LEARNING THROUGH VARIATION, PART 1:
A PEDAGOGICAL TOOL*

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This paper forms Part One of a two part series in these proceedings on learning through variation. In Part One, we will explore some recent ideas that have been put forward about learning and the essential role that variation plays in it. The type of variation we are exposed to can either limit or widen the learning space, and hence the possibility of learning. Repetition may be helpful if it is done with purposeful variation. When we encounter variation our response depends on what is brought into focal awareness and what is brought together in our consciousness. Learning may also be defined as a change in what we are able to see behind the representation of a concept. There is a difference between the object of learning as intended by the teacher and as experienced by the learner. We will illustrate these concepts by relating them to everyday experience, as well as two examples from the teaching of engineering, and in Part Two (Pang, Linder and Fraser 2004) examine research dealing with learning through variation, in order to identify the challenges that they pose, and see how they can promote student learning in all types of science-related teaching situations.

INTRODUCTION

“That’s why I’m trying to get this understanding thing in the bag. Because, if I can understand it, I can do a problem. Rather than if I am sitting there doing hundreds of problems and getting tired and irritable and whatever.” This is a quote from one of the students our colleague Jenni Case interviewed for her PhD (Case 2000). Jenni comments further about this student:

He had been advised by senior students to do as many problems as possible, and he now regretted taking this advice. He now took one problem and tried to get as much understanding out of it as possible, by shifting from looking for a single method of solving a problem, to wanting to explore as many different ways as possible (Case and Gunstone 2002).

This resonates with a number of studies done around the question: Why do Japanese and Chinese students always do so relatively well in standardised Mathematics tests? One of these was the TIMMS study, in which a large number of mathematics lessons in the USA, Germany and Japan were video recorded and then analysed.

Runesson and Marton (2002), quoting Stigler and Hiebert (1999), describe the findings of this study as follows: In the American and German classrooms they studied the teacher gave a method for solving a particular type of problem, then gave the students lots of problems, all of the same type, just with different numbers. In contrast, the Japanese teachers in the study used the lesson period to help the students explore different ways of solving one problem, even using incomplete solutions as a means to gaining insight into how to solve problems of the type being examined.

In this paper we explore the issues that are raised by the studies quoted above, both of which introduce the subject of variation. In Part One of this paper we will cover a number of aspects of variation that have been reported in the literature (much of it distilled from a monograph edited by Marton and Morris [2002]), and relate these to our experiences of life and teaching, together with our students’ related

* For Part 2 of this paper, see Pang, Linder and Fraser (pp. 764-771).
learning, with some examples drawn from teaching senior-level chemical engineering in a university context. Part Two of this paper will continue the exploration by looking at research into the application of variation as a pedagogical tool in two subjects in quite different contexts: high school economics, and introductory university physics.

**VARIATION**

A number of workers in the field (especially Ference Marton and his co-workers) have postulated that variation is central to all learning (Marton and Booth 1997; Bowden and Marton 1998; Marton and Pang 1999; Runesson 1999; Trigwell 2000; Marton and Morris 2002). These authors go so far as to say that we only learn because of variation, and that variation is necessary for discernment to occur. When a particular aspect of a certain phenomenon is varied while other aspects of the phenomenon remain constant, the varying aspect is what will be discerned or come to the fore of our awareness.

We understand from this that when we encounter something different to what we have experienced so far, it challenges us to re-examine our pre-conceived ideas and to revise our understanding. If we do this, we will have learned something, and the new experience will be incorporated into our thinking. This presupposes a view that learning involves coming to an understanding of what is being learned, rather than simply memorising it.

It seems that for the American students in the study quoted above the variation was in the numerical formulation of the problems (they were exposed to the same type of problem, with different numbers), and they therefore had the opportunity to learn how to use a particular method. In contrast, for the Japanese students the variation was in how to approach the problem (they were exposed to the same problem, but different approaches), so they had the opportunity to learn how to find a solution.

For example, say we are learning how to solve a particular type of problem. We then encounter a problem of the same type that we cannot solve in the way or ways we already know. This challenges us to find a different way of solving this type of problem. But if in our learning we never encounter a problem that cannot be solved the way or ways we already know, we lose the opportunity to learn a different way of solving such problems. This has been described as narrowing the space of learning (Tsui 2002).

Note that this applies to all learning, including our research. We have found it very helpful to apply these ideas about variation to our research as well as to our teaching.

All of this leads us to ask the question: How were we taught Mathematics, or Science, or Engineering, or Economics? If your experience is anything like ours, we were largely taught by doing lots of problems of the same type, with just the numbers varying. It also challenges us to consider the sorts of problems we are giving to our students to solve, and how these will impact on their learning.

**LEARNING SPACE**

Let us now consider the effect that variation has on the learning space that our students experience, which is the possibility for learning that is formed by the learning experience that we have created for them (i.e. all the possible learning outcomes in that situation).

The type of variation that we use in our teaching will open up different learning spaces for our students (Tsui 2002). When the variation is limited, the opportunity for learning is diminished and learning space will be narrow, but when the variation encompasses a number of different aspects of the phenomenon being studied, then the opportunity for learning is expanded and the learning space will be wide. Runesson (1999) describes each different aspect of the phenomenon that is varied as a new dimension of the learning space. Tsui points out that if we relate the phenomenon to everyday life as well, we open up another dimension in the learning space and it may be said to have been thickened.

Runesson (1999) illustrates this with teaching students about squares. You can simply define a square as a figure with four equal sides and four equal angles. Or you can vary the length of the sides (and show that it is still a square), or the angles (in which case they are no longer equal and it is no longer a square), or the
number of sides (where you can still have the same angles and lengths, but it is no longer a square). Each aspect that is varied is a separate dimension of variation around the properties of the square and opens up the possibility of learning more about what makes a square a square.

By widening the learning space we increase the possibility of learning for our students. Not that all our students will inevitably take advantage of this opportunity, but at least they have the chance to do so. If we narrow the learning space, we are denying them the possibilities for learning that could otherwise have been provided.

How each student engages with any particular learning situation depends on a number of factors, many beyond our control as teachers. These factors include their physical and emotional state (have they had enough sleep, or is the room very hot, or have they just had some bad news?), their previous experience of the subject being learned or the teacher who is teaching it (whether good or bad), their view of what learning is (for example, understanding or memorisation), and distractions caused by other pressures they are facing (heavy workload, personal problems, social engagements, or concurrent sporting events).

Tsui (2002) also draws on observations of learning in both first and second language contexts to point out how the learning space is narrowed when learning takes place in a student’s second language. This is because of linguistic constraints, and is highly relevant to the situation in which we find ourselves in South Africa (and in Sweden, see discussion by Airey 2003), with many of our students learning in a second or even third language.

REPETITION

From what has been said above, it may be thought that repetition is bad. But repetition of itself is not necessarily bad, it is just the nature of the repetition, which depends on the purpose and focus of the repetition. If repetition is done without changing the focus of the problem, then it becomes simply rote learning. However, if the repetition focuses on different aspects of a phenomenon each time, then it is purposeful repetition, and can lead to understanding (Linder and Marshall 2003).

Kwan et al. (2002) give an example of a teacher who showed his students a video of a sloth (a slow-moving animal that lives in the jungles in South America) a number of times, but each time he had them focussing on a different aspect of the sloth – its colour, its appearance, its movements. Because of the different focus, in each repetition the students learned something new about the sloth.

Runesson and Marton (2002) also discuss learning how to throw a ball at a target, with reference to a study done by Moxley (1979). Moxley found that children who had only practiced hitting a target from one angle performed worse than children who had practiced hitting the target from different angles, when they were all required to hit the target from an angle neither group had practiced. The group that practiced from different angles had a dimension of variation of angle from the target opened up for them, so they were more able to cope with an unknown angle than those for whom this dimension was closed.

Relating this to repetition, we can say that by using the same ball, and trying to hit the target from the same spot each time, there is no variation in the learning space, just repetition of the same act. The skill of throwing the ball at the target will be highly developed (as far as is possible for the person concerned), but only from that particular position using that particular ball. If, however, the size of the ball, and the mass of the ball, and the distance from the target, and the angle of the target were all varied, the repetition takes place in a much wider multi-dimensional learning space. This is clearly a much richer learning experience, leading to a more complex ball-throwing skill.

Something else that strikes us from this example is that developing any skill takes practice, and this is just as true of mental skills as it is of physical skills. From this point of view, there is a need for repetition in the form of practice, to ensure mastery of the particular skill being learned.

If, when learning to solve a particular type of problem, all that varies is the numbers, the learning space is limited, and all one can learn is how to perform a certain procedure for solving those problems. If, on the
other hand, what varies is the approach to solving such problems, then a whole new dimension of variation is opened up, and with it the opportunity to learn different approaches to solving these problems.

RESPONSE TO VARIATION

The teacher who changed the focus of what his students saw in the video of the sloth was bringing different aspects of the sloth into focal awareness for his students. This brings us to consider what happens when we encounter variation.

According to Marton and Booth (1997), there are three possible responses to variation:

- Certain things are transcended. This means that they are completely overlooked, and not even noticed at all. For instance, before one of us bought a maroon Golf motor car, he had never noticed any of the maroon Golfs on the road (in other words, they were transcended), but once he owned one, suddenly he was seeing maroon Golfs everywhere he looked.

- Some things are taken for granted. Things that are taken for granted have already been noticed previously. Having become familiar, we no longer consider them worthy of further examination. A famous example of this is in one of the Sherlock Holmes stories, where all the witnesses swore that nobody entered or left the house, but the milkman had in fact done so in full view of them all. Sometimes things that are taken for granted have already been incorporated into our thinking, and form part of the implicit assumptions we make about a particular situation.

- Other things come into focal awareness. When something comes into focal awareness, or into the foreground, it is noticed in a way that it was not seen before. Those things that are not in focus recede to the background or margin. In the example we used above, maroon Golfs came into focal awareness for one of us once he owned one.

Another example from the experience of life of one of us may help to explain this more clearly. He used to visit game reserves as a child, but his family were only interested in the animals. So he never noticed the birds (they were transcended). But later in life, when he visited the same game reserves with his adult friends who were keen on birds, he started to notice the birds and began to distinguish one kind of bird from another (they came into focal awareness). As he learned more about them, he was able to recognise many of the birds for himself, and eventually he could tell what type of bird it was not only from its appearance, but also from a silhouette, or from how it flew. Moreover, he could also tell the differences between different birds of the same type, but only when he had been exposed to their distinguishing features.

When we teach our students a particular subject, such as engineering, we take for granted a certain level of skill in, for example, mathematics, in other words we assume that it is there and build on it in our teaching. But most of us never even consider the possibility that language may impact on our students ability to learn something like engineering – this is transcended, until it may be brought into our focal awareness by something like the study by Tsui (2002) that we quoted above.

The challenge for us as teachers is how to bring particular aspects of a phenomenon (or concept, or system) into focal awareness, thus opening up a fresh learning space for our students. What is the equivalent of owning a maroon Golf for our students in bringing things into focal awareness for them?

BRINGING TOGETHER

Having something in your focal awareness is an important step towards learning something new, but for the learning to occur, we need to be able to discern variation, and for this to take place, it is important that different aspects of a phenomenon or concept are brought together in our consciousness. The way this is done is captured by two related concepts, namely contemporaneousness and simultaneity (Marton and Pang 1999).
Contemporaneity refers to bringing together in your mind aspects or instances of a phenomenon that you have experienced in the past with what you are currently experiencing of it. Being aware of something from the past in the present enables you to distinguish variation in your experiences of the phenomenon. We do this all the time when we make comparisons or judgments. The performance of a piece of music is only good by reference to other performances of the same piece of music that we have experienced before.

We could only distinguish different types of birds either by being able to see them at the same time (which is not always possible) or by remembering what one type of bird looked like when we saw another type (thus making them contemporaneous).

Simultaneity is when different dimensions of a phenomenon are brought together (discerned and focused on) at the same time. This opens up a new way of discerning the phenomenon. For example, we can see light as just having a wave nature. Or, we can see it as just having the characteristics of a particle. When we discern these different dimensions of light simultaneously, we see light in a completely new way. Simultaneity involves seeing the links between different aspects of the same phenomenon, or similar aspects of different phenomena. For us it is also seeing the relationships between different variables in a system, and how they depend on one another, with a recognition of which may vary independently and which are dependent.

The order in which variation is introduced also affects what is simultaneously in focal awareness. Mok et al. (2002) illustrate this by comparing how two teachers approached teaching the concept of “some” to their students. One went from the concept of “one” to the concept of “how many” for more than one object, and finally to the concept of “some” as being any number more than one. The other went from “one” to “some” and then to “how many”. They found that students exposed to the “one – how many – some” sequence did not grasp the meaning of “some” as well as those who experienced the “one – some – how many” sequence. In the first case “one” and “some” were not simultaneously in focal awareness for most of the students, whereas they were in the second case. Tsui (2002) also points to the importance of how variation is sequenced.

**REPRESENTATION**

Another way in which we can conceive of learning and variation is in terms of the representation of a concept. One way of exposing students to variation in their learning is to use different representations. Tsui (2002) points out how important it is to use variation of representation to highlight critical aspects of a phenomenon. This could be in terms of words, or sounds, or symbols, or equations, or pictures, or diagrams, or domains. For example, control systems may be represented in the time domain, or the Laplace domain, or the frequency domain. These different representations each emphasise different aspects of the characteristics of control systems.

We can also use different forms of presentation for the representation we have chosen. We can present something: orally, or in writing, or graphically, or as still images, or as a video images, with or without sound. For example, a tank may be presented as a freehand sketch, or as a 2-dimensional line drawing, or a 3-dimensional line drawing, or as a picture. In control, the frequency domain can either be presented in a Bode diagram (which is a plot of amplitude ratio versus frequency plus a plot of phase angle versus frequency) or in a Nyquist diagram (which is a polar plot of amplitude ratio and phase angle, with frequency as a parameter that varies along the curve). These different presentations highlight different aspects of the frequency responses of control systems.

This can be taken further by examining what a student sees behind any particular representation of a concept or system and its presentation. This has been termed appresentation (Marton and Booth 1997). Learning may then be expressed as a change in what a student sees behind the representation of the concept or phenomenon. For example, when a student sees a flow diagram of a process, what does he or she see behind the presentation? This could just be what each unit on the flow diagram stands for, or
what the function of each unit is, or how to design the various units, or how the different units relate to one another, or what would happen if something changes on one of the units.

These three concepts are related to variation as follows: the way in which a representation is presented to the learner will alter the focus of attention and this in turn will affect the learning that may take place, while the way in which it is appresent to the learner may vary with the history of each learner’s understanding of similar phenomena or systems or concepts.

Take a piece of music, for example. This may be represented and presented by the sounds of the piece of music, in which case many people would readily recognise what piece of music it is if it is well known. But the same piece of music could also be represented and presented as a musical score. To the uninitiated, what is presented is merely a set of meaningless squiggles. Someone who knows a little about this form of representation would be able to recognise the symbols for longer and shorter notes, and which notes are higher or lower by the position of the notes relative to the lines. More knowledge of music would enable a person to recognise the rhythm of the music and the key in which it is written, as well as what tune it is. So the more you see behind the representation (the more you appresent), the more you know about it. For us, a good example of appresentation is Mozart who could compose a whole piece of music in his head (including all the instruments) before even committing any of it to paper. Note that in this example there seems to be a one-to-one correspondence between the representation of the music and its presentation.

OBJECT OF LEARNING

The final concept we would like to introduce that relates to the subject of variation is the object of learning, or the learning outcome. Lo and Ko (2002) distinguish three different objects of learning:

- Firstly, we have the nominal or intended object of learning. This is what the teacher in a particular lesson or course intends the students to learn. Thus it is the objective or outcome that the teacher has in mind that the students should achieve or master.

- Next, there is the realised or enacted object of learning. This refers to the potential space of learning that is jointly created by the teacher and the learners in terms of what is possible to learn. Teachers who have the same intended object of learning may deliver it in a variety of ways, and it has been noted that even seemingly subtle differences in delivery can result in vastly different learning experiences for the students. These differences seem to manifest in terms of the sort of variation that has been used in the learning situation (Mok et al. 2002). Lo and Ko (2002) give evidence that this can even occur when two teachers have planned a lesson together. The American and Japanese teachers had the same intended object of learning (to teach their students how to solve particular types of mathematical problems), but the way they presented their lessons resulted in very different enacted objects of learning. As discussed earlier, this is due to the different type of variation used by the teachers in their lessons.

- Lastly, there is the experienced or lived object of learning. This is how each student actually experiences the learning situation, which can be a strong function of their prior experiences (which can completely undermine the teacher’s intentions). Because of this, the lived object of learning will not necessarily be the same for all students in the same learning situation.

The challenge for us as teachers is to facilitate the learning of our students in such a way that the lived object of learning approaches the intended object of learning as closely as possible. One of the ways we can do this is to deliberately open up a number of dimensions of variation in relation to what we are wanting our students to learn, so as to increase the opportunity for learning for them. As noted above, this will not guarantee that the learning we desire to see occurring will take place, but the chance of this happening will be much greater.
CHALLENGES

Having opened up what we trust may be a new way of seeing learning for you (it certainly has been that for us), the challenge is how to make effective use of this in our teaching, and especially to ensure congruity between our intended object of learning and our students’ lived object of learning. How can we make effective use of variation to challenge what our students take for granted, and open them up to seeing things they haven’t seen before?

Students respond to any learning situation that we design for them in a variety of ways. Ingerman et al. (2004) have tentatively distinguished four different ways of engaging with a particular learning situation (in this instance, a computer simulation):

- **Focus on the situation.** In this the students focus is in the level of the situation. They do not focus on the simulation or the phenomenon, but on the situation, so their primary concern is the features of the situation, such as completing the tasks they have been given. This may be termed compliance.
- **Focus on the presentation.** Here the students focus on the level of the simulation, but simply see the simulation as a way of visualising the phenomenon being studied and nothing more. This may be termed visualisation.
- **Focus on manipulating the phenomenon.** In this case the students focus is on the level of the simulation. They see the simulation as a way of manipulating the physical system, but do not use it to gain understanding. This is manipulation.
- **Focus on exploring the phenomenon using the simulation as a conceptual tool.** Here the focus is on the level of the phenomenon. Students see the simulation as a representation of the phenomenon, and use it as a conceptual tool to explore the characteristics of the phenomenon they are studying. This leads to a richer experience of the system or phenomenon, and deeper insight into how it functions. This is exploration.

In this situation, the possibility exists for each of the students to move to the highest level of exploration, so there is a wide space of learning. But clearly, not all students take up this opportunity. How can we most effectively encourage students to move from compliance to exploration, at the same time challenging them not to take things for granted and helping them to see different features of the phenomenon simultaneously?

EXAMPLES

We would like to flesh this out a little by describing two examples from the teaching of engineering in the Chemical Engineering Department at the University of Cape Town (UCT).

The first example is a course in Process Dynamics and Control, which students take in their fourth and final academic year of study for the B.Sc.(Eng) degree.

A traditional course in process control would start by introducing students to Laplace Transforms, then develop models of a variety of typical processes and show how these respond to various forcing functions using Laplace Transforms. Then one would introduce controllers and their characteristics before developing the responses of whole control systems to particular forcing functions. Next one would examine the stability of control systems, using various means such as root locus or frequency response, and finally design controllers to meet certain performance criteria.

Now you can simply view this as a logical sequence in which to teach the subject, and teach each bit of it as a technique to be mastered. Alternately, you can see the Laplace domain and the frequency domain as different representations of the same system. These representations then enable you to look at the same system from different vantage points, which opens up a dimension of variation and the opportunity to see the same material from different perspectives.

Another possibility is to use another completely different representation to help students learn control, namely the use of real-time simulation. The intention is that the students would be able to gain understanding of control by seeing how individual units and whole control systems respond to different changes in input.
The hope is that when students see a diagram of a process, or an actual piece of equipment, they will not only be able to see what this represents, but that they will also be able to see behind this how that system functions, especially dynamically. Unless they can do this, they cannot begin to be able to do something like a hazard and operability study of a process flow diagram.

The second example is a distillation column simulation which was done by third year students as an experiment in their laboratory course. This was set up to enable them to explore what happens in a distillation column as different column parameters are varied. The compositions and temperatures on all the trays are available and can readily be plotted, so you can see what happens when you vary the reflux ratio, or if you put more trays in the column. For example, if there are too many trays, you can see the dead sections where nothing is changing.

It was most disappointing to find that the students seemed unable to engage with this experiment in the way that the lecturer did, nor did they see what he saw in the column. Most of them were stuck in the compliance mode noted above. We have recently redesigned this experiment with the help of a pair of fourth-year project students, with the following aims:

- What could we do to sensitise the students to what we would like them to see (bringing these issues into their focal awareness)?
- How can we help them to tap into their previous knowledge of distillation columns (contemporaneousness)?
- How can we help them to see the effect of varying different column parameters at the same time (simultaneity)?

The first thing that was done was to remove the initial phase of this experiment, which involved setting up the simulation in the HYSYS flowsheeting package that was used. It was felt that this distracted from the main aim of understanding distillation, so instead, the simulation was given to the students already set up for them. In order to help the students become familiar with how to use the simulation package to manipulate the experiment, the students were required to complete a set of preliminary tasks. These tasks were developed as a result of piloting the experiment with some groups of fourth-year students and finding that they took no notice of the helpful hints that were given.

It was also noted that the previous experiment involved varying too many of the column operating parameters. So the scope was narrowed down, with the main focus being on varying the number of trays and the location of the feed tray (these were done simultaneously), and a subsidiary focus on the effect of changing the number of trays and the reflux ratio on the column capital and operating costs. All this could be completed by the students in a period of an hour (the previous experiment took a full three hours to complete).

Students were also reminded about the main features of a distillation column in the introduction to the experiment. This was in order to help them remember what they had been learning about distillation in their course on it. In contrast, the previous experiment was done at different times of the year by different groups of students, so that students had not necessarily had any previous exposure to distillation when they did it.

The revised experiment was run with nine pairs of third year students, at a time when they were all towards the end of a course in which they learned about distillation columns. The students were observed during their engagement with the simulation, and immediately afterwards they were interviewed about their experience of it. These interviews were transcribed for analysis. It was found that the observations largely helped in terms of seeing how better to put the questions, and that the most helpful information for analysis came from the interview transcripts.

Most of the students stated explicitly that they needed previous knowledge about distillation to engage properly with the simulation. Some of them, in contrast with this, indicated that they thought this simulation might be a good introduction to distillation, in that it would give them an experience of distillation to refer back to when being taught the theory.
The analysis of how students engaged with the revised distillation experiment revealed that the areas of number of trays and feed tray location seemed to come much more strongly into their focal awareness than the other ones dealt with in the simulation. We postulate that this is because of the repetition they had to undertake in those specific areas (and the resulting longer time spent on these tasks) as well as the simultaneity involved, rather than because these were the first tasks undertaken. It would seem from these results that we were successful in bringing two particular areas, namely the effects of number of trays and feed tray location, into the students’ focal awareness. It would also appear that both the use of repetition and simultaneously examining these two parameters contributed to both of them coming more strongly into their focal awareness.

Another point to note is that by locating the experiment at a time when they had recently learned about distillation, their previous learning was accessible to them.

The analysis also showed that students did not seem to think they had learned anything new, despite the fact that when questioned more closely they admitted that concepts about distillation had been re-inforced and also that there were things they had not seen before such as dead zones. This highlights the limited views many students have about what constitutes learning – it certainly does not extend to something like playing around with a simulation.

The revised distillation column simulation would be suitable for use as a tutorial exercise in the course on distillation columns. This is possible because we can now run multiple copies of HYSYS simultaneously (as opposed to the single license we had before) and this exercise can be strategically placed in the course to optimise the students’ learning.

CONCLUSIONS
In Part One of this paper we have laid out the basic theory of learning through variation, and briefly shown how it may be applied to teaching of a discipline such as engineering. In Part Two of this paper we will provide two in-depth studies of applications of such an approach to high school economics and introductory university physics.

We trust that you will be able to take these ideas and use them to improve both your teaching and the learning of your students.

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REFERENCES


