td>13:46:20</td>
<td>Winnie becomes so confident in her understanding that she challenges Dion’s assertion that the y-value is 0. She challenges him by asking him whether the y-value is always 0. From this challenge to Dion it is apparent that she knows the answer to this question but is merely asking to test Dion’s knowledge or to make her own point.</td>
<td>Winnie is using her own conceptual frame to extract Dion’s reasoning in an effort to influence his understanding.</td>
</tr>
<tr>
<td>13:47:18</td>
<td>Here we see Winnie’s confidence further grows by Dion’s admittance of his own incorrect assertion. She says something to the effect that she wanted Dion to admit that she was right and that he was “wrong”.</td>
<td>Although one senses that Winnie’s statement here reverts to a grudge match it is worthwhile to mention that she realizes that her assertions must be socially accepted and how better than to convince her most fervent opponent</td>
</tr>
<tr>
<td>13:57:20</td>
<td>On an enquiry by the teacher whether she understands Winnie responds by stating that she was fine because she explained it.</td>
<td></td>
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</tbody>
</table>