# TABLE OF CONTENTS

**Guest Speakers Long Papers**  
When Praxeologies Move from an Institution to Another one: The Transpositive Effects  
Corine Castela  
Empowering Socioeconomic Development Efforts in Africa through the Indigenization of Science, Mathematics and Technology Education  
Meshach Oguniyi

**Mathematics Long Papers**  
In-Service Mathematics Teachers’ Interpretation of Students’ Errors  
Million Chauraya & Samuel Masingaidze  
Arithmetical Discourse Profile in College Mathematics Classroom: An Issue worthy exploring?  
Nancy Chitera & Jill Adler  
A Modeling and Models Approach: Improving Primary Mathematics Learner Performance on Multiplication.  
Emmanuel Dlamini, Hamsa Venkat & Mike Askew  
Identifying Stages of Numeracy Proficiency to Enable Remediation of Foundational Knowledge Using the Learning Framework in Number  
Mellony Graven, Debbie Stott, Zanele Mofu & Siviwe Ndongeni  
An Exploration of Learners’ Learning of Mathematics by Using Selected (VITALmaths) Video Clips: A Case Study  
Thomas Haywood & Marc Schäfer  
Exploring Frameworks for Identifying Learning Dispositions: The Story of Saki  
Diliza Hewana & Mellony Graven  
Mathematical Literacy: Are we Making any Headway?  
Mark Jacobs & Duncan Mhakure  
Mathematical Knowledge for Teaching in Africa – A Review of Empirical Research  
Arne Jakobsen & Reidar Mosvold  
Mathematics Education in South Africa: The Problems and the Perceived Causes  
Marie Joubert  
The Relevance of Mathematics Teacher Identity in the Context of a Mathematics Teacher Development Programme (MTEP)  
Nyameka Kangela & Marc Schäfer
ML Teachers’ Non-recognition of Realistic Constraints in Solving a Problem Set within a Real Life Context
Cathrine Kazunga & Sarah Bansilal 146

Grade 11 Mathematics Learners Approaches to Working with Vertical and Horizontal Shifts of Parabolas
Happy Kunene & Sarah Bansilal 159

Jim’s Dream: In Search of a Professional Identity of a Beginner Mathematics Teacher
Ajayagosh Narayanan & Marc Schafer 171

Seeking Synergy: The Need for Research at the Literacy/Numeracy Interface
Sally-Ann Robertson 185

Applying a Linguistic Complexity Checklist and Formulae to the 2013 Grade 4 Mathematics National Assessments
Lucy Sibanda & Mellony Graven 197

Confronting, Navigating and Resolving Research Tensions
Debbie Stott 211

Teachers’ Pedagogical Content Knowledge in the Teaching of Grade 3 Mathematics Concerning Equivalent Fractions in Some Rural Schools of Limpopo Province
Kgalushi Themane & Kakoma Luneta 222

Representation of the Equal Relationship in the Development of Mathematical Thinking: A Case of Grade One’s
Zingiswa Mybert & Monica Jojo 230

Science Long Papers

Mentoring Physical Science Subject Advisors on Acid-base Titration
Washington T. Dudu 239

Mentoring Physical Science Subject Advisors on Acid-base Titration
Washington T. Dudu 240
Norman Lederman & Judith Lederman

The Nature of Interactions of the Components of Topic Specific Pedagogical Content Knowledge
Elizabeth Mavhunga

‘We use guided inquiry and open discovery in our lessons’: Investigating the Extent to Which this is True in the Practice of In-service Science Teachers in Malawi
Dorothy Cynthia Nampota

Effects of High School Students’ Chemical Concept Understanding Level on Achievement in Kreb’s Cycle
Ikhifa Onyenenue & Chukunoye Ochonogor

Technology and Indigenous Knowledge Long Papers

The Effect of Computer Simulations on the Speed of Writing Tests
Sam Kaheru & Jeanne Kriek

An Experiment with Peer Instruction in Computer Science to Enhance Class Attendance
Maria Keet

Emancipating Secondary School Teachers from their Technology Knowledge and Pedagogical Challenges: An Action Research Study
Tomé Mapotse & Mishack Gumboa

Teachers and Learners Level of Computer Literacy to Use Educational Technologies at Some Secondary Schools in Attridgeville Township
Olika Moila & Moses Makgato

Pre-service Technology Teachers’ Misconceptions about the Concept of a Lever
S. Ramaligela, A. Mji & Ugorji Ogbonnaya

Examining the Impact of Dialogical Argumentation on Grade 9 Learners’ Beliefs about Weather and Indigenous Knowledge
Alvin Riffel

Adapted Mathematical Knowledge for Teaching Measures: Reliable, But Still Challenging
Reidar Mosvold, Janne Fauskanger & Arne Jakobsen
SAARMSTE EXECUTIVE 2014/2015

President  Prof Mercy Kazima (University of Malawi)
Past President  Prof Mellony Graven (Rhodes University)
Secretary/Treasurer  Prof Margot Berger (University of the Witwatersrand)
Chapter Representatives  Dr Carlos Lauchande (Pedagogical University)
                      Prof Lyn Webb (Nelson Mandela Metropolitan University)
AJRMSTE Editor  Prof Fred Lubben (Cape Peninsula University of Technology & University of York)
Research School Chairperson  Prof HamsaVenkatakrishnan (University of the Witwatersrand)
Secretariat (ex-officio)  Ms Caryn McNamara (University of the Witwatersrand)

SAARMSTE 2015 LOCAL ORGANISING COMMITTEE

Conference Chair  Prof Emília Nhalevilo (Pedagogical University)
Treasurer/Secretary  Dr Carlos Lauchande (Pedagogical University)
Proceedings Editor Chair  Prof Danielle Huillet (Eduardo Mondlane University)
Deputy Proceedings Chair  Prof Amália Uamusse (Eduardo Mondlane University)
Programme Chair  Dr Marcos Cherinda (Pedagogical University)
Deputy Programme Chair  Prof Eugénia Cossa (Eduardo Mondlane University)
Mathematics Convener  Dr Ribas Guambe (Eduardo Mondlane University)
Science Convener  Prof Armindo Monjane (Pedagogical University)
Technology Convener  Dr Daniel da Costa (Pedagogical University)
IKS Convener  Dr Alberto Cupane (Pedagogical University)
Marketing  Ms Aissa Mithá (Pedagogical University)
Fundraising  Dr Ismael Nhêze (National Institute for Education Development)
                      Dr Ribas Guambe (Eduardo Mondlane University)

Editor: Danielle Huillet
Cover: Marcos Cherinda
Forward LOC Chairperson - Prof Emília Nhalevilo

In Mozambique the past few years were agitated by discovery of natural resources in provinces which are still the less developed in the country. This poses the challenges of how well and fair explore and distribute the richness while, at the same time being mindful of the needs for sustainable development. This landscape-resources versus social justice and sustainability—though I rose in the context of Mozambique, constitutes a challenge also in Southern African countries and even abroad. Our much-loved leader, Nelson Rolihalha Mandela, used to say that “Knowledge is Power”. Science, Mathematics and Technology education should be a top contributor for empowerment and equity in our societies, for social justice.

We brought to this conference the theme, *Science Mathematics and Technology education for empowerment and equity* to strengthen our vision that education should indeed empower people; it should provide them with critical and emancipatory skills to pave the way for social justice. This is relevant because we still assist how our people and particularly children are deprived from the basic human rights, from food, relevant school curriculum to language of instruction just to cite some. Knowledge *must* be power.

From the topics of our distinguished guest speakers to a variety of the cherished presentations in these proceedings, we read about application of knowledge to solve our day to day problems, teaching methodologies, didactic materials, language of instruction, Indigenous Knowledge and sustainability. We expect that the materials presented will be a very worth contribution for the empowerment and equity of our people.

This year, Mozambique is hosting for the third time the SAARMSTE conference. Being Island of Portuguese speakers in a community majority English, we are honored by our teachers and researchers whom along the years of SAARMSTE make their effort to hear and to be heard on issues of Science, Mathematics and Technology Education. I wish this conference in Maputo will encourage more compatriots to join this community of research in education.

Finally, I wish to thank all presenters, reviewers, editors, group coordinators and funders, for their hard work in contributing to the success of the SAARMSTE 2015.

Hosi Katekisa Africa! Kanimabo!

Emilia Afonso Nhalevilo
Chairperson of the LOC
Message from SAARMSTE President - Prof Mercy Kazima

It is my great pleasure to welcome you all to the 23rd annual SAARMSTE conference hosted by Pedagogical University and Eduardo Mondlane University in Maputo, Mozambique. The annual conference is a major event of SAARMSTE which brings together delegates from Southern Africa and all over the world, hence provides an opportunity for sharing and learning from one another as well as networking.

The theme of this conference is “Mathematics, Science and Technology Education for Empowerment and Equity”. This is appropriate for us to deliberate on because we are living in a world of many inequities including distribution of wealth. Furthermore, many developing countries are faced with challenges of producing qualified and relevantly skilled graduates for employment in industry. Mathematics, Science and Technology Education can help to provide such skilled graduates and empower learners hence address the inequity issues. As researchers in Mathematics, Science and Technology Education, we all have a role to play; therefore this is appropriate theme for this conference.

Organising a conference is a huge task and demands a lot of time from the organisers. I therefore thank the local organising committee for their time and dedication, and I thank the Pedagogical University and Eduardo Mondlane University for hosting the conference. I also thank authors, presenters and all delegates to the conference, not forgetting the reviewers and editors who helped shape the papers in this publication.

Lastly but not least, I thank all those that have funded the conference. Their generosity and support has made the conference possible as well as publication of these proceedings.

I wish you all a great conference.

Mercy Kazima

SAARMSTE President

January 2015
Message from the Proceedings Chair

We have great pleasure in presenting these proceedings to the participants of the 23rd Annual SAARMSTE Conference, the third one held in Maputo, jointly organized by the Pedagogical University and the Eduardo Mondlane University.

The Local Organizing Committee received more than 200 papers and abstracts, showing the great interest of researchers and practitioners of Mathematics, Science and Technology Education to participate in the SAARMSTE Conference this year. In order to allow Mozambican researchers and teachers to attend this conference, we also accepted papers in Portuguese. All papers and abstracts were divided in four areas: Mathematics, Sciences, Technology and Indigenous Knowledge. Each of these areas had a convener, who received the blind copies from the proceedings chair and sent them to the reviewers.

All long papers were reviewed by a team of reviewers collaborating with the SAARMSTE Journal (African Journal of Research in Mathematics, Science and Technology Education), in order to ensure a rigorous process of peer review. Short papers, round tables, symposia and snapshots abstracts were reviewed by reviewers collaborating with the SAARMSTE Journal, SAARMSTE members and authors of long papers. We are very grateful to all of them without whose dedication we could not have put together the Long Papers Proceedings and the Book of Abstracts.

Some papers and abstracts were accepted. Others were accepted with conditions and authors were advised to make changes according to reviewers’ suggestions in order to have their papers or abstracts accepted. When authors disagree with the decision, another reviewer was appointed to give an opinion that, together with the first reviews, was taken into account to take a final decision.

Papers in the Long Paper Proceedings and in the Book of Abstracts have been organized into categories and are presented in alphabetic order with respect to the author or first author surname within each category.

Mistakes and errors, as well as omissions of papers that should have been part of the proceedings because of poor communication with the LOC are deeply regretted.

We have enjoyed working with everyone of the contributors to the proceedings: Presenters and reviewers.

We hope that readers will find in these proceedings useful contributions to our knowledge and understanding of Mathematics, Science and Technology Education research.

Proceedings Chair

Danielle Huillet
Guest speakers

Long Papers
When Praxeologies Move from an Institution to Another One: The Transpositive Effects

Corine Castela 1
1 Laboratoire de Didactique André Revuz, Normandie Université, Université de Rouen, France
1 corine.castela@univ-rouen.fr

This paper focuses on three key concepts of the anthropological theory of the didactic, after giving an idea of the basic principles of the theory, namely: institution, praxeology and transposition. An understanding of praxeology, being a general model for socially-acknowledged cognitive resources with examples coming from different contexts, is presented. I looked at examples from mathematics and automatic control in the first part and dressmaking in the second part. In both parts, the movements of praxeologies between different institutions and their transpositive effects are discussed. General models of such processes are also provided. I conclude by questioning an anthropology issue of reference to academic mathematics without that of workshop mathematics.

Introduction: The Anthropological Theory of the Didactic

The anthropological theory of the didactic (hereafter ATD) is at the same time a theory in the scientific usual meaning of the term, that is, an organised body of knowledge, socially legitimated, and the most prominent dimension of a research program in mathematics education developed by a community of researchers, mainly from European and American French-and-Spanish-speaking countries. This program has been initiated by Yves Chevallard in the 1980s with the study of didactic transposition processes (Chevallard 1985, 1989), the anthropological perspective being introduced in 1992 (Chevallard 1992). Although this principle has never been explicitly acknowledged, I dare claim that a socio-cultural conception of humans underpins the ATD. The second point is that the research program focuses on the social aspects of the didactic reality. It is not interested in the individuals who learn or teach some piece of knowledge. The ATD addresses the issues of the social resources and constraints that establish a framework for an individual’s didactic activity, as learner, teacher, internship advisor etc. If we consider one knowledge field, this choice is a restriction within education researches related to this very field. Yet, the anthropological approach of the didactic opens up a very large domain to investigate, far beyond the education area in the restricted meaning as usually considered, in accordance with the following reasons. Firstly, transmission of socially produced knowledge characterises human nature, the didactic is everywhere dense in any human society. Secondly the knowledge production is always socially situated; hence, the didactic research must address socio-epistemological issues related to the social footprint on the knowledge at stake in the educative processes. And lastly, as a generalisation of the foregoing, the ATD assumes that what is going on in the classroom between teacher and students is mainly determined by conditions and constraints deriving from various social organisations, from local ones up to society and civilisation. From these principles it derives that some of the ATD key concepts are neither specific to mathematics, nor to education. They especially provide powerful tools to an epistemo-anthropology, in other words, to any epistemological research interested in the issues of production and movement of knowledge within and between various social contexts. I
discuss some of them in the following sections: first, the institution and the subject, next the praxeology, and lastly the transposition processes.

**The Institutions and the subjects**

I define an institution \( I \) as a stable social organisation that offers a framework in which some different groups of people carry out different groups of activities. These activities are subjected to a set of constraints, - rules, norms, rituals - which specifies the institutional expectations towards the individuals intending to act within the institution \( I \). An individual has to satisfy these expectations, at least, to a certain extent depending on the institution. Hence, using the ATD vocabulary, the individual (s/he) is subjected to the institution’s expectations and becomes an institutional subject (from Latin *sub-jectus*: literally thrown under). This meaning is very different from the Kantian one, which considers the subject as a responsible agent, usual in several theoretical frameworks that give priority to psychology and individuals, even when, like the cultural historical activity theory (CHAT, Leontiev 1978), they take socio-cultural dimensions into account.

Institutions tend to constrain their subjects but conversely they provide the resources (material and cultural) necessary for activities to take place. Epistemologically, the existence of institutions is an absolute precondition for the development of human culture. They foster collective processes for facing and solving human problems; and they favour the dissemination of inventions/innovations, even when they do not create specific schools for that.

**Examples of institutions**

Institutions, referring to the production of mathematical knowledge (without claiming to be exhaustive), could be the international mathematics research organisation and its sub-institutions according to the domain or the country; each laboratory, each mathematics journal, each congress.... Institutions referring to mathematics, science and technology education: ICMI and ICME, SAARMSTE, PISA (Program for International Students Assessment), each Department of Education, each National Curriculum, each educational institution, each classroom of a mathematics, science or technology teacher some weeks after school started. With these examples, we can see that various-sized institutions may be concerned with a topic, the more local ones being embedded in and partly determined by some of the more global ones.

In order to diversify the context of these examples, I will draw on Wolf-Michael Roth’s research (Roth 2014) about a training program in a Canadian college where students train to become licensed electricians. Let us note that Roth’s framework refers to CHAT and to Radford’s notions of objectification and subjectification, both approaches focusing on the individual development within socio-cultural contexts. This theoretical choice is common among vocational mathematics education researches (see Educational Studies in Mathematics 86). From the ATD point of view, what are the institutions considered in Roth’s analysis of this vocational course? Within the college, mathematics units, science units and shop units; outside, workshops where the students work as apprentice and at a higher level, the Canadian control system of electrical installations, based on the Electrical Code of Canada framing the professional practices. During the program, a student has to become a subject of each of these institutions, that is, has to submit to their specific expectations regarding what students must do and how. Within CHAT framework, Roth addresses the issue of how one individual tackles the experience of crossing boundaries between different socio-cultural contexts. An ATD research would have focused on these contexts considered as some institutions and on
the system of constraints and affordances that each one creates for the activities of each category of its subjects (e.g. students and teacher or supervisor). Such a deep exploration of institutions is considered as necessary to address the issue of inter-institutional transitions, in ATD words, boundary crossings in CHAT ones. Without developing further, I suggest that ATD and CHAT are complementary theories, based on a rather similar conception of human, the first one focusing on sociological objects, the second one on psychological ones.

**Praxeology**

Coherently with the foregoing, ATD is interested in the processes and products of what we may consider as the institutional cognition, that is to say, in how institutions develop their socially acknowledged capitals of practices and knowledge. In other words, the aforementioned epistemo-anthropology is a subfield of the ATD research program. The key notion of praxeology is the basic unit proposed by this theory to analyse the institutional cognition.

What exactly is a praxeology? [...] one can analyse any human doing into two main, interrelated components: *praxis*, i.e. the practical part, on the one hand, and *logos*, on the other hand. [logos refers to human thinking, rational discourse]. How are $P$ [*Praxis*] and $L$ [*Logos*] interrelated within the praxeology [$P/L$], and how do they affect one another? The answer draws on one of the fundamental principle of ATD [...] according to which no human action can exist without being, at least partially, “explained”, made “intelligible”, “justified”, “accounted for”, in whatever style of “reasoning” such as an explanation or justification may be cast. *Praxis* thus entails *logos* which in turn backs up *praxis*. For *praxis* needs support – just because, in the long run, no human doing goes unquestioned.[...] Following the French anthropologist Marcel Mauss (1872-1950), I will say that a praxeology is a “social idiosyncrasy”, that is, an organised way of doing and thinking contrived in a given society. (Chevallard 2006, p.23)

The practical block (or know-how) associates a type of tasks $T$ and a technique $\tau$. $\tau$ is a “way of doing” which is endowed with certain efficiency for a certain subfield within the set of $T$ tasks.

The *logos* block contains two levels of description and justification of the *praxis*. The first level is called a “technology”, using the etymological sense of “discourse” (*logos*) of the technique (*technè*). The second level is simply called the “theory” and its main function is to provide a basis and support of the technological discourse. (Bosch & Gascón 2014, p. 68)

This four components model is usually represented as follows: [$T$, $\tau$, $\theta$, $\Theta$], $\theta$ being the technology of $\tau$, $\Theta$ the theory supporting $\theta$.

To exemplify the praxeological model, I will consider a mathematical type of tasks we can meet in strictly mathematical contexts as well as in engineering sciences. This type of tasks is the following one: *Breaking up a rational fraction into partial fractions*. Let us note that, except when I give an example of effective calculation, everything below belongs to the logos block, mostly to the technology.

**Mathematics praxeologies to break up a rational fraction into partial fractions**

*Description of the technique in the general case*: (1) Make the denominator monic (leading coefficient 1), and use the Euclidean algorithm to reduce to a problem where the degree of the numerator $r$ is less than the degree of the denominator $d$. (2) Factorize the denominator as a product of powers of distinct monic irreducible polynomials. (3) Write the fraction as a sum
of partial fractions of the form $R/Q^k$ where $Q$ is one of the irreducible factors, $k$ is at most equal to the multiplicity of $Q$ in $d$ and the degree of $R$ is less than the degree of $Q$. (4) At first every $R$ is unknown. The coefficients need to be determined. One way of doing this is to take a common denominator, multiply out, equate coefficients and solve the resultant system of linear equations.

The fact that every rational function may be written as such a sum of partial fractions, uniquely determined, is a theorem. Even for rational functions on the fields of real or complex numbers, the proof needs many results as the fundamental theorem of algebra, division in the ring of polynomials, Bezout theorem, that is, the polynomials arithmetic theory. Moreover, mathematical induction is necessary. This is a great part of the praxeology theory. But this non-constructive theorem does not provide a technique to determine the polynomials $R$. However, step 4 will do since we know that the system has a unique solution.

Example: We want to express $\frac{3x+1}{(x-1)^2(x+2)}$ as the sum of its partial fractions $\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$. 

$$\frac{3x+1}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

$\Leftrightarrow 3x + 1 = A(x - 1)(x + 2) + B(x + 2) + C(x - 1)^2$ $\Leftrightarrow (*)$

$(* \Leftrightarrow 3x + 1 = (A + C)x^2 + (A + B - 2C)x - 2A + 2B + C \Leftrightarrow \begin{cases} A + C = 0 & A = 5/9 \\ A + B - 2C = 3 & B = 4/3 \\ -2A + 2B + C = 1 & C = -5/9 \end{cases}$

Two theorems validate this technique, that is, prove without further checking that $\frac{3x+1}{(x-1)^2(x+2)} = \frac{5}{9(x-1)} - \frac{4}{3(x-1)^2} - \frac{5}{9(x+2)}$. $\theta_1$: two rational functions with the same denominator are equal if and only if their numerators are equal. $\theta_2$: two polynomials are equal if and only if they have same degree and same coefficients.

Appraisal of the technique: This technique $\tau_1$ is obviously tedious in some cases with many coefficients to find, if you do not have any software to solve the system. Indeed, mathematicians are aware of this heaviness of the “equating coefficient-wise technique” and look for other techniques. For example, in the following excerpt of a calculus on-line textbook$^1$, we find an appraisal of $\tau_1$ when the denominator is a product of $n$ linear terms together followed by the presentation of another technique $\tau_2$:

For $n \leq 2$, this procedure $[\tau_1]$ is possible and quick. However, for $n \geq 3$ the procedure becomes messy because we first need to do a lot of tedious term multiplication to find coefficients, and then we need to solve a tedious system of linear equations. [...] The approach we will now use is based on the key idea that if two polynomials are equal, their values at every number are equal. (ibid, p.7)

Using $\tau_2$ on the example gives the following: $3x + 1 = A(x - 1)(x + 2) + B(x + 2) + C(x - 1)^2$; so plugging in the values $x = 1$ and $x = -2$, we get every term but one equal to 0 (this point is important to understand as a motivation of the values choice), so that we have $4=3B$ and $-5=9C$. To get $A$, there is no special value that would eliminate $B$ and $C$, we choose $x=0$ and get $1=-2A+2B+C$. Hence $A = 5/9$.

According to the on-line textbook, this technique is clearly “preferable (from a speed perspective) when $n \geq 3$” in the case of linear factors. In a French on-line text-book$^2$, we find

---

2 Analyse 2, M. Hasler, Université d’Antille-Guyane
another case of *appraisal of a technique* \( \tau_3 \), using a combination of variable substitution and polynomial long division: “

This method is especially interesting when there is a pole of high order \((\geq 4)\) and few other factors \(B(x)\) [the denominator] or when the pole is around 0 from the beginning. (p.29) (the translation is mine)

*A missing motivation of one step of the technique*: Now, as a transition to the corresponding praxeology in automatics, we can ask a question: why is it important to make the denominator monic? In the aforementioned Chicago text-book we find no explicit motivation of this choice. The author only argues that: “Every non-zero polynomial can be expressed as a constant multiple of a monic polynomial. Since constants can be pulled out of integration and differentiation problems” (p. 2), there is no loss of generality by restricting to monic polynomials. Indeed, the fact that linear monic polynomials have 1 as a derivative and hence that the antiderivative of rational functions \(1/(x-a)^k\) is easy to calculate is one motive of the restriction to monic factors. It remains implicit in both text-books, although well known among mathematicians.

**The automatics praxeology to break up a rational fraction into partial fractions**

The following example is from the PhD study carried out by A. Romo Vázquez (2009) and supervised by M. Artigue and C. Castela. This research addressed the issue of the mathematical preparation of engineers. The central part of the research was twofold, consisting of an analysis of engineering projects on the one hand (practical activity carried out as part of the training of engineers at one French University Vocational Institute) and of Engineering Sciences and Mathematics courses on the other hand. Due to the central role played by the Laplace transform in one of the projects, the courses study focused on this notion and compared the way it was taught in textbooks of different institutions (2nd and 3rd university years), one written by a mathematics lecturer, the other two by automatics lecturers, one being an on-line course for higher technicians⁴. The example we consider now is from the latter (see Castela & Romo Vázquez 2011 for a detailed study).

Some explanations about the automatics issues are necessary. The problem at stake is automatic regulation of systems (e.g., heating installation with controlled temperature): if a quantity must be kept constant, an electronic gauge measures its value; when some variation is recorded, an appropriate regulation process is triggered to come back to the desired value. The less time is necessary to get the quantity back to this value, the more efficient is the control system. The temporal evolutions of the different systems involved are described by differential equations (linear ones in the considered textbook), turned to algebraic ones by the Laplace transform and easily solved, with a rational fraction \(F(p)\) as a solution. At last you have to get back to the temporal function, that is, to inverse the Laplace transform. The on-line textbook recommends using a table of Laplace transforms, especially adapted to automatics requirements. The type of tasks *Breaking up a rational fraction into partial

---

⁴ The Laplace transform is a widely used integral transform in mathematics with many applications in physics and engineering. It is a linear operator of a function \(f(t)\) with a real argument \(t (t \geq 0)\) that transforms \(f(t)\) to a function \(F(p)\) with complex argument \(p\). The most important result is that it transforms \(f'(t)\) to \(pF(p)\).

⁴ [http://public.iutenligne.net/automatique-et-automatismes-industriels/verbeken](http://public.iutenligne.net/automatique-et-automatismes-industriels/verbeken)
fractions appears when complicated rational fraction $F(p)$ are involved. In what follows, I give an idea of the praxeology (technique and technology) proposed by the textbook.

Description of the technique: In fact, the author assumes that the mathematical techniques are familiar to the students. The only point that he specifies is that $F(p)$ denominator must be written under the following canonical form $k(1+\tau_1 p)(1+\tau_2 p)(1+\tau_3 p)\ldots$ with decreasing values of the $\tau_i$. For instance, $3p+2$ is transformed into $2(1+1.5p)$ and not into $3(p+2/3)$. This is a significant change to the original mathematical technique.

Motivation (raison d’être) of this special factorisation: If $F(p) = \frac{1}{1+1.5p}$, the corresponding original function is $f(t) = K(1-e^{-t/1.5})$. $1.5$ is called the time constant of this function. The system reactivity, hence its quality, is directly dependent on the time constants $\tau_i$, more precisely on the higher value; therefore, this value must appear clearly in the calculation.

Explanation of the relation between time constant and reactivity: if $f(t)$ represents the controlled quantity and $K$ its desired constant value, it is known that after $7\tau$, here $7\times1.5$ seconds, the exponential will be equal to 0, that is, considered as negligible in Automatics. Hence the transitional regime lasts $7\times1.5$ seconds.

Validation of this claim: $e^{-t/\tau}<0.01$, hence (the textbook does not use $\Leftrightarrow$) $e^{t/\tau}>100$, $t>\tau ln(100) \approx 7\tau$

What needs does the technology of a technique intend to satisfy?

As mentioned previously, (Romo Vázquez 2009) analyses the Laplace transform chapter in one mathematics and two automatics textbooks from tertiary vocational courses for engineers and higher technicians. The first one, in a classical mathematics style, is focused on the comprehensive accurate presentation of concepts, theorems and proofs. The Laplace transform technique to solve non-linear differential equations is alluded to, without any examples related to engineering sciences. As shown in the above example, the automatics textbooks are very different. They give a lower priority to mathematical proofs and instead, they develop another kind of knowledge about techniques, strongly correlated with the vocational context. Actually there are many things to know about Laplace transform and the derived techniques but all these technological elements satisfy diverse needs. Drawing on the aforementioned textbooks, Romo Vázquez and I have differentiated six of them: describing the technique, validating it i.e. proving that this technique produces what is expected from it, explaining the reasons why this technique is efficient (knowing about causes), motivating the different gestures of the technique (knowing about objectives), making it easier to use the technique and appraising it (with regard to the field of efficiency, to the using comfort, relatively to other available techniques). Such technological elements are present in both previous examples of mathematics and automatics praxeologies. This list should not be taken as exhaustive. For instance, drawing on some other researches (e.g. Covián Chávez 2013, addressing land surveyor training), I currently consider one more need: controlling the technique implementation. Even if a mathematical proof justifies that the technique, when it works and is adequately implemented, produces the expected solution, an individual may make errors when using it. The institution where this individual works needs that he or some supervisor can verify the process.

This analysis grid of the praxeology technological component has been developed within vocational contexts. It is in full convergence with results obtained by most of researches addressing the issue of occupational mathematics (as recent examples, see Educational Studies in Mathematics 86). But it is not only relevant for this kind of context. I use it to
describe knowledge involved in mathematical problem solving, for mathematicians as well as students. From the ATD point of view, problems, even research ones, contain elements of genericity, that is, may be related in some way to others already encountered and solved problems. This means that mathematics researchers, even if they need much inventiveness, draw also on previously developed praxeologies. In research journals as well as in scholarly books, the praxis part of praxeologies is often peripheral and the technological one focuses on the validating need, satisfied by mathematical proofs; sometimes causes are explained, sometimes they are not (think of analytical proofs for geometrical theorems). According to the contemporary mathematics epistemology, these technologies directly derive from theories. Yet I claim that other knowledge about mathematical techniques are necessary to use them efficiently and that this knowledge is produced and disseminated in the local mathematics research institutions, such as research teams, laboratories, seminars. This knowledge appears in some tertiary textbooks (see the aforementioned example of mathematics praxeology), not all. This part of the technologies satisfies practical needs within problem solving and generally derives from experiencing the technique implementation, that is to say, not from a mathematical theory. Hence this is an empirical kind of knowledge.

Most of this practical part of mathematics praxeologies is not taught. Yet students need such knowledge, in France at least from high school (for detailed argumentation, see Castela 2004, 2009). So the responsibility for building this practical knowledge on their exercise and problem-solving experiences rests on the students. But some sociological and didactic French researches have given solid evidence that this requirement has a strong differentiating effect, students from disadvantaged backgrounds being largely unaware of the learning objective of exercises and problems. Hence, it is up to the mathematics teachers to introduce, in the classroom, the idea of practical mathematics knowledge and to develop their students’ skill to build by themselves such knowledge. This means that, at least within teacher training institutions, the pre-service secondary school teachers should work on examples of mathematics praxeologies with practical as well as theoretical technology as fully institutionally acknowledged mathematics. That is why, working myself in a teacher training institution, I give specific importance to the analysis grid of the praxeological technology developed with Romo Vázquez.

**When a praxeology moves from a research institution to another one: The transpositive effects.**

We are now going to work on the original ATD symbolic representation \([T, \tau, 0, \Theta]\) for a praxeology, in order to incorporate the institutional cognitive process in the model and to make it visible in the diagram. A praxeology is an institutional idiosyncrasy, that is, an organised way of doing and thinking acknowledged by this institution as legitimate. The anthropology of the institutional cognition I have previously called epistemo-anthropology is not only interested in praxeologies developed by a given institution \(I\) but also by the production and legitimating processes in this institution. Institution and processes are therefore added in the following diagram: \([T, \tau, 0, \Theta] \rightarrow I\).

(Romo Vázquez 2009) considers four institutions: two scientific research institutions, in mathematics \([I_r(M)]\) and in automatic control \([I_r(AC)]\) and the related teaching institutions in different vocational courses \([E(M), E(AC)]\). The social responsibility of the first two is to produce new praxeologies in their own field, with a particular emphasis on systematic validation according to each field’s specific scientific epistemology. In the small case of the Breaking up into partial fractions praxeology (“Bup” praxeology in the following) we have considered here and more generally for the differential equations solving praxeologies related
to Laplace transform, $I_r(M)$ nowadays acts as a praxeology producer, even if Heaviside’s operational calculus has been a precursor, acknowledged by the electromagnetism research institution at the end of the 19th century, if not by the mathematics one. $I_r(AC)$ uses $I_r(M)$’s praxeologies but the movement from the mathematics institution to the automatic control one changes the praxeologies, this is the phenomenon of transposition that Yves Chevallard has introduced at first in the case of didactic transposition (1985, 1989). Then the movement goes on from $I_r(AC)$ to $E(AC)$, with new possible transpositive effects, due to the fact that students are beginners and to the working context perspective. We have seen that in the “Bup” praxeology, the type of tasks and the technique have changed, due to the requirement about the time constants. If we consider the technology, we may assume that there is no change regarding the mathematical theoretical validation of the technique itself. But we find new elements coming from the fact that the mathematical task has been embedded in an automatic control type of tasks.

The diagram in Figure 1 gives a general representation of the possible transpositive effects when a praxeology $[T, \tau, \theta^r, \Theta]$ produced by a research institution $I_r$ moves to be used in an institution $I_u$. $I_u$ may be an educational institution working with the praxeology to teach it. What does this diagram say? At first, the asterisk expresses that every component of the original praxeology may evolve. This transformation is an object of institutional transactions completed in a specific institution $I^{*r}$, created and controlled by $I_r$ and $I_u$. $I^{*r}$ is more or less vanishing, the transactions are more or less difficult and controversial, depending on several factors: the extent of the transformations, the distance between the two institutional epistemologies (e.g., $I_r$ is mathematics and $I_u$ is an experimental science), the importance for $I_u$ that $I_r$ validates the new technique (e.g., $I_u$ is a profession as nursing with high security requirements), the importance for $I_r$ that the transposed praxeology be not too far from the original one (it is frequent that when $I_u$ is an education institution, mathematicians have a critical look on what is taught). At last, this diagram says that a practical technology $\theta^p$ is developed and acknowledged by $I_u$ on specific empirical bases.

![Figure 1. From $I_r$ to $I_u$, the transpositive effects model](image)

(Romo Vázquez 2009) shows that the transactions between institutions about the logos block $[\theta^r, \Theta]$ of the Laplace transform praxeologies may result in different forms of transposition according to the considered vocational training institution. The crucial issue that underpins the transpositive choices is whether or not engineers or higher technicians should know the most possible about the mathematical validation and explanation of the techniques that they will be supposed to use. In other words, does the training institution try to reduce the presence of black boxes or not? If so, these praxeologies are taught by a mathematics lecturer, if not they may be directly included in an engineering science course. Romo Vázquez meets the first case in a very high level engineering school; as aforementioned, $E(M)$ textbook is focused on the mathematical theory, exhaustively presenting proofs. In this case, transposition totally remains under the mathematics’ epistemological control; changes in $[\theta^r, \Theta]$ are limited. The two $E(AC)$ textbooks provide examples of the second case; they come from different vocational training institutions with emphasis on the occupational
competencies. The analysis (Castela & Romo Vázquez 2011) reveals that the $[0^\circ, \Theta]$ block is partially vanishing: the mathematical proofs of some theorems (mainly the easiest ones, with low theoretical needs) are presented; for other theorems, the proof is omitted but the textbook refers to its existence as a mathematical guarantee; at last, some claims should need validation but their problematic dimension is completely hidden. The counterpart to these choices relative to the mathematical component of the technologies is that in $E(M)$ textbook, the practical component is totally missing, while it is especially developed in $E(AC)$ textbooks, with emphasis on the technological elements satisfying motivation and appraisal needs.

An epistemo-anthropological approach of custom dressmaking in Argentina

Up till now, we have considered some mathematical praxeologies, ‘mathematical’ being understood as produced and acknowledged by mathematics research viewed as an institution and we have addressed the issue of how they change when moving to other institutions in order to be used. In what follows, I intend to show that the praxeological and institutional approach is also relevant for occupational contexts, even with few school mathematics involved. I draw on a research realised in Argentina by C. Elguero (Elguero 2009, Castela & Elguero 2013) in the custom dressmaking context.

We will consider two institutions: One is a system developed by an experienced acknowledged tailor H. Zampar, whose pattern drafting techniques are disseminated all over Argentina through numerous books, a website and a web of training schools. The second one is Gladys’s workshop; Gladys being a confirmed master ‘craftswoman’ is charged with the on-the-job training of several apprentices. Roughly, what are the main problems that dressmakers tackle? Firstly, making a piece of clothing, that is, a spatial complicated object, hence they have to draft the flat patterns from which they will draw the spatial form. Secondly, tailoring the pattern to the customer’s morphology, knowing that for some practical reasons, some of the necessary measurements are not taken on the customer’s body; they need therefore to infer the missing measurements from those they have. Zampar’s system provides techniques to solve these problems for any piece of clothing, for man, woman and child. Gladys uses some of them but she also chooses techniques from several other well-known systems, thus building her own mixed system that she teaches to her apprentices.

Examples of pattern drafting praxeologies

The following example is drawn from a Zampar’s book (2003) for children dressmaking. The type of tasks is Drafting the back pattern for a shirt. We consider only a few steps of the technique.

The first step is to draw a rectangle with following measurements: Height= Back length+2cms, Width= $\frac{1}{4}$(bust circumference+4cms). The coefficient $1/4$ is partly explained in an online course written by another expert: human body is modelled by a cylinder, its right and left parts are considered symmetrical. This should give a coefficient $\frac{1}{2}$ but in Zampar’s book, for a child shirt, back and front too are assumed symmetrical. Zampar insists on the raisons d’être of the additional 4 and 2 cms: the first ones provide ease for breathing and the second ones anticipate the future child’s growth.
The second step is to divide the rectangle in three horizontal unequal parts, the upper one with a height of one tenth of the basic rectangle height. Zampar does not explain why he takes this coefficient, nor does he for virtually all relations he uses in the different praxeologies. Yet some scarce ones derive from geometrical reasons. For instance, the point A in Figure 2 is obtained reporting one sixth of the neck circumference. Within Elguero’s investigation, no one in the workshops could explain this formula. However this comes from an approximation of the relation between the circumference of a circle and its radius, with \( \pi \approx 3 \). In both institutions we consider, this mathematically validated formula is not differentiated from the other ones.

The second type of tasks follows the first one; it is Drafting the sleeve pattern for a shirt, in the same context of child dressmaking. We focus on one step: on the basic rectangle, drawing an upper horizontal rectangle (A in Figure 3-left) in which will be further drafted the sleeve head (Figure 3-right). Zampar’s technique for this rectangle height (medida 3) is to take \( 2/3 \cdot BD \) where BD is measured on the back pattern (Figure 2), that is, not on the customer’s body. Gladys uses this technique. When asked to explain the formula, Gladys and her assistant answer that they have never addressed such issue. Gladys adds: “Es una fórmula para que la manga te salga con la medida justa\(^5\). (Elguero 2009, p. 91). She does not know about the causes of this relation. Yet she knows about the objectives as we can see in the following:

Gladys: Yo antes enseñaba la manga de otra manera [ese refers to another system]. Pero por mi escuela yo tengo que actualizar con todo lo nuevo que sale en moldería y buscar lo más práctico para las alumnas. La manga que yo enseño, no requiere que vos tomes la medida del contorno de sisa en el cuerpo, sino que medís directamente en el molde de espalda, entonces es más exacta, porque es una línea lo que medís…Las instrucciones son mucho más sencillas que las que enseñan en otros sistemas\(^6\). (Elguero 2009, p. 90)

---

\(^5\) It is a formula for the sleeve leaves you with just the right measure. (our translation)

\(^6\) I previously taught sleeve [she refers to another system] otherwise. But at my school I have to update with anything new that comes in dressmaking and seek the most practical for students. The sleeve I teach, does not require that you take the armhole circumference in the body, but you measure directly into the back pattern then it is more accurate because it is a line which you measure ... The instructions are much easier teaching than in other systems. (our translation)
We see that Gladys’s training responsibility leads her to appraise the different techniques for the same type of tasks on criteria of simplicity and accuracy, that is to say, of ergonomics, efficiency and perhaps ease of teaching/learning. This appraisal motivates a change of technique. It should be noted that she too proposes an explanation of the improved accuracy obtained by this technique: measuring a segment on the pattern is more accurate than measuring a circumference on the customer’s body. So we note that Gladys has chosen to adopt Zampar’s technique after using another one, she uses it and teaches it as it stands, and develops her own technology that we assume she passes on to her apprentices.

**Movements of praxeologies between professional institutions**

It is time now to address the issue of whether the model of praxeological movements and transposition effects presented in Figure 1 is relevant in the case of the aforementioned pattern drafting praxeologies.

At first we focus on Zampar’s system. Are Zampar’s praxeologies transposed forms of praxeologies produced by research institutions? What would be the research field of such institutions? Possible answers would be ergonomics, anthropometry, applied anthropology. Castela & Elguero (2013) give some specific examples of laboratories collaborating with dressmaking industries. But there is not the least sign of a contact between such a laboratory and H. Zampar for the pattern drafting techniques. With respect to mathematics, the scarce types of tasks involved are to draft a cylinder pattern and to calculate the radius of a circle from its circumference. We should therefore consider Zampar’s system as a professional dressmaking institution ($I_p$) with praxeology producing activities.

How do Zampar and his collaborators produce the techniques and relations they use? This issue has remained unaddressed, Elguero’s research being not interested in this level of dressmaking institutions that produce and disseminate pattern drafting praxeologies. If we consider the model $[T, \tau, 0, \Theta] \rightarrow I_p$, we see that, although Zampar’s books give access to the praxis and technological components of the praxeologies, the production and legitimating processes represented by the arrow remain fully unexplored as well as the institutional theory supporting these processes. I should here emphasise that in such context, the ATD concept of theory does not refer to the organised set of knowledge considered by the scientific meaning of the word. $\Theta$ is the institutional knowledge that supports the technology $\theta$, or rather, the technologies of several praxeologies. For instance, the theory of Zampar’s praxeologies...
should express the technology of the techniques which are used to produce and legitimate the pattern drafting techniques and their related technologies. If this pattern drafting system is totally produced and empirically validated in the course of Zampar’s and collaborators’ dressmaking work, how do they manage to do so? How do they explain that it is possible? If not, that is if they sometimes have a specific activity, away from any relation with customers and from any economical preoccupations, focused on the invention and systematic validation of new praxeologies, what can they tell us about this kind of dressmaking research? I propose to represent these two configurations as follows, with the first case on the left, the second on the right, $I_p'$ being a research institution, ‘offshoot’ from $I_p$:

![Figure 4. Professional institutions as praxeology producers](image)

Let us now consider Gladys’s workshop, another professional institution $i_p$ at a more local level. $i_p$ demonstrates some independence from Zampar’s system, choosing techniques from several well known books. In the examples, the chosen techniques are used as they stand. It seems that the issue of their validation is not raised again by Gladys. Although (Elguero 2009) does not provide any explicit evidence of such a hypothesis, I will assume that Gladys believes in Zampar’s guarantee. In other words, $i_p$’s theory should express some principle like: when a technique $\tau$ comes from an institution $I_p$ with acknowledged professional expertise, we take the $\tau$ validity for granted. This may be compared with the ‘black-boxing’ phenomenon that we have encountered in the automatic control textbook, where mathematical proofs are sometimes only alluded to. To this, Gladys could possibly add that throughout her long workshop experience with Zampar’s techniques, she has never faced any problem. This means that the theoretical component changes from $I_p$ to $i_p$; likewise, as some technological elements of the original praxeology may vanish in the workshop when others are added, in relation with the local working conditions, especially with the training educational context. The diagram in Figure 5 represents a general model for praxeological movements between professional institutions with different status within the professional field, assuming that every component may possibly change:

![Figure 5. Transpositive effects from one professional institution to another](image)

This model aims at generality; however, it is influenced by the dressmaking example. It should at least consider, as does Figure 1, the possibility that $I_p$ be concerned with what happens to its praxeological production when disseminated in the workshops $i_p$. In other words, this is a proposal that should be adapted to other contexts of investigation.

**Conclusion: Towards an anthropology of mathematics**

If we look back on the path taken in this text, we should see that the first group of examples considers academic mathematics as a reference. The addressed issue is the following: What does a mathematical praxeology become when used by other scientific or occupational
domains? In other words, which mathematical praxeologies are embedded in the praxeologies of these domains? This is a classical approach in mathematics education. What is the meaning of the word ‘mathematics’ in that case? Mathematics is a body of knowledge, or preferably of praxeologies, produced through specific research activities; both, knowledge and activities, are acknowledged as mathematics by international mathematics institutions.

Mathematics is at the same time a body of knowledge, a field of activities and an institution. This looks very much like a closed world. When someone of this world, that is, a mathematician, begins to investigate on mathematics education, especially but not only in vocational education, he needs tools to distance himself with the ‘alma mater’. Since the beginning, this has been Chevallard’s objective with the anthropological theory of the didactic. I contend that the work I have presented here around the notion of praxeology provides a powerful tool to investigate the mathematics dimension of human social activities in any context, without referring to academic mathematics.

Indeed, in the second group of examples, we have some evidence that the dressmakers’ pattern drafting techniques only scarcely draw on mathematical ones. Of course, we have not considered here the calculations with rationale numbers involved in the formulas: this was the central issue addressed by Elguero’s investigation (2009); Castela & Elguero (2013) analyse in detail the related dressmakers’ praxeologies in relation to the mathematical usual ones. But, in the present text, I have intentionally focused on the modelling part of the techniques. These techniques provide solution to a very broad class of problems related to the plane development of spatial surfaces. As a mathematics teacher with a rather high level (up to PhD in mathematics), I never met any consistent mathematical theory solving this problem for the complex forms involved in dressmaking. Perhaps such theory does exist, but it is obvious that dressmakers, independently from the scholarly mathematics institutions, have produced knowledge about these problems and found solutions. Will we consider this knowledge and these solutions as mathematics? The answer is clearly no, if we take academic mathematics as a reference. Yet, producing a plane development of a spatial surface has certainly some mathematical dimension, since such problems are considered for polyhedrons. Hence, we should consider at least dressmakers as providers of solutions to mathematical problems; why not as workshop mathematicians? This is the kind of issues that anthropology of the mathematics should address to deconstruct the whole concept of mathematics. The contemporary anthropology no longer considers human evolution as a long path culminating in the occidental human being; an anthropological approach to the mathematics should also get rid of any reverence vis-à-vis academic mathematics considered as one of the highest achievement of mankind. It is a condition for addressing afresh the issue of what is mathematics, a condition that is not always satisfied by the researchers referring to ethnomathematics.

This anthropology of the mathematics should investigate social practices without too narrow restrictions on what is an interesting object. That is why I consider the praxeological model as previously presented as an interesting tool. It highlights dimensions of the institutional cognition that would be neglected otherwise, especially when the reference to acknowledged mathematics is too strong. Such a research program is directed towards epistemological and anthropological goals, intending to unearth the diversity of human mathematics praxeologies. And, as a conclusion, I would like to emphasise the fact that I consider as not at all obvious the question of whether any such “ethno-praxeologies” should be incorporated into school curricula (about this discussion, see Pais 2011).
References


Empowering Socioeconomic Development Efforts in Africa through the Indigenization of Science, Mathematics and Technology Education

Meshach Ogunniyi
School of Science and Mathematics Education, University of the Western Cape, South Africa.
mogunniyi@uwc.ac.za

The paper alludes to recent efforts made by many African countries to make science, mathematics and technology (STM) education relevant to learners’ life-worlds. It contends that authentic socioeconomic development of Africa ultimately depends on decolonizing all facets of its education systems particularly STM education that has been used in the last 400 years to subjugate the African peoples. Colonization is construed here not only as a vehicle of cultural imposition, it is also regarded as a mechanism for social control. The aftermath of colonization in Africa as elsewhere has been the production of a people whose psyche and sense of interiority have become subservient to the dictates of the colonizers. This characteristic is usually called “colonial mentality”- i.e. the manifestation of a grotesque sense of inferiority complexity. The paper posits indigenization of STM education as probably the best way to overcome this complexity and to develop instead a sense of self and cultural pride. It contends further that SMT education, the very tool used as a secret weapon of socioeconomic domination and cultural bastardization in Africa could be turned into a mechanism for emancipation and self-actualization. In conclusion, the paper shows how a dialogical argumentation instructional model (DAIM) was used as a tool to harness the potential of STM education into a liberatory pedagogy which increased the participants’ awareness about the educational and cultural values of indigenous knowledge as well as empowered them to develop a sense sociocultural identity.

Whereas educational arguments have been raging for decades over when to introduce one school subject or the other to the African child, not much has been done to ensure that what is taught relates to his/her everyday experience or stimulates his/her intellectual interest. For instance, there has been arguments about when to introduce English, French, Portuguese or Spanish as the language of instruction, which aspects or version of history, religion, music or art should be reflected in the curricula and so on, SMT have always been considered as universal and culture-free. This paper challenges that myth and places school SMT in their rightful place in the arguments.

Bishop (1990) contends that until the mid-1970s mathematics was considered a universal and culture-free subject after all, two plus two is four, the sum of all triangles is 180 degrees and a negative number times a negative number results in a positive number, a flat perfect circle is 360 degrees and so on. These mathematical truths or abstractions are universally valid because based on formal argument known in logic known as modus tollens if the premises are valid the conclusion cannot be otherwise. However, when this same argument is extended to science dealing not only with abstraction but also physical reality the conclusion reached might not be as exact as the abstraction. The validity of the conclusion would to a large extent depend on the accuracy of the experiments, observations, data, analysis, inferences drawn from data analysis, etc. In other words, the truth of deductivism rests a priori on the rules of syllogism. It is also important to note that the outcomes of many experimental tests do not necessarily imply a conclusive prove of a scientific generalization (e.g. hypothesis, laws or theories). All that can be said about such favourable outcomes is that as far as the test
implications are concerned a generalization has received further corroboration or confirmation but not complete proof. To proof a scientific generalization implies knowing all the facts if that were ever possible. To know or collect all the facts by undertaking an innumerable number of experimental tests, as inductivism suggests, would imply awaiting the end of the world; so to speak, because there is an infinite number of them (Hempel, 1996).

The usual argument is that since mathematical facts are valid anywhere in the world and they are culture free they can be taught without paying any close attention to the sociocultural environment of learners. Bishop (1990) has a different opinion about this According to him, “Of all the school subjects which were imposed on indigenous pupils in the colonial schools, arguably the one which could have been considered least culturally-loaded was mathematics...“western mathematics”...is one of the most powerful weapons in the imposition of western culture” (p. 51). Of course, the same is true of science and to some extent technology as well.

Nature of science

The assumed universality of science (mathematics and technology inclusive) is premised on its precise description; explanation and prediction of natural events in comparison to other thought systems lacking such exactitude (Nola & Irzik, 2005). It is further argued that in light of the consistency of the internal contents of science (e.g. facts, concepts, laws, theories and the like) it should not only be construed as just one form of possible knowledge, but rather coincides with knowledge or truth itself in the pure objective sense. However, that assumption ignores the real nature of science (NOS), the way science develops, and how it is conceived in the mind. It also ignores the fact that scientific concepts, laws and theories- the very pillars of science are the products of creative imagination rather than invented purely out of share application of logic, algorithmic procedures or even the combination of the elements of rationalism and empiricism as some logical empiricists would like us to believe. In fact, the idea that science is only one form of knowledge but knowledge itself is tantamount to “scientism” (Habermas, 1971, p.4). Although there are disagreements about how scientific concepts, laws and theories are formulated (e.g. induction versus deduction), there is a general consensus among scholars that these entities are products of the human mind, though the actual processes involved are not clearly known. In other words, the processes must of necessity involve not only the application of syllogism alone but also metaphysical and axiological musings as well as intuition, practical wisdom, etc. (Habermas, 1999).

It is important to note however, that despite the erroneous image of science that has been perpetuated for decades certain salient facts are worth mentioning namely, that: (1) whatever changes take place on account of new data, science is fairly a stable enterprise; (2) though the changes in science are warranted from time to time, they do not necessarily imply a wholesome discarding of all that is presently known; (3) there might be variations in the way scientific findings are explained but these do not depend on the whims and caprices of the scientists concerned as the tests and findings of other scientists serve as checks and balances; (4) scientific revolutions, as Thomas Kuhn (1970) has indicated, do not take place readily as scientists tend to hold their grounds to defend their hypotheses or theories; (5) continuity rather than radical changes is a persistent feature of science; (6) although scientists use a variety of methods in solving a problem certain commonalities are discernible in the way they tackle such problems (e.g. formulation of a hypothesis, testing the validity of the hypothesis through experiments, careful observation, collection and analysis of data, a display of open-mindedness); (7) drawing inferences from analyzed data; (8) publicising findings in seminars and conferences); etc. (Good & Shymansky, 2001; Kuhn, 1970; Popper, 2001).
It is apposite to state that there is a general naïve belief, especially in the science education literature that scientific laws and theories are provable or verifiable. However, unlike mathematical theorems which are provable on account that they are solely constrained by syllogistic reasoning, scientific laws and theories are neither provable nor verifiable. To attempt to prove or verify them as Hempel (1966) and Popper (1968, 2001) have warned, is to run the risk of infinite regress because these rational principles are limited to what we know now. The more experimental tests we run to verify them the more new insight we gain about their limitations under certain conditions. For example, the hypothesis which asserts that, “water boils at 100° centigrade is only valid under a given environmental condition. The fact is that water does not always boil at 100° centigrade. It depends on whether or not it is in an open or closed vessel and at a particular elevation e.g. at the sea atmospheric level compared to a mountain top. The same applies to other scientific generalizations e.g. law of gravity, law of thermodynamics, law of genetic dominance, etc. They hold under certain contingent conditions (Magee, 1973). In light of this, it suffices to say that scientific laws and theories, the most powerful instruments used to explain natural regularities are only provisionary in nature and as more are known they are modified accordingly to accommodate the new knowledge.

Besides the above, the interpretations and the meanings derived from the contents and applications of science are not necessarily the same for all people. No matter how precise a given scientific fact, concept or generalization may be, its truth is still probabilistic, tentative, dubitable and revisionary rather than absolute or infallible. This is not the same as stating that scientific knowledge is unreliable or false as some extreme post-modern critics of the realist perspective would have us believe. Rather, my view is that the strength, beauty and the truth of science (mathematics and technology inclusive) rest on the fact that it is falsifiable and in constant need of being updated as new knowledge accrues. The implication of all this is that science, the quintessence of modern civilization is not infallible knowledge and must be considered as such when compared to other knowledge corpuses.

The brain scientist and Nobel laureate Sir John Eccles has attested eloquently to the tentativeness and falsifiable nature of scientific generalizations (laws and theories). Karl Popper recalls how the distinguished scientist became depressed on realizing that his tenaciously held hypothesis about the electrical synaptic transmission of both the nerve stimuli and nerve inhibitions would have to be replaced by the more popular chemical transmission theory proposed by Sir Henry Dale in Cambridge. At the time he considered the failure of his theory to account for certain transmissions as his own personal failure as a scientist. In his own words, Eccles said:

That was my trouble. I had long espoused an hypothesis which I came to realize was likely to have to be scrapped, and I was extremely depressed about it… At that time I learnt from Popper that it was not scientifically disgraceful to have one’s hypothesis falsified. That was the best news I had for a long time…Now I can rejoice in the falsification of a hypothesis I have cherished as my brain-child, for such falsification is a scientific success. (Popper, 2001)

Later on it was discovered that while certain synaptic transmissions are chemical in nature others are electrical. In other words, neither Eccles nor Dale theories applied to all synaptic transmissions. Both were guilty of the same error: namely, “overhasty generalization without waiting for all the relevant data” (Popper, 2001, p.13).

It is important to state that despite the erroneous image of science that has been perpetuated for decades, certain salient facts are worth mentioning namely, that: (1) whatever changes
take place on account of new data, science is fairly a stable enterprise; (2) though the
changes in science are warranted from time to time, they do not necessarily imply a
wholesome discarding of all that is presently known; (3) there might be variations in the way
scientific findings are explained but these do not depend on the whims and caprices of the
scientists concerned as the tests and findings of other scientists serve as checks and balances;
(4) scientific revolutions, as Thomas Kuhn (1970) has indicated, do not take place readily as
scientists tend to hold their grounds to defend their hypotheses or theories; (5) continuity
rather than radical changes is a persistent feature of science; (6) although scientists use a
variety of methods in solving a problem certain commonalities are discernible in the way they
tackle such problems (e.g. formulation of a hypothesis, testing the validity of the hypothesis
through experiments, careful observation, collection and analysis of data, a display of open-
mindedness); (7) drawing inferences from analyzed data; (8) publicising findings in seminars
and conferences); etc. (Good & Shymansky, 2001; Kuhn, 1970; Popper, 2001).

The challenges and prospects of inclusive STM curricula

Fifty years ago Schaffner decried the decontextualized science curricula and instruction
commonly taught in an average science classroom. He argued that portraying science as a
collocation of infallible facts or a set of operations for problem solving to the exclusion of the
understanding of the general principles and methods of science did not adequately reflect the
true image of science. To him:

In order to comprehend science, one must be acquainted with its structure; its metaphysic
or absolute presupposition such as the uniformity of nature and the amenability of
substance to mathematical description, its methods of observation, hypotheses, and
experiments, which in a sense follows from the presuppositions; and finally, its laws, the
fruit of the application of method to nature or substance. (Schaffner, 1964)

Although great strides have been made in many African countries to make SMT curricula
more relevant to learners’ sociocultural environment, a cursory review of such efforts would
easily reveal their indelible colonial stamp. Whereas considerable efforts have been made in
the social sciences and humanities to reflect some aspects of the African worldviews and
cultural ways of knowing and doing things, very little has been done to make SMT curricula
relate to the daily lives of African learners e.g. by exploring the scientific processes evident
in their indigenous knowledges and cultural practices.

All that has been done since the independent era in the 1960s in Africa therefore, has been to
tweak or tinker here and there these presumably sacred school subjects. Many of the so-called
new SMT curricula are, more often than not, transplants from western countries or other
formerly colonized countries. For example, the Integrated Science, Modern Mathematics and
Introductory Technology in the early 1970s came largely from Britain and the West Indies. A
recent example in South Africa has been the outcomes-based curriculum which came largely
from Australian Science Outcomes designed for the Victorian Curriculum and Standards
Framework (CSF) and National Curriculum and Profiles (NCP) in science, which in turn
drew inspiration from the British and American outcomes-based curricula of the 1980s. The
irony of course, is that such curricular transplants was neither all that successful in their
native soils nor have they now been so in the new African soils. The issue here is not to
question the merits of the outcomes-based curricula per se, but to recognize the inevitable
cultural impact of their native soils and the cultural undertones in the new environment. This
is understandable considering the fact that they were in the first borrowed from the West and
as such are not free of Eurocentric perspectives. In other words, its positivist portrayal of
NOS does not take the complexity of an average multicultural South African classroom into
consideration.

Every classroom is unique in the sense that the learners’ backgrounds vary quite widely. Added to this of course, are the challenges that a teacher faces in an attempt to teach an inclusive school SMT in which the diversity of learners is recognized and of which IK forms are an integral part. Yet as stated earlier, IK is fundamental and foundational to learners’ apprehension of the physical world they live in. However, such an inclusive or indigenized school SMT curricula are framed by a pluralistic way of knowing and interpreting human experience or what Ogawa (1995) calls multiple sciences. However, as explained earlier, these various forms must not be confused with the relativistic perspectives so much abroad in the extant post-modern literature (e.g. see Good & Shymansky, 2001; Nola & Irzik, 2005).

In light of the foregoing, the aim of the paper is neither to deify modern science in comparison with other ways of knowing and interpreting experience as the extreme realists and positivists like to do, nor to deny its significant contribution to human progress in the last four centuries as the extreme relativists tend to do e.g. by focusing solely on the diverse environmental problems associated with scientific and technological activities. Rather, it is to present a moderate and robust image of science as a human enterprise which learners can relate to in or outside the school environment. In the same vein, the paper attempts to present a more positive image of indigenous knowledge than has usually been the case since the colonial era. In other words, it is not about deifying, eulogizing or romanticizing indigenous knowledge in the extreme relativists’ perspective but to present it as a legitimate way of knowing and worthy of scholarly consideration by the STM education community.

Both science and IK have their merits and limitations in addressing human problems. Of particular interest in this regard is to explore how the science-IK curriculum can serve as a useful platform for engendering a more robust understanding NOS and the nature of indigenous knowledge systems (NOIKS). A corollary to this is to explore how the ensuing dialogue could to facilitate the development of valid images of both systems of thought. However, as indicated earlier, this aim can be thwarted if science is presented as abstract and culture-free enterprise or if IK is regarded as mere witchcraft or a bunch of superstitions as has been the case since the independence era.

No one doubts the challenges involved in the integration of school SMT with indigenous knowledge and practices. For example, the Revised National Curriculum Policy Statement first implemented by the Department of Education (DOE) and later by the Department of Basic Education (DBE) is a notable attempt to incorporate IK into the various school subjects particularly school SMT (DBE, 2011; DOE, 2002). In response to this policy, the Science and Indigenous Knowledge project (SIKSP) has been equipping several cohorts of educators and teachers with knowledge and pedagogical skills to integrate indigenous knowledge with science in their classrooms. Later on this paper will highlight the challenges that were encountered and the significant gains made by the project to achieve the goal of indigenizing school science and in making it relevant the daily lives of learners using a dialogical argumentation instructional model (DAIM).

Indigenous knowledge (IK), unlike science which has acquired some elements of universality, is one that is acquired formally and informally by living in a particular local community. For the same reason and the issue of colonialism alluded to earlier, the inclusion of IK into the STM curricula has been a herculean task which many teachers find difficult to do. But as the first knowledge encountered by the children in a local environment IK is foundational to their understanding of the physical world as well as the development of their affective skills and sense of moral responsibility. Although this view has been expressed in
many a curricular reform policies in Africa since the independence era in the 1960s very little has been done afterwards. It is also worthy to note that though the past decade (2000-2010) was designated the “indigenous decade” very little was accomplished in terms of indigenizing school STM or to reducing their colonial undertone. The poignant question is, “To what extent have STM curricula reforms in African countries embraced the philosophy of indigenization?”

**Using the indigenizing STM education for African empowerment**

There role of STM education in the development of potential scientific and technological humanpower is well attested to in the extant literature. However, the way science is taught presently in most African countries (including South Africa) has tended to alienate the majority of learners from wanting to pursue science or related careers. It is apposite to indicate upfront that science teaching in most African countries is still largely based on expository chalk-and-talk approach. Such an instructional approach tends to leave little or no room for inquiry activities or classroom discourse. Other than some cosmetic reforms that have been made since independence, the science curricula and way they are enacted in the classroom have remained the same way they were left by the colonial masters. In other words, science teaching in Africa is no more than a replica of the Eurocentric colonial relic. Although no one denies the universality of science, or more correctly, universal application of science, the way science is taught or learned is certainly not universal (Cobern (2000). Individuals and groups do not necessarily do things the same way. In the same way many people have different ways of knowing and interpreting the physical world and as such there are many sciences and the ways they are taught. Consequently there are many voices in any science classroom (Ogawa, 1995) which a teacher cannot ignore if he/she is to hold the attention or maintain the intellectual interests of his/her learners. The question one might ask is, why include indigenous knowledge in the school curriculum?

Bishop (1990) provides a succinct historical account not worth repeating here of how the internationalized mathematics and science have dominated the curricula in the colonized countries namely through trade, administration and education especially the language of instruction. I shall also like to add religion to the list as a factor that ought to stand on its own because Africans are very religious people. As Mbili (1996) puts it, Africans live so to speak in a religious drama. In fact, the central core of African philosophy is religion. The missionaries capitalized on this but in their zeal to introduced the new religion to the African peoples they also relegated not only the multiple religious beliefs in Africa to the background they inadvertently deprecated even the good cultural values of Africans which they substituted with western cultural values. The result of this approach has led to the emergence of assimilies with a grotesque appetite for western goods and the acquisition of material wealth at all costs. That is not all. The same approach has brought about the production of the present generation of corrupt, morally depraved and visionless political leadership driven by greed, cupidity and an insatiable craving for power.

The moral sanctions within the African society before the Europeans arrived at our shores have thus been thrown into the wind so to speak. All the politicians need do to control the African public is to put on the veneer of religiosity, though inwardly and by practice, they are ravenous wolves in sheep clothing, so to speak. The subjugation of the African cultural values and their replacement with theirs has brought about dramatic changes not only in names, lingua franca but also the way Africans are expected to think, live and behave. The indigenous languages were relegated to the languages of the colonial countries. The European rationalistic thinking and the very core of western STM became the order of the education
system, administration, business and public life in general. Objectivism replaced subjectivity, logic replaced practical wisdom, the world of people was replaced by the world of things, power to control people replaced social and environment harmony, and so on. In the same vein, the contributions to the development of science and mathematics over the centuries, especially before the 17th century e.g. by the Egyptians or more correctly, Africans, Indians, Babylonian, Dravidians, Chinese, etc. were suppressed and school science and mathematics were wrapped and coined solely as western epistemology.

In recent years calls have been made to rewrite the history of science and mathematics as has been done in history and African literature. However, the road is not likely to be easy. Yet, the justification for the inclusion of indigenous knowledge in school science, math and technology and other school subjects is a moral imperative. This is because the indigenous knowledge of a given community to a large extent defines and shapes the worldview and life worlds of that community. Indigenous knowledge or what Hountondji would rather call endogenous knowledge, is the knowledge experienced by a given cultural group and which now forms “an integral part of its heritage, in contrast to exogenous knowledge which is perceived, at this stage at least, as element of another value system” (Hountondji,1997:18). Oggunniyi (2004) goes further to define indigenous knowledge as knowledge that is peculiar to a particular people group that has not been borrowed from another culture or if borrowed at all, has become so assimilated in the new culture that it is difficult if not impossible to determine its original source or foreignness.

As stated earlier, many African countries in pursuance of equity, justice and to make science more relevant to learners’ traditional worldviews many African countries have formulated and implemented indigenous policies. Likewise, the Education Department in many countries have implemented science curricula which demand science teachers to integrate school science with indigenous knowledge (IK) in the classroom context. However, as Okebukola (2013) points out at the Fourth International Conference on Science and Indigenous Knowledge Systems, the visibility of IK policies in Africa with few exceptions have been generally low.

The case of South Africa

For the past two decades South African learners have consistently under-performed in school science both in the national examinations and international mathematics and science tests e.g. TIMMS (Reddy, 2006). The reasons for this are not unrelated to the issues already mentioned in the preceding section in terms of how STM curricula and instructional practices have tended to alienate learners from wanting to pursue the scientific fields and related careers. Despite the efforts that have been made by government to address this anomaly not much improvement has resulted. The question is, could this persistently poor performance of learners in SMT be because the way they are presented in the typical textbooks as a body of abstract facts with no direct relevance to the learners’ environment? Many researchers have stressed that unless STM are taught in an interesting and relevant manner learners will continue to perform poorly in these subjects. It has been noticed in a number of studies that for one reason or the other even the bright learners do not want to pursue scientific careers and tend to opt for other subjects such as accounting, computer science, law and other subjects which they believe would lead to better salaries or at least improve their condition and quality of life (e.g. Aikenhead, 2006; Aikenhead & Elliot, 2010; Oggunniyi, 2004).

Although the factors responsible for learners' poor performance are many the most conspicuous and frequently mentioned in the extant literature is the way science is taught. The teacher as a knowledge broker is therefore key to breaking the vicious cycle of failure
and poor performance in science among South African learners. Many classroom-based studies have shown that the majority of South African science teachers teach by the traditional chalk-and-chalk method thereby given little or no opportunities for their learners to contribute to classroom dialogues. They also teach science in a decontextualized and Eurocentric manner. These teaching approaches tend to alienate learners whose cultural identities differ from the Euro-centric science. Other than memorize science concepts to pass examinations most of the are not able to pursue further studies in science because of their poor conceptual understanding. Besides, school science and the mass media convey a mythical image of science and scientists (Aikenhead & Elliot, 2010). In earlier studies less than one-third of grades 7-9 learners in South Africa scored above 50% of selected critical science concepts selected from the new curriculum (Ogunniyi, 1999; Ogunniyi & Mikalsen, 2004; Ogunniyi & Taale, 2004).

There is a general belief in many African countries (including South Africa) that IK has little to do with western science (Ogunniyi, 2004, 2007a & b; Govender, 2011; Van Wyk, 2002). Learners have been told and brainwashed on many occasions that IK is a bunch of superstitions. For instance, I remember that before I started school, I knew the names of many indigenous plants, their dietary and medicinal and other practical uses. However, at school I was introduced to a different form of taxonomy. The constant brainwashing experience I had ultimately succeeded in eroding the indigenous knowledge I brought into the science classroom. As a schoolboy, my understanding of the natural world- tempered at home and among my friends- was far removed from the world presented by the teacher or the science textbooks handed to me at school. The concepts of science, especially, were abstract and alien to me. This is because the science prevalent within my own indigenous community was relegated to the background of the one presented at school. For instance, the calculus, mechanics, systematics and ecological knowledge within the Yoruba cosmology, to name just a few, were thought to be irrelevant to school science.

At high school we were prepared for the Cambridge School Certificate Examination. Though living in tropical Africa we studied the European biomes and imagined how plants grew in such a cold country like the United Kingdom. We wondered how the “greenhouses” looked like and why they were so called. I remember our American chemistry teacher, who in response to learners’ suggestions about the similarities between the indigenous ways of making certain things e.g. soap, detergents, dyes, perfumes, etc. would say, “I came to Africa to teach chemistry not indigenous nonsense.” Of course, we would giggle and laugh about this but the consequence was that gradually we lost most of our IK. In fact, we began to believe that IK and related practices which shaped our lives and gave us our sense of self and sociocultural identity was no more than a form of witchcraft or a bunch of superstitions to be gotten rid of as soon as possible. We branded ourselves as civilized.

The average Yoruba child (and I believe every African child) came into the classroom with an encyclopaedic knowledge of natural phenomena, but this had to be suppressed or jettisoned to survive in an environment where canonical science was the only standard accounts acceptable at school. I was reminded of this, once again, when I recently witnessed the Afropessimism and the furor that followed in the wake of a report by the World Economic Forum (WEF) which ranked the quality of science and mathematics education in South Africa as the last out of 148 countries. To add insult to injury, the forum’s Global Information Technology Report, which looked at “how prepared an economy is to apply the benefits of information and communications technologies (ICTs) to promote economic growth and well-being”, placed South Africa 146th out of 148 for the overall quality of
education. Although the news media did not pull their punches, there are plenty of reasons why South African learners performed so poorly. The blame was placed on the early introduction of calculators at school, making thinking redundant; the little attention paid to basic education; the poor implementation of a reasonably good curriculum policy; the lack of political will; the inadequate number of qualified science teachers and insufficient teaching facilities; etc.

While admitting possible negative impact of many of the inadequacies listed above, there are however, certain critical flaws in the WEF report and the method that was been adopted in writing it. The Department of Basic Education (DBE) also took issue with the WEF report. Likewise, critics have argued that the rankings are “subjective, unscientific, unreliable and lack any form of technical credibility or true cross-national comparability”. They have pointed out that no standardised tests were actually conducted to assess the quality of mathematics and science education in the surveyed countries. Instead, the rankings were based on an annual “Executive Opinion Survey” in which the WEF asked business leaders to assess the quality of mathematics and science education in the country and score it accordingly. In short, these sorts of “League of Nations” assessments reports tend to generally lack cultural validity. They are like square pegs in a round hole. It is the same lopsided hegemonic and inequitable power division between the dominant voice over the silenced majority in a world where might is always right. They hardly count as objective scientific assessment. The next section provides an example of a project that has attempted to equip teachers with instructional skills on how to make the indigenization of science a reality in their classrooms.

The Science and Indigenous Knowledge Systems Project (SIKSP)

The central focus of SIKSP since its inception in 2004 has been to equip prospective and practising science teachers with the knowledge and instructional skills to teach a culturally relevant school science using an innovative pedagogy known as Dialogical argumentation Instructional Model (DAIM)- an approach which is not so familiar to most South African science teachers. DAIM attempts to create a classroom environment where learners are able to work together, learn together, express their views freely without intimidation, clear their doubts and affirm their sense of sociocultural identity in the spirit of Ubuntu (see Diwu & Ogunniyi, 2012; Ogunniyi, 2004, 2007a & b; 2011); and (2) to evaluate the teachers' perceptions of DAIM and to implement an indigenized science curriculum.

In our consideration of the integration of two distinctly different thought systems namely, science and indigenous knowledge (IK), the central concern of this study, we drew on what Hempel (1966) calls “the bridge principle” (p. 72). According to him, the function of the bridge principle is to provide a meaningful link or connection between two distinct entities or constructs e.g. between theoretical and empirical concepts. The entities of a theoretical concept e.g. moving molecules, their masses, momenta, and energies cannot be observed directly compared to the observable physical entities such as the temperature or pressure of a gas that can be measured by a thermometer or a pressure gauge. Kant also alludes to the same principle in his effort to find a meaningful connection between rationalism and empiricism. Rationalism contends that knowledge of the material world is attained through innately endowed concepts apart from direct experience. Empiricism on the other hand contends that knowledge of the world is acquired solely by practical sensible experience. Kant argues further that contrary to the rationalists’ stance, knowledge is not simply reducible to innately endowed concepts or principles, but on balance, and contrary to the empiricists, “knowledge is not the reflex of experience” (Foley, 2000, 83). In the same vein, it was our view is that the
same bridge principle is also applicable to attempts to integrate science with IK.

We also drew on the Aristotelian contiguity association theory of how two conflicting ideas in a person’s mind can be harmonized through a logical process that involves some form of wrestling, recalling and interactions before they finally coalesce into one unified idea (Runes, 1975). The same idea is reflected in Piagetian personal constructivism. However, this does not address conductive, axiological or value-laden aspects of human reasoning (Govier, 1987) or what Aristotle would call phronesis, i.e. practical wisdom or wisdom-in action which the worldviews of indigenous peoples. It was in light of this that we deployed Ubuntu, a central African worldview theory that exemplify both logical and non-logical (not necessarily illogical within its cultural framework) or metaphysical aspects of human reasoning. In other words, Ubuntu goes beyond the Aristotelian form of syllogistic reasoning. Ubuntu stresses, a priori, the relatedness, reciprocity, synchronicity, complementarity, connectivity and unity in diversity of ideas by an individual or the community to resolve conflicting ideas (e.g. Oggunyi, 1988, 2004, Hountondji, 1997; Mbiti, 1996; Ntuli, 2002). Ubuntu construes conflict resolution beyond the logical Aristotelian, the Piagetian and even the Vygotskian notions of constructivism. Besides, Ubuntu construes the individual not only a microcosm of society but also an inseparable part of it.

To fulfil the function of the bridge principle, and to cater for the logical and non-logical aspects of human reasoning, we combined the Aristotelian contiguity association theory with Ubuntu, which for ease of reference, we designated as the Contiguity Argumentation Theory to stress the argumentative nature of human reasoning (CAT) (1st Oggunyi, 1988, 2004, 2007a & b). CAT underpins the dialogical argumentation instructional model (DAIM) that we used to drive all the activities of the study (e.g. Oggunyi, 2004, 2007a, 2013).

**Argumentation as an instructional tool**

Argumentation is a dialectical tool used in in all cultures for resolving conflicts (Habermas, 1999) as would be the case in any attempt made to integrate science and IK. It is an effective tool for creating the needed intellectual space for dialogue. It provides the crucial opportunity for people to resolve conflicting ideas, clear their doubts and express their views freely without feeling intimidated (e.g. Oggunyi, 2007a & b, 2011; Erduran, Simon & Osborne, 2004; Osborne, 2010; Simon & Johnson, 2008). Argumentation is an important aspect of scientific discourse. Often argumentation arises in the context of conjectures and refutations within a scientific community of practice (Popper, 1968, 2001). Although the process of how argumentation brings about changes in people’s perceptions and value orientations is not fully known, there is substantial evidence in the extant literature to show that that it plays a vital role in the process (Leitao, 2000).

**Toulmin’s Argumentation Pattern (TAP)**

In looking for a handy argumentation theory that could be used to explore the logical arguments mobilized by the participants in the classroom discourses we adopted a modified version of Toulmin’s Argumentation Pattern (TAP) developed by Simon & Johnson (2008) Argumentation Pattern (TAP). TAP has been frequently used by several science educators in the past decade to facilitate changes in the beliefs of teachers and students about one subject matter or the other. Essentially TAP consists of a claim- a statement awaiting confirmation; data or evidence; warrants or justification of the claim on the basis of the evidence; backings or underlying assumptions; qualifiers or conditions in which the claim is valid and rebuttals or a contradictory statements to the claim (e.g. Erduran, et al, 2004; Osborne, 2010; Simon & Johnson, 2008). Despite the limitations that have been pointed out in the literature about TAP...
e.g. its assumptions about the passivity or the ineffectiveness of the opponent in directing the course of an argument, overlaps amongst the constituent elements, its unsuitability for analysing non-logical aspects human reasoning and decision-making, etc. (e.g. Ogunniyi, 2004, 2007; Erduran, et al. 2004; Leitao, 2000) we found it useful enough for analysing simple logical arguments like the ones adduced by the participants in the classroom discourses. A modified version of TAP developed by Simon & Johnson (2008) that we used is as follows:

Level 1: Non-oppositional arguments or arguments with simple claims versus counter-claims.
Level 2: Arguments supported with claims, data, warrants or backings but with no rebuttals.
Level 3: Arguments consisting of a series of claims supported with data, warrants, backings and only occasional weak rebuttals.
Level 4: Arguments supported with at least one strong rebuttal.
Level 5: Arguments supported with claims, data, warrants, backings and with more than one strong rebuttal.

Contiguity Argumentation Theory (CAT)

As stated earlier, CAT is a pragmatic theory derived from the amalgamation of the Aristotelian contiguity association theory and Ubuntu. Essentially, CAT consists of five main cognitive and affective states aroused by contextual changes namely: dominant; suppressed; assimilated; emergent; and equipollent. These cognitive-affective states are in a state of dynamic flux and may change from one form to another as the context changes. More details about CAT have been published elsewhere and will not be repeated here (Author, 2004; 2007a). The categories of CAT used as a unit of analysis are as follows:

Dominant: The most prevailing worldview (scientific or otherwise) in a given context
Suppressed: The worldview subdued by, or becomes subordinate to the dominant one.
Assimilated: The worldview that capitulates or is subsumed by a more dominant one.
Emergent: A worldview that arises from a new experience.
Equipollent: Two distinctly different but co-existent worldviews exerting equal cognitive force on a person’s worldview

Dialogical Argumentation Instructional Model (DAIM)

DAIM construes argumentation as starting from the individual (intra-argumentation), then to the small groups (inter-argumentation) and finally to the whole group (trans-argumentation) where the final collaborative consensus is reached. As stated earlier, we used TAP explicitly to analyze the logical aspects of classroom discourses while we used CAT to explore both logical and non-logical arguments in the classroom discourses as well as trace possible perceptual shifts among the participants (Fig. 1). One important rule which we stressed in all these activities is the Habermas’ (1999) notion of a fair argument namely: (i) no person who could make a relevant contribution may be excluded; (ii) all participants have equal opportunities to make contributions; (iii) participants are truthful in what they say; and (iv), the contributions are freed from internal or external coercion (Author, 2004, 2007a & b). As learners participate in dialogical argumentation in the process of performing cognitive tasks they are able to deploy all forms of reasons at their disposal (including scientific and IK-based arguments) to make their views known. It is during this same process that they are able to clear their doubts, revise their views and reach collaborative with their peers. Fig. 1 below
summarizes the essentials of DAIM.

![Figure 1. CAT-based dialogic argumentation instruction model (DAIM)](image)

In a recent study involving a cohort of 23 SMT science teachers and educators the participants were asked to indicate to what extent DAIM to which they had been exposed for two-three years have influenced their views about integrating IK with SMT in their classrooms. Table 1 and subsequent excerpts reflect their experiences of the participants.

The participants’ overall perceptual views reveal that 63% and 38% instances in which IK was considered valuable among SE and T respectively. Conversely, 36% and 64% instances of initial opposition to the new curriculum emerged among SE and T respectively. However, less than a quarter (23%) compared to 77% instances concerning poor training emerged among SE and T respectively as one of the reasons for opposing the new curriculum and so on. The themes that emerged from the participants’ responses to an item of the Reflective Questionnaire are presented in Table 1 as well as a sample of excerpts below.
Table 1. Emerging themes from the participants’ reflective diaries on item 2

<table>
<thead>
<tr>
<th>Emerging themes</th>
<th>SE</th>
<th>T</th>
<th>Tot</th>
<th>SE</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2a) What views of the newly implemented science-IK curriculum did you hold before and after being exposed to DAIM-based activities?

I was ignorant or unaware of IK before joining SIKSP. 4 1 26. 1 15 7 73.3
I looked low on IK, regarded it as unscientific before joining SIKSP. 4 9 13 8 69.2
I had knowledge of IK before joining SIKSP. 6 1 85. 7 7 14.3
Science alone shaped my worldview before I joined SIKSP. 6 1 35. 1 17 3 64.7
SIKSP has enabled me to value IK now. 4 9 30. 13 8 69.2
I now support the integration of science and IK as result of DAIM. 6 1 35. 1 17 3 64.7
DAIM is an enabling tool for integrating science and IK. 6 4 60. 10 0 40.0
Lack of training & exam-driven curriculum is still an obstacle to implementing the new science-IK curriculum. 0 5 5 0.0 0 0

2b) Were you once opposed to the implementation of the new curriculum? What specific aspects of DAIM have influenced your view over time about the new curriculum?

IK is now seen as valuable after an exposure to DAIM. 5 3 62. 8 5 37.5
I was initially opposed to the new science-IK curriculum because of the way it was implemented. 4 7 11 4 63.6
Opposed new curriculum because teachers were not trained or well equipped to implement it. 3 1 23. 0 13 1 76.9
Still opposed because of inadequate training and resources available for teachers. 2 6 8 0 75.0
I was opposed to IKS before, but now want to try it. 2 5 28. 7 6 71.4

N = 23 (SE = 11; T = 12); f = frequencies; Tot = total frequencies; SE = Science Educators; T = Teachers
Besides the numerical representations above, the emerging themes derived from the participants’ responses seem to reveal some cognitive shifts on their part relative to their perceptions of the new curriculum before and after their exposure to DAIM. The nature of these perceptual shifts becomes more visible when the statements made by the individual participants are considered. Analysis of the data using CAT shows the perceptual shifts among the participants about their valuation of an integrated science-IK curriculum.

The italicized phrases excerpts below are indicative of the nature of such perceptual shifts. The italicized phrases of excerpts below are indicative of the nature or pattern of the perceptual shifts among the participants concerning the new curriculum. For ease of reference, the shaded letters indicate probable patterns of perceptual shifts in terms of CAT categories: $D Sw$, a dominant scientific worldview; $SI Kw$, a suppressed IK worldview; $DI Kw$, a dominant IK worldview; $SSw$, a suppressed scientific worldview; $As w$, an assimilated worldview; $E w$, an emergent worldview; and $EQ w$, an equipollent worldview:

Lorna: A 49 year-old female primary science teacher with 30 years of teaching experience said: “Before the workshops I thought that Western Science is dominant ($D Sw$) over IKS… ($SI Kw$). In my view IKS was all about witchcraft…($SI Kw$). After attending the workshops I realized that IKS is not something new to me, perhaps the terminology…($E w$) These workshops are so valuable to me because it made me realize once again how precious IKS is ($E w$).”

Brenda: A 34 year-old female physical science teacher with 13 years of teaching experience said, “My experiences in the SIKSP activities [referring to DAIM] have reformed my way of thinking, doing things completely ($E w$). When I started I did not appreciate indigenous knowledge nor did I realize its richness ($SI Kw$) until I matured in these workshops ($E w$). I have now reached a level where I am confident of integrating these two worldviews harmoniously ($EQ w$).”

Diamond: A 47 year-old male science/math educator with 22 years teaching experience said, “Before being part of the IKS group I was a bit skeptical about the role which IKS can play in our everyday life…($D Sw$). Having attended the workshops and seminars I have grown to understand ($E w$) that knowledge from both IKS and modern science are all the same ($EQ w$). Further reflections have lead me to conclude that two knowledge system can actually co-exist ($EQ w$).”

Similar perceptual shifts were made by the other participants. One common theme reflected in Table 1 and the excerpts above is that the participants’ awareness about what IKS stood for increased considerably as a result of being exposed to DAIM. The experience also enhanced their self-image, and sense of sociocultural identity. Instead of being ashamed of IK and consequently their cultural heritage, they saw it as a legitimate way of knowing and interpreting experience.

Conclusion
In line with the aim of the new South African SMT curriculum to make school learning relevant to the socio-cultural environment of learners DAIM appears to have motivated the participants to want to implement the new curriculum. Many of the participants claimed that DAIM enhanced their understanding of the similarities and differences between science and IKS and to know when they are compatible or otherwise; increased their awareness about the educational and cultural values of IKS as a legitimate way of knowing and as a source of pride rather than something to be ashamed of; helped to disabuse their minds of their previously erroneous views about the new curriculum and to wanting to implement it in their
classrooms. Although I am aware of the difficulty involved in making people to revise their beliefs, as they tend to hold tenaciously to such beliefs as a plethora conceptual change has shown (Gunstone & White, 2000; Leitao, 2000), nevertheless it has been my belief and experience over the years that the beginning of that process seems to have begun among this cohort of participants. However, much still needs to be done to determine more explicitly the trajectories of these perceptual shifts and what specific factors are responsible. If the enthusiasm shown by the participants in the study is anything to go by, then it is safe to say that DAIM is worth being considered as a potential tool for integrating SMT with IK in a classroom context. This positive outcome holds promise towards attempts directed at decolonizing STM curricula and instruction in Africa and perhaps other formerly colonized countries elsewhere.

References

Magee, B. Karl Popper. New York: The Viking Pres.


Mathematics

Long Papers
The study reported in this paper investigated mathematics teachers’ interpretations of some common students’ errors in algebra. The study was motivated by the desire to understand how qualified and experienced mathematics teachers interpreted their students’ errors. Correct interpretation of students’ errors is fundamental for teachers if they are to help students deal with the errors. The research participants were forty-two in-service mathematics teachers enrolled for the Bachelor of Education degree in mathematics at one university in Zimbabwe. The research instrument was a questionnaire with examples of typical students’ errors in algebra. The teachers were asked to explain how each error could have happened. The findings indicated that the teachers’ explanations were in six categories, namely: mathematically incorrect explanations; mathematically correct or plausible explanations; mathematically imprecise explanations or blaming students; explanations or illustrations of what should have been done; explanations that linked errors to teaching; and descriptions of what the students did in making the error. These categories formed the analytical tool for analysing the teachers’ interpretations of the errors. The results showed significant numbers of teachers gave mathematically incorrect explanations; mathematically correct explanations; and mathematically imprecise explanations. The variety of explanations given by the teachers for each error suggests that interpreting errors correctly is a challenge for some mathematics teachers. Failure to interpret errors correctly constrains responding to errors in ways that support students in dealing with the errors. In view of the findings the study recommended that error analysis be incorporated in in-service teacher professional development programmes if teachers are to develop the capacity to interpret and respond to their students’ errors in productive ways.

Introduction

Students’ learning of mathematics involves making errors. Errors are common in the teaching and learning process of the subject as well as in students’ written work. Mathematics teachers respond differently to their students’ errors. Some teachers respond by ignoring errors while others make some efforts to engage with the errors. Contemporary thinking on students’ errors in mathematics proposes that teachers embrace errors rather than avoid them (Borasi, 1987; Brodie, 2011). Such thinking is based on the justification that errors in mathematics are pervasive and systematic (Nesher, 1987), and often are a result of mathematical thinking on the part of the students, and hence are reasoned and reasonable for the students (Brodie, 2011). This view on errors suggests that teachers need to respond to students’ errors in ways that involve understanding the students’ thinking behind the error as a way of accessing learner thinking, which in turn can inform teaching. Such ways of dealing with students’ errors requires that teachers shift their understanding of students’ errors; from viewing errors as obstacles to learning mathematics to an understanding of errors as integral to learning, and as possible sources of learning mathematics (Borasi, 1994). The study reported in this paper sought to establish a group of in-service mathematics teachers’ interpretation of students’ errors in mathematics. Understanding how mathematics teachers interpret students’ errors was seen as one way of accounting for the ways in which teachers’ respond to students’ errors in instructional situations.
The nature of errors in mathematics

Research on errors in mathematics highlights various pertinent issues relating to the nature of errors and teachers’ conceptions of errors. In the Data Informed Practice Improvement Project (DIPIP) errors are defined as “systematic, persistent and pervasive mistakes performed by learners across a range of contexts” (Brodie, 2012, p. 2). DIPIP is an on-going in-service mathematics teacher professional development project based at the University of the Witwatersrand in Johannesburg, South Africa (Brodie & Shalem, 2011; Brodie, Shalem, Sapire, & Manson, 2010). DIPIP works with students’ errors in mathematics as a learning focus for the participating teachers (Brodie, 2012). Errors are different from slips. Slips are mistakes that are easily corrected (A Olivier, 1996). Errors have the following characteristics, among others, they are a world-wide phenomenon and are made by students of any age, country, or ability (Gagatsis & Kyriakides, 2000); they are difficult to deal with (Brodie, 2012); they arise independently of the teaching methods used (Peng & Luo, 2009); they are persistent even when corrected (Smith, DiSessa, & Roschelle, 1993); and “they occur among learners within and across contexts – suggesting that they are systemic rather than the result of individual learner or teacher failure” (Brodie, 2012, p. 2). These characteristics suggest that errors are integral to teaching and learning mathematics, and cannot be ignored in instructional contexts. According to White (2005) it is difficult for teachers to escape from students’ errors, hence it is important for teachers to correctly interpret students’ errors before taking corrective measures.

Theoretical explanations for errors have been provided mainly from the constructivist perspective. Before the advent of constructivism errors were negatively viewed as digressions, a result of some confusion on the part of the student, and as unfortunate events that had to be eliminated and avoided at all times (Gagatsis & Kyriakides, 2000; White, 2005). From a constructivist perspective errors are explained as: results of gaps in comprehension that threaten students’ construction of knowledge and the coherent structure of mathematics (Legutko, 2008); the result of applying previously acquired and correct knowledge to mathematical situations where the knowledge is inapplicable (Gagatsis & Kyriakides, 2000; Radatz, 1979); a result of misconceptions which are “consistent conceptual frameworks based on earlier acquired knowledge” (Nesher, 1987, p. 33) and make sense to students as they make conceptual links to knowledge they acquired previously (Lourens & Molefe, 2011); and a result of prior conceptions or misconceptions that students use to interpret phenomena, events and situations in their construction of knowledge in the classroom (Erlwanger, 1973; Smith, et al., 1993). A misconception is defined as “a student conception that produces a systematic pattern of errors” (Smith, et al., 1993, p. 119). These conceptions of errors highlight the centrality of students’ conceptual structures and how these structures are deployed in learning new knowledge. As students are faced with new situations they draw on their prior knowledge or experiences to make sense of the new situations. The basic cognitive argument is that in making attempts to work with previously acquired knowledge in novel situations students make errors as students’ prior knowledge becomes inadequate for explaining phenomena and solve new problems (Smith, et al., 1993). Thus errors are seen as reasonable and sensible for students (Brodie, 2012; Lourens & Molefe, 2011). These ideas highlight the need for teachers to engage with students’ errors in ways that enable them to identify the students’ thinking or conceptions behind any observed errors. Such knowledge will enable teachers to deal with students’ errors in ways that support students in accessing the correct mathematical knowledge. This paper is based on the argument that teachers’ interpretation of students’ errors can support or constrain their ways
of engaging with errors in instructional situations. In the study we investigated a group of mathematics teachers’ interpretation of some common students’ errors.

**Literature review on teachers’ conceptions of students’ errors**

Research on teachers’ interpretation of students’ errors is an area that has received substantial attention. Research in this area can be classified into two categories. The first category relates to studies that sought to investigate teachers’ interpretation of, and reasons for common students’ errors in mathematics (e.g. Brodie, 2013a; Erlwanger, 1973; Gagatsis & Kyriakides, 2000; Hall, 2002; Jacobs, Lamb, & Philipp, 2010; Legutko, 2008; McNamara & Shaughnessy, 2011; Radatz, 1979). The second category are studies that investigated teachers’ perceptions of students’ errors (e.g. Gagatsis & Kyriakides, 2000).

The study by Gagatisis and Kyriakides (2000) established that the teachers who attended an in-service professional development programme interpreted students’ errors more in terms of the underlying mathematics rather than in terms of students’ knowledge and abilities. Generally teachers tend to interpret students’ errors in terms of student related factors, which is actually blaming students for errors. An example of a student related factor is students’ capabilities in mathematics. Limited students’ capabilities are seen as accounting for the common errors in mathematics. Knowledge factors relate to students’ knowledge of mathematics. Students’ limited knowledge of the mathematics is often given as the one of the main reasons for learners’ errors in mathematics. Few teachers see teachers’ teaching and the nature of mathematics as contributory to learners’ errors in mathematics. While the tendency by teachers to account for students’ errors by blaming students is well-known, such a way of explaining errors usually misses the potentially valid mathematical reasoning that may be behind the error. Failure to access the reasoning behind each error constrains the chances of dealing with errors in ways that support students’ learning of mathematics. Accessing the students’ reasoning behind errors is a process that requires identifying and interpreting errors correctly (Brodie, 2013a).

Legutko (2008) argues that teachers should engage with errors if they are to support students in dealing with the errors. She provides a possible framework for analysing students’ errors that includes analysing the mathematical reasoning behind errors and possible strategies for addressing the errors. Radatz (1979) analyzed possible reasons for students’ errors. He identified the following as possible causes of students’ errors: language difficulties; difficulties in interpreting spatial information; deficiency in mastering prerequisite skills, facts and concepts; incorrect associations with other mathematical ideas; and application of irrelevant rules and strategies. In her paper, Olivier (1989) presented an analysis of some students’ misconceptions in mathematics and argues that teachers should be able to: predict errors, explain how and why students make these errors, and help students to resolve any misconceptions behind the observed errors. Gagatsis and Christou (1997) found that most primary school teachers attributed errors to students’ psychological situations, limited capabilities and lack of knowledge. The ideas in the above papers indicate a need for teachers to be able to identify and interpret correctly students’ errors if they are to be in a position to help students in overcoming the errors.

From the DIPIP project Brodie (2013a) showed how teachers shifted from identifying to interpreting errors and from interpreting to engaging with errors. In another paper from the DIPIP project Chauraya (2011) analysed how a group of mathematics teachers deepened their understanding of students’ errors and their own knowledge of the mathematics content related to particular errors. Other research findings within the DIPIP project show the importance of teachers correctly interpreting their students’ errors before engaging with the errors in ways
that support students in dealing with the errors (e.g. Brodie, 2013b; Brodie & Shalem, 2011; Lourens & Molefe, 2011).

The study reported in this paper sought to find out how a group of experienced mathematics teachers interpreted some common students’ errors in mathematics. We argue that it is important for teachers to interpret errors correctly if they are to substantially help students in dealing with the errors.

Methodology

The study used a survey research design. The research participants in this study were forty-two in-service mathematics teachers enrolled for the Bachelor of Education (B. Ed.) degree in mathematics at one university in Zimbabwe in the year 2013. All the participants had teaching diplomas from the various teachers’ training colleges in Zimbabwe and all had majored in mathematics. The B. Ed. Degree is offered to teachers with teaching diplomas who want to upgrade their qualifications to degree level, majoring in particular teaching subjects. The respondents consisted of twenty-six males and sixteen females. Most of them had a post-diploma mathematics teaching experience of between four and ten years, although the maximum teaching experience was twenty-three years. Twelve of the respondents were Heads of Mathematics Departments in their schools.

Data was collected using a questionnaire adapted from the study by Gagatsis and Kyriakides (2000) with some adjustments. While Gagatsis and Kyriakides studied teachers’ attitudes towards students’ errors, the focus of our study was on teachers’ interpretation of common errors in Algebra. In the questionnaire the teachers were provided with five sample student errors in mathematics and were required to give an explanation or reasons for each error. The aim of this section of the questionnaire was to find out how the teachers interpreted the errors. This is similar to Part B of the questionnaire used by Gagatsis and Kyriakides (2000) although the examples of errors were different. In this paper we present the teachers’ interpretation of three of the five errors.

Data Analysis

In a thematic analysis of how the teachers interpreted each of the five errors, we observed that the explanations fell into six different categories as follows: mathematically incorrect explanations; mathematically correct or plausible explanations; mathematically imprecise explanations or blaming students; explaining or illustrating what should have been done; attributing errors to teaching; and descriptions of what the student did. The categories were developed into a data analysis framework by asking colleagues in mathematics education to code the data separately and then agreeing on the categories. Table 1 below summarises the framework.
Table 1. Error Analysis Framework

<table>
<thead>
<tr>
<th>Nature of Explanation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematically incorrect explanation</td>
<td>An explanation that does not match the error, or is mathematically incorrect</td>
</tr>
<tr>
<td>Mathematically correct or plausible explanation</td>
<td>A correct explanation for the error, or a partially correct explanation that explains part of the error</td>
</tr>
<tr>
<td>Mathematically imprecise explanation or blaming students</td>
<td>An explanation that lacks mathematical clarity, or blames students for the error</td>
</tr>
<tr>
<td>Explaining or illustrating what should have been done</td>
<td>An explanation of what the student should have done or a presentation of a correct solution</td>
</tr>
<tr>
<td>Attributing errors to teaching</td>
<td>An explanation that describes the error as a result of teachers’ teaching</td>
</tr>
<tr>
<td>Descriptions of what the student did</td>
<td>A description of what the student did</td>
</tr>
</tbody>
</table>

Results

The following tables summarise the reasons given by the teachers for each of the five common errors presented in the questionnaire. In each table we have included the number of teachers who made explanations in each category.

Interpretation of an error involving expansion of brackets

Table 2 below summarises the teachers’ interpretations of the error \((a + b)^2 = a^2 + b^2\).

Table 2. Reasons for a binomial expansion error

<table>
<thead>
<tr>
<th>Error</th>
<th>Reasons for the error</th>
<th>Examples of teacher interpretations</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a + b)^2 = a^2 + b^2)</td>
<td>Mathematically incorrect explanations (11)</td>
<td>e.g. ‘students confusing (a^2)-(b^2) with (a^2+b^2), ‘lack of knowledge of order of operations’, ‘poor background of indices’, ‘incomplete knowledge on factorisation’</td>
</tr>
<tr>
<td></td>
<td>Mathematically correct or plausible explanations (3)</td>
<td>e.g. ‘Over-generalising statements such as ((ab)^2 = a^2b^2) or ((a \times b)^2 = a^2 \times b^2)’</td>
</tr>
<tr>
<td></td>
<td>Mathematically imprecise explanations or blaming students (21)</td>
<td>e.g. ‘lack of knowledge of expansion of brackets or expressions’, ‘misconception of expansion’, ‘student has a problem in removing brackets on quadratic expressions’, ‘violation of mathematical rules’, ‘pupil is lazy to follow the expansion procedures’, ‘failure to apply the distributive law’</td>
</tr>
<tr>
<td></td>
<td>Explaining or illustrating what should have been done (2)</td>
<td>e.g. ‘((a+b)^2 = a(a+b) +b(a+b) = a^2+ab+ab+b^2)’ or ‘((a+b)) needs to be multiplied by itself twice’</td>
</tr>
<tr>
<td></td>
<td>Result of a teaching problem (1)</td>
<td>e.g. ‘teacher did not emphasize on expansion problems of this type’</td>
</tr>
</tbody>
</table>
Describing what the student did or thinks

| (4) | e.g. ‘student squared a and b separately without expanding the bracket by multiplying the given expression twice’ or ‘students think that each unknown in the brackets has to be raised to the power which is outside the brackets’ |

The error \((a + b)^2 = a^2 + b^2\) is common among students in multiplying algebraic expressions that involve brackets. The error is usually associated with over-generalisation or simplification of expressions such as \((ab)^2 = a^2b^2\). Only three teachers gave correct interpretations of the error. The majority of the teachers (21) blamed students for the error without precisely articulating the mathematics that the students could have been working with. For example the explanation ‘lack of knowledge of brackets’ does not specify what mathematical knowledge the students lacked. Blaming students for errors without articulating the mathematical knowledge that the students require in dealing with the error does not help teachers in supporting students to deal with the error. Such blaming of students for errors constrains engagement with the errors in productive ways.

The next most common explanations (11) were mathematically incorrect explanations. Although in these explanations teachers made reference to some mathematical ideas, there was no or very little connection between the explanation and the error. Explanations such as students had ‘incomplete knowledge on factorisation’ or ‘lack of knowledge of order of operations’ are not linked to the error. Such explanations do not help teachers in understanding the students’ reasoning in making the error and they constrain the teachers’ choice of strategies for helping the students.

Four teachers gave explanations that were descriptions of what the students did or thought in making the error. The descriptions did not specify what was wrong in what the student did; therefore do not help in providing information that can be useful in taking corrective measures.

Two teachers explained or illustrated what the students should have done. This is a common strategy used by teachers in dealing with students’ errors. The limitation of such explanations is that they do not address what the student did wrongly and therefore in a classroom situation the students are left in the dark as to what is wrong in their answers, and may continue to make similar errors in future. Only one teacher associated the error with teaching by blaming teachers for the error. The explanation however was vague as to what the teacher did or should have done.

Overall the teachers’ interpretations of the error were largely incorrect, mathematically vague or blaming students for the error. Such ways of interpreting students’ errors show limited understanding of the error and therefore constrain productive ways of dealing with the error.

**Interpretation of error involving conjoining**

Table 3 below summarises the teachers’ interpretations of the error: \(x + y = xy\). The error is common in mathematics and is usually explained as the tendency by students to conjoin, and/or to associate the addition sign with the need to get an answer that is a single term. The error may also be explained as generalising addition of numbers to addition of algebraic variables.

The table shows that the majority of the teachers (28) gave mathematically correct or plausible explanations. The interpretations revolved around students’ mixing addition with
multiplication, or students failing to differentiate between like and unlike terms. Such interpretations of errors can inform teachers on how to help students in dealing with such errors.

The next most popular category of explanations was that of imprecise explanations or blaming students (10). The explanations highlight a deficit in knowledge by students, without specifying the correct mathematical knowledge required in simplifying the expression. Associating errors with a deficit in students’ knowledge is common among teachers’ interpretation of errors.

**Table 3. Reasons for a conjoining error**

<table>
<thead>
<tr>
<th>Error</th>
<th>Reasons for the error</th>
<th>Examples of teacher interpretations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x + y = xy)</td>
<td>Mathematically incorrect explanations (1)</td>
<td>e.g. ‘Lack of knowledge in multiplication of terms’,</td>
</tr>
<tr>
<td></td>
<td>Mathematically correct or plausible explanations (28)</td>
<td>e.g. ‘just as the addition of say 1+2=3 which is a single answer, pupil seeks to get single answer by eliminating the plus’, ‘failure to identify like and unlike terms’, ‘confusing addition with multiplication’, ‘failure to note the difference between addition of numbers and that of letters’, wrong generalisation of laws of indices (a^x \times a^y = a^{x+y}), ‘wrong generalisation of law of logarithms which states (\log M + \log N = \log MN)’, ‘pupil failed to understand terms that cannot be added together e.g. (g+p), but they can be multiplied to give a term (g \times p = gp) but (g + p \neq gp).’</td>
</tr>
<tr>
<td></td>
<td>Mathematically imprecise explanations or blaming students (10)</td>
<td>e.g. ‘lack of mastery of the algebraic processes’, ‘failure to apply knowledge on addition of symbolic terms’, ‘failing to interpret the meaning of basic operations and their applicability to algebra’, ‘confusing concepts’, ‘child might have not the concept of operations of signs’, ‘pupil failed to identify the concept of operation’, ‘pupil failing to perform the operation of addition in algebra’, ‘pupil cannot apply concepts taught on addition of unlike terms’, ‘failure to understand concept of addition and multiplication of algebraic terms’,</td>
</tr>
<tr>
<td></td>
<td>Explaining or illustrating what should have been done (1)</td>
<td>e.g. ‘(x \text{ and } y) are two different terms added together and these cannot be simplified further’</td>
</tr>
<tr>
<td></td>
<td>Result of a teaching problem (0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Describing what the student did or thinks (2)</td>
<td>e.g. ‘student multiplied (x) and (y) not knowing that the two cannot be combined when adding them since they are different’, ‘student multiplied the two unknowns instead of adding’</td>
</tr>
</tbody>
</table>

Only one teacher gave a mathematically incorrect explanation. One other teacher gave an
explanation of what students should have done. Two teachers described what students did or thought in making the error. Conjoining is a common students’ error in algebra and it is important for teachers to identify and correctly interpret errors that occur through conjoining. A large scale study carried out in the United Kingdom found out that thirty one percent of the students gave an answer of $7n$ to the task $3n+4$ (Herscovics & Linchevski, 1994). In this study we found that the majority of teachers gave correct or partially correct interpretations of the error. However a significant number of the teachers gave mathematically imprecise explanations or blamed students for the error.

**Interpretations of an error in solving a quadratic equation**

Table 4. Reasons for quadratic solution error

<table>
<thead>
<tr>
<th>Error</th>
<th>Reasons for the error</th>
<th>Examples of teacher interpretations</th>
</tr>
</thead>
</table>
| $2x^2 - 3x + 1 = 5$  
$(2x - 1)(x - 1) = 5$  
$2x - 1 = 5$  
$x - 1 = 5$  
$x = 3$ or $6$ | Mathematically incorrect explanations (7) | e.g. ‘student confused the use of the quadratic formula and factorisation method of solving quadratic equations’, ‘failure to understand the meaning of a bracket and using the wrong concept’, ‘student does not know factorisation method’, ‘student forgot to exclude 5 since 5 has not been moved to the other side when factorising’, ‘student is able to factorise but the error is on simplification of equations on 2x-1’, ‘student failed to find the factors so that the result will mean two factors should replace the unlike term and when multiplied obtain the last term after multiplying by two’,  |
| | Mathematically correct or plausible explanations (6) | e.g. ‘child did not realise that quadratic equations can only be solved by factorisation when they are equated to zero’, ‘student is not aware of the standard form of the quadratic equation which is $ax^2+bx+c=0$’, ‘generalisation of solving quadratic equations in factorised form, i.e. $(A)(B)=0$ where the pupil is used to say either $A=0$ or $B=0$’,  |
| | Mathematically imprecise explanations or blaming students (18) | e.g. ‘failure to rearrange the equation’, ‘student lacks the knowledge that factorisation can only be done when the RHS=0’, ‘failure to understand the method of solving equations’, ‘equating the two numbers after factorising to any given number to the other side of the equal sign’, ‘lack of techniques in solving equations especially quadratic equations’, ‘misunderstanding of quadratic expressions’, ‘lack of knowledge’ |
| | Explaining or illustrating what should have been done (4) | e.g. ‘should have first equated the quadratic equation by creating a 0 on one side and then factorize’, ‘pupil has to first collect like terms, i.e. for 1 and 5 that is when 5 crosses the equal line it changes its sign to -5 hence $2x^2-3x-4=0$ becomes the required equation’, ‘failed to equate the expression to zero e.g. $2x^2-3x-4=0$, then look for the product of 2 and -4 whose sum is -3 which are to be used to rewrite -3x in order to factorise accordingly’, ‘a quadratic equation is of the form $ax^2+bx+c$ and should be equated to zero before factorising, i.e. $2x^2-3x+1-5=0$’,  |
| | Result of a teaching problem (3) | e.g. ‘the teacher’s problem on the part of setting a quadratic equation with a 5 on the other side instead of zero which maybe the pupils are used to’, ‘a result of the
teacher giving students equations which are equal to zero, so the student generalised, ‘the teacher may not have emphasised the need to have all terms in a quadratic equation on one side before factorisation’

Table 4 above summarises the teachers’ interpretations of a common students’ error in solving quadratic equations. Two teachers did not give any explanation for the error. The error can be explained as a generalisation of solving quadratic equations by factorisation. It arises from failure to realise that solving quadratic equations by factorisation is based on an application of one of the properties of the number zero (0), i.e. that for any two real numbers A and B, AxB=0 means that either A=0 or B=0. Without an adequate understanding of this principle, students are likely to make errors like the one in Table 4.

In their interpretations of the error, none of the teachers connected the error to a property of the number zero. The majority of the teachers (18) gave mathematically imprecise explanations. Explanations like ‘student lacks the knowledge that factorisation can only be done when the RHS=0’ and ‘failure to rearrange the equation’ do not include reasons why the right hand side (RHS) of the equation has to be zero ‘0’. The explanations in this category show an algorithmic understanding of what was to be done.

The next most popular category of explanations was that of mathematically incorrect interpretations (7). These interpretations are not connected to the error; hence do not explain the error. Explanations that are not linked to the error may indicate that the teachers did not understand the error.

Six teachers gave interpretations that were correct or partially correct. These explanations show that the teachers interpreted the error in terms of the algorithmic knowledge required to complete the task. The explanations underscore the need for the right hand side of the equation to be zero before factorisation could be done, without saying why the right hand side has to be zero. An interesting explanation in this category was one that referred to ‘... standard form of the quadratic equation which is ax^2+bx+c=0’. In teaching quadratic equations teachers normally refer to this form of a quadratic equation without explaining why the equation has to be in this form.

Explanations in the category of illustrating what was to be done (4) also show a focus on the steps that needed to be followed in order to answer the task. Three teachers attributed the error to teaching. One teacher in this category suggested that there was a problem with the task since the right hand side is not equal to zero. Another teacher described the error as a result of generalising from how teachers teach quadratic equations. Normally students are introduced to quadratic equations through equations that have zero on the right hand side, without explaining why the right hand side has to be zero.

Students therefore develop the misconception that whatever number is on the right hand side, factorisation of the left hand side can be done.

The last category of descriptions of what the student did or was thinking shows again an algorithmic understanding of what needed to be done. Two teachers gave explanations in this category.
Discussion and Conclusion

In this study we sought to find out how a group of in-service mathematics teachers interpreted some common errors made by students in algebra. It was our view that for teachers to be able to respond productively to students’ errors in instruction, they need to be able to interpret each error correctly. According to Jacobs et al. (2010) teachers should be able to notice and interpret students’ errors before they respond. This process sometimes happens in a moment during instruction. Such in the moment decisions can either be productive in helping the students or may be unproductive if the teacher fails to correctly interpret the errors. Thus mathematics teachers need to be able to interpret students’ errors correctly if they are to respond in ways that help students in dealing with the error. Research shows that engaging teachers in professional development programmes helps teachers in making correct interpretations of their students’ errors (e.g. Brodie, 2013a; Gagatsis & Kyriakides, 2000; Jacobs, et al., 2010). The findings from these studies show that development of the capacity to notice and interpret students’ errors correctly is not easy.

The teachers’ interpretations of the first error in this study, \((a + b)^2 = a^2 + b^2\), show that the majority of the teachers could not interpret the error correctly and precisely. Only three teachers gave correct interpretations of the error. The error is common and is explained in terms of students’ generalisation of mathematical knowledge that is valid in some areas of mathematics, but not valid in this context. Linking the error to other domains of mathematics helps in identifying generalisations and connections that students make in learning mathematics, some of which may be erroneous. Teachers who can interpret such errors correctly are better positioned to help their students in dealing with the error. The majority of teachers gave mathematically imprecise interpretations or blamed students. Imprecise interpretations indicate that the teachers were not able to fully understand the mathematical thinking that led to the error. Blaming students without articulating the possible causes of the error is also unproductive as it shows that the teachers noticed the error but were not able to interpret the mathematics that the students were working with in making the error. A significant number of teachers gave mathematically incorrect interpretations. Such interpretations of students’ errors are unproductive since the teachers may end up working with mathematical ideas that do not address the error in their efforts to help students overcome the error.

The error ‘\(x + y = xy\)’ was interpreted correctly by the majority of teachers. The error is common in students’ learning of algebra and as most teachers correctly interpreted, the error arises from working with other systems of mathematics in a context in which they do not apply. However it is worrying that a significant number of teachers (10) gave mathematically imprecise explanations that did not articulate how the error could have occurred. A number of explanations in this category blamed students for the error. Interpretations of students’ errors that are focused on a knowledge deficit on the part of students miss the basic fact that in making errors students are working with some mathematical knowledge that is valid according to their thinking. Errors are reasoned and reasonable for students (Brodie, 2013a). Teachers therefore need to understand the students’ mathematical reasoning in making the error without blaming the students for the error.

The error in solving a quadratic equation was difficult for the teachers to interpret. Only six teachers gave correct or partially correct interpretations. Other interpretations that were given by the teachers emphasised the algorithms for solving quadratic equations by factorisation or by using the quadratic formula. What was evident in the teachers’ explanations was that the teachers seemed to view the two methods as not connected. The quadratic formula arises
from solving quadratic equations by completing the square, which is factorisation. This connection was not evident in the teachers’ explanations, something that may indicate algorithmic understanding of the topic without understanding the underlying connections among the different ways of solving quadratic equations. Without an understanding of the underlying connections among various algorithms for completing mathematics tasks, teachers may find it difficult to respond to students’ errors in ways that address the actual error.

A worrying observation in this study was the numbers of teachers who gave mathematically incorrect interpretations across all the three errors. The teachers’ explanations were not linked to the errors. If teachers cannot interpret their students’ errors correctly, then they are at a disadvantage when making decisions on how to help students deal with the error. Such situations may mean that students’ errors remain unaddressed. The incorrect interpretations may also be an indication that interpreting students’ errors correctly is not easy, and teachers who fail to make correct interpretations of errors may teach in ways that avoid errors rather than engaging with errors.

The findings in this study raise some implications for error analysis by teachers. While generally it is important for teachers to engage with errors rather than avoid errors, this may not be realised if mathematics teachers do not develop the capacity to interpret their students’ errors correctly. We suggest that mathematics teacher in-service programmes need to incorporate capacitating teachers in correctly interpreting their students’ errors. Errors always arise in teaching contexts and it is vital that teachers be able to interpret those errors correctly if they are to make instructional decisions that help students overcome the errors.

References


In this paper I draw on a research project conducted to describe and discuss the arithmetical discourse profile of both mathematics teacher educators and student teachers in college mathematics classrooms in Malawi. I focus particularly on what type of arithmetical discourse profile is produced in a college mathematics classroom and what could be the implications of such a discourse on the development of the arithmetical discourse of the school learners. I use principles of Arithmetical discourse Profile (Ben-Yehuda, Lavy, Linchevski, & Sfard, A. (2005) in order to identify and describe the arithmetical discourse profile produced in college mathematics classrooms. An analysis of the data based on lesson observations suggests that the arithmetical discourse profile produced in college mathematics classroom is never objectified. The student teachers are not given space to be flexible and make transitions from one mode to another. Through my descriptions and discussion, I argue that while literate discourse was used by both the mathematics teacher educators and the student teachers, there is no wider applicability of the literate procedures that were displayed which means that the discourse produced in college mathematics classrooms is strictly defined set of rules that cannot be changed even at a personal level.

Introduction: Mathematics as a discourse

In this study mathematics is seen as a discourse, where discourse refers to a “specific type of communication” (Sfard, 2012, p.1). Mathematics has its own registrar (Halliday, 1978). Apart from that Sfard (2012) indicates that Mathematics has its own “special keywords …., used in distinctly mathematical ways; has unique visual mediators; has distinctive routines, that is, patterned ways in which mathematical tasks are being performed; and has generally endorsed narratives.....”p 2., Because of this specific way of communication, mathematics is referred to as a discourse.

The objective of learning in a mathematics classroom is to help learners develop the mathematical discourse so that they can participate properly in mathematics community (community of all competent participants of mathematical discourse, (Sfard, 2012. p.1). The development of mathematical discourse can be presented as a succession of several different forms of mathematical discourses, since within mathematics there are also other discourses such as a discourse of algebra, or of geometry (Sfard, 2012). Kim, Ferrini-Mundy, & Sfard (2012) explains that the development of mathematics is a process in which mathematical discourse builds its own successive meta-discourses. Now with different discourses within mathematical discourse, it implies that mathematical discourse consists of “hierarchies of increasingly complex, increasingly reified, and possibly mutually incommensurable discourses” (Kim et al 2012, p.87).

As indicated above, Sfard (2012) categorizes the mathematical discourse by mainly three objects which are; special words (vocabulary) such as triangle; visual mediators such as symbols and diagram and general endorsed narratives. Mathematics discourse has its own specialized register (Haliday, 1978) and has its own thematic patterns (Lemke 1990). As a result, its exposure will have to be different to the teaching of all the other academic subjects.
Teachers will have to pay specific attention to different categories of the mathematical discourse explicitly rather than implicitly. How do mathematics teachers expose the mathematical discourse to students? Is it done explicitly or implicitly? In addition, in schools, the main objective is to help learners produce endorsed narratives at the end of the day. Therefore when we talk of learning, we are talking of bringing the learner closer to the established form of discourse.

**Implications in teacher training institutions**

The discussion above has its own implications for mathematics teacher educators in teacher training institutions. Firstly how do teachers help to bring the learner closer to the established mathematical discourse in a proper way that would help the learners to become experts as well? Where do student teachers learn this discourse and how it should be presented? One area where they learn the discourse and how it should be presented is in a college mathematics classroom through their teacher educators. Which implies that the type of mathematical discourse that teacher educators produce and how it is presented to the student teachers is very crucial. Student teachers in a college mathematics classroom learn and develop familiarity and confidence with the mathematical discourse that is required for school mathematics teaching. In a teacher training institution, how does this type of discourse come out considering that the teacher training institutions focus on two major issues, mathematics discourse itself and the mathematics teaching discourse?

As the teacher training institutions focus on the two major issues, how do the teachers educators balance the complexity of the mathematical discourse and the discourse for teaching of mathematical discourse. Teaching mathematics discourse involves teachers to learn how to scaffold the hierarchies of the complex discourse of mathematics. The teacher educators have a huge job of displaying the mathematical discourse that is acceptable in the discourse of teaching and learning. In other words the complex mathematical discourse gets mixed with the teaching discourse which is intertwined together as they teach. To what extent do teacher educators display the mathematical discourse in the how to teach discourse? In addition to that, the mathematical discourse to be produced need not be compromised, what type of mathematical discourse comes out and how is it projected as they display the discourse for teaching mathematics?

Therefore this study considers the following questions: In college mathematics classroom, as both teacher educators and student teachers are involved in the discursive routines of mathematical discourse how much recognition is given to the type of discourse produced? Does it matter in a college mathematics classroom how the mathematical discourse is produced, including the number words, the mediation aids, the routines and the endorsed narratives? How does this impact on the literate mathematical discourse of the student teachers in a college mathematics classroom?

Clackson (1991) argues that mastery of any academic subject depends on critically mastering its language (in this case discourse). The mathematical discourse is more than just a special vocabulary. Mathematical discourse as pointed out above is categorized by its specialized words, mediators, routines and the endorsed narratives (Sfard, 2012). As the student teachers are learning how to teach mathematics, it means they are learning how to master the interconnected use of the particular categories of the mathematical discourse, which includes their semantic relations of meanings. In a college mathematics classroom, student teachers also have to learn by listening to, practicing the discourse of how to teach the mathematical discourse. They must learn to talk, write and reason according to the accepted way of literate mathematical discourse. How much of these are the student teachers exposed to in a college
mathematics classroom? Further, what type of mathematical words, mediators, routines and the endorsed narratives are produced in a college mathematics classroom as they listen, read, speak and write? To what end are the student’s teachers involved and how does this affect the development of the literate mathematical discourse.

In this paper, attention will be given to the lively use of mathematical discourse, ie when teachers speak to the student teachers, when student teachers speak to each other, and when students speak with their teacher. What type of discourse and how does a teacher model the mathematical discourse.

The Study
The sample in this study included two teacher training colleges in Malawi, one from the central region and the other from the southern region, both of which are multilingual colleges. Four mathematics teacher educators, two from each college, were selected purposefully (Patton, 1990) based on the following criteria: Every mathematics teacher educator had to have tertiary mathematics qualification to ensure that they have at least a high level mathematics qualification; Each teacher had to have at least three years of teaching experience at college level and therefore was well experienced, which ruled out the possibility that their discourse practices might be due to lack of teaching experience; They were also selected on the basis of their willingness to participate in the study.

Theoretical Framework
Ben-Yehunda et al (2005) and Sfard (2012) categorizes mathematical discourse into two, namely Colloquial and Literate mathematical discourses. Colloquial mathematical discourse is defined as the everyday or spontaneous discourse. This type of discourse is normally used by people who are not experts in mathematics such as the tailors, vendors and many more. However, in a classroom context, this discourse is also used by both the teachers and students especially when the students have not yet mastered the mathematical discourse. On the other hand literate mathematical discourse is a scholarly discourse. Ben-Yehunda et al (2005) asserts that the literate mathematical discourses are the objective of school mathematics learning. This implies that developing the mathematical discourse and be able to use it appropriately gives students access to the mathematics community. Thus mastering the mathematical discourse is gaining access. With this view, therefore, it is very important for student teachers then to develop the discourse.

This literate mathematical discourse has special features that can be used to identify it. These features include (i) use of words that count as mathematical; (ii) use of mathematical visual mediators; (iii) special discursive routines; and (iv) endorsed narratives produced throughout the discursive activity. Ben-Yehunda et al (2005) argues that within literate mathematical discourse, its discursive routines (such as well-defined types of requests, questions and tasks) are particularly strict and rigorous. These routines are invisible to the participants of the discourse.

In a college mathematics classroom, the mathematics teacher educators are regarded as experts in the field. Therefore they are the ones who are supposed to endorse the narratives as presented by the student teachers. This article uses Ben-Yehunda et al (2005) and Sfard (2012) arithmetical discourse profile framework in order to analyze the mathematical discourse being produced and projected in a college mathematics classroom.

Stages of analysis as used in this paper
The underlying principles in Ben-Yehuda’s theory of Arithmetical Discourse Profile (ADP)
are its descriptive and explanatory approaches towards the mathematical discourse. Based upon these principles Ben Yehuda et al produces a two dimensional approach to ADP namely Subject (author) dimension and Object dimension. As presented in figure 1.

**Figure 1.** The structure of the Arithmetical Discourse Profile (ADP) (Ben Yehuda, p190)

The focus of this study is on the object dimension where it explains about the type of mathematical discourse that student teachers produce in a college mathematics classroom. Therefore below I present the framework that focuses on the object dimension.

The Object Dimension of the arithmetical discourse analysis considers the object of mathematical discourse and arithmetical communication in action. This is described briefly below as used in this study.

(i) *mathematical words used*

One of the characteristics of the mathematical discourse is the specialized or mathematical words that are used. According to the framework of Ben-Yehuda there are 3 areas concerning the mathematical words, including: how the number words are being produced, operation words used and production of the words that announce the results. Ben-Yehuda et al (2005) argues that these arithmetical words can be produced and projected differently. Depending on how these words are produced and projected, the discourse can be described as objectified or not. Objectified implies that “number words are being produced as if of themselves” (p. 197), and if they are not objectified implies that “the number words produced are always the outcome of a formerly implemented calculation or a straightforward vocal counterpart of a numeral” (p. 197). When the discourse is objectified, the words for operations are predominantly structural and impersonal, on the other hand if it is not objectified the words of operations are operational and personal. Structural type of discourse implies that the presentation of the discourse is read as describing the structure of the composite number, while operational, it means that for each number word produced there is a performer which goes hand in hand with personalization. The third part of the analysis concerns the expressions for the results of operations. With the objectified type of mathematical discourse, the words expressing the results is structural impersonal otherwise it is operational.

(ii) *visual mediators*

Since in most cases mathematical is abstract, there is the use of certain visual means that act
as mediators between what is being said and the people around. The most common visual mediators in a literate mathematical discourse is the use of arithmetical symbols (p. 200). Other possible mediators include the concrete objects and the icons. There is no specific format of how these mediators can be used, it depends on how the participant to use which type of mediation so that he/she is able to communicate mathematically. Therefore the way the participants will use the mediators will differ significantly. The way one uses the mediators as he/she mentions the mathematical words as described above reveals whether the discourse is objectified or not. For example Ben-Yehuda explains that the use of written symbols or numerical symbols goes hand in hand with the objectified type of discourse.

(iii) Routines

According to Ben-Yehuda et al the word routine refers to a set of meta-rules that specify both when and how repetitive discursive action is employed (p. 203). This study only considers that how of the routine as used by the participants. The how of the routine pays attention to the student’s ability to perform a given routine procedure in the specific situation in which the use of the procedure is being tested (p.204). In this type of analysis the performances can be classified as (i) flexible, where the participant is able to use more than one routine response (p.205); (ii) Corrigible which focuses on how the student retraces in search of possible mistakes, how he/she switches the mediation in order to correct the mistake and (iii) proficient in each meditational mode separately.

(iv) Endorsed narratives

Ben-Yehuda et al explains that endorsed narrative is an explorative activity which is the gist of literate arithmetical discourse. Three types of explorative actions that complement each other; (i) Derivations which are discursive procedures resulting in new narratives; (ii) substantiations actions through which the expert in the classroom decides to endorse previously constructed narratives and (iii) memorizations, which are process that enable the students to remember and recall narratives whenever necessary.

This discussion is presented in the analytical framework given in figure 2.
Figure 2. The structure of the Arithmetical Discourse Profile (ADP) as used in this study

Findings
The findings in this article are presented in four areas namely: Words Used; Mediators Used; Routines and Endorsed Narratives

Words used
Using the analytical framework given in figure 2, the words used were analysed by considering three areas, number words, operation words and words expressing the results as presented below.

Number words used
Two observations are made regarding the use of number words. Firstly, the number words are being used as they are given or as a result of an implemented action. For example both the teacher and student teachers produced number words which are straightforward numeral that is given or the outcome of an implemented calculation. These types of number words being produced were observed from both the teachers and student teachers talk as they explain the tasks that they were supposed to do. Given below are three extract that shows this type of words.

In the first extract the teacher was explaining how to multiply 0.248 by 100.

Extract 1
L: now let us try to multiply the very same number by a hundred (writes 0.248) then times, one hundred what is the answer
[......]
L: ok, twenty four point eh, two four eight zero zero, then two decimal places (24800.), one two, three which will give us twenty four point!

Ss: eight

L: now how many decimal places as a decimal point shifted from here going to that side

Ss: two

L: two, but we have multiplied by ten here raised to the power what!

Ss: two

L: (writes ten squared in place of 100), raised to the power two, which is hundred, now how can you conclude ah the process, how can you conclude this strategy, (silence), yes (pointing to a student)

[.........]

L: whole numbers, so even if you, you don’t write the place value, no problem you are going to multiply these numbers as whole numbers, as if whole numbers, after that that’s when you consider the number of decimals, then we shift decimal place ah from the decimal point from the right hand side to the left hand side according to the number of decimals, so even if you don’t write ah the place value chart, ah no problem, aha yes

[.....]

L: but its good here to indicate, its good here to indicate, but when multiplying that’s when you ignore the decimal points after that then you place the decimal point (pause).

The second extract is where the student teacher was solving the problem of division on a chalkboard.

Extract 2

L: alright now let us come to division (rubbing the board, writind division of decimals), division of decimals, division of decimals (writing 51.45 / 4.9), ok lets say we have fifty one point four five divide by four point nine, ... solve the problem, ah is it difficult

Ss: no

S12: (student going to from, receiveis chalk from the lecturer)for the first time we have to remove our decimal points, here to remove the decimal point we have to multiply (writing 51.45), here we have two digits, it means we are going to multiply by a hundred and we do the same with this (writing 51.45 * 100 / 4.9 * 100) so here we get five one four five by four nine, we are remaining with a zero, we put a zero (writing 5145/490), now, we have the long division, now we are dealing with these numbers as whole numbers, now we can divide four hundred and ninty into five hundred and fourteen

Ss: one

S12: one (writes on top of four), now, one times zero, zero (writes), one times nine, nine (writes), one times four (writes four), then we subtract four minus zero

Ss: zero

S12: zero (writes), here we take one from five and here have eleven, eleven minus nine
Ss: two
S12: it will be two then we drop five, now four hundred and ninty into two hundred and forty five
Ss: zero
S12: so here we write zero then point, now we have come to our ... now we have introduced a decimal point, just because it fails, then here we put what, zero then four hundred and ninty into two thousand four hundred and fifty, its how many times
Ss: five
S12: five, so we multiply five times zero, zero, five times nine
Ss: forty five
S12: forty five (writes five only), five times four twenty, plus four twenty four, so its we subtract here the answer is zero, now our answer is ten point five (writes)
L: thank you, is he right

From the first extract, we see the teacher indicating “let us try to” ..... “which will give us” ..... “you are going to multiply” ..... “when you consider the number” ..... “then we shift decimal place ah from” ..... “when you ignore” ..... “then you place the decimal point”. Similarly, in extract two the student teacher uses the number words either as they are or the outcome of an implemented action. For example the words like “we have to remove our decimal points” ..... “here to remove the decimal point we have to multiply”, ” here we have two digits, ..... we are going to multiply ..... we do the same with this ......... here we get five one four five by four nine,........we are remaining with a zero, we put a zero ........, we have the long division, ..... now we are dealing with these numbers as whole numbers, ...... now we can divide four hundred ...... then we subtract four minus zero......here we take one from five ........ then we drop five, ...... we write zero ......, now we have come to our ... now we have introduced a decimal point, ....... we put what, zero ........ we multiply five times ........ In these two extracts there are no indications where both the teacher and the student teachers tried to use number words that did not correspond directly to the numerals given. This is just an example of the extract that shows these words, I have attached the appendix where this is shown throughout the lesson and even in the other lessons of different teachers. Another example is in Mr Luhanga’s class where the lecture was explaining how to teach addition of mixed numbers with different denominators asking the student teachers to change the mixed number fraction two four fifth .......as in extract 3

Extract 3
L: ..... first thing that we are supposed to do in trying to teach the addition of mixed numbers with different denominators is to add the whole numbers, when we add the whole numbers, we will have our seven (6 + 1), is that right,
Ss: yes
L: next we must add the fractions and these are proper fractions with different denominators, therefore there is need for us to use the idea of equivalent fractions, we must come up with equivalent fractions of what has been given (writing on the board), so you need to identify equivalent fractions of four over five and half (writing what he says simultaneously), now what are the equivalent fractions to those two and explain how you get them, yes
S1: **we must multiply** the denominators by of each fraction by the denominator of the other

L: is he correct

S: no

L: yes

S2: **we have to multiply** the first fraction both numerator and denominator by two then second fraction to be multiplied by both denominator and numerator by five.

L: ok the first fraction which is four fifth, multiply both the numerator and the denominator by two, **this fraction we also multiply** the numerator as well as the denominator by five, so we have four by five times two, times two, **to get eight over ten** plus five down ten (writing on the chalk board). This time around we have the same denominators therefore **just have to add the numerators which will give us thirteen down what**

Ss: ten

L: thirteen over ten and that five the mixed number will be one, three over ten, so **this must be further added** to what has been found earlier upon and therefore **we end up with** that is adding the two answers (writing), **we are going to have** seven plus one and three tenth **to get eight three tenth** (writing), eight and three tenth, that is addition of mixed numbers using different denominators apart from this

This extract also shows that the number words produced are always either the outcome of a formerly implemented calculation or a numeral that has been given.

The second notable observation is that, the number words were produced as if they are physical entities that could be moved (Ben-Yehuda 2012). For example in extract two, there are words like “…we have to remove…”, “…we have to multiply …”, “…we do the same with…” “… then we drop five …”, “…we’ve introduced decimal point…” “ … we put….” and many more. This according to Ben-Yehuda et al (2012) treated the number words as symbols (p. 197) and not as if they were mere names of independently existing entities. Thus the arithmetical talk being produced here is never objectified in the college mathematics classroom.

**Words of operations**

From the three extracts above, it is evident that the operational words being produced/used are mainly operation and personal rather than structural and impersonal. For example, in extract one usually says “….we multiply …”, in extract two “… we remove…” “… we drop…”. According to Ben-Yehuda et al, the word operation here means the operation in terms of somebody’s action and because in an operation utterance must have a performer, and so the utterances are personalized. The other presentation of structural refers to a presentation that can be read as describing the structure of a composite number, where there is no need for performing. In an objectified discourse, the discourse produced is structural and impersonal. This confirms what has already been said above that the discourse is never objectified.

**Expressions for the results of operations**

The common expression for the results used include “… which will give us …”, “…. which is …”, “…. to make …”, “…. the answer is …” and “…. we get …”. The use of such words support the claim given above that the discourse is never objectified
*What does this situation tell us about the discourse produced in a college mathematics classroom?*

It has been observed that there is no degree of objectification in the discourse being produced. The discourse is highly personalized especially on the operations. Secondly, they both focus on what they did or have done with the numbers and not what the numbers produced as if of themselves. This can be because of a number of reasons. Firstly, the students are all learning in a language which is not their first language. Secondly, it might be the nature of the problem that they have been given or the way they have been prompted by the environment or the textbooks being used. However, it has been noted that the phenomenon has been repeating itself throughout the lessons. Thus numbers are not spoken in the object-level. As a result the discourse is framed as a story of their actions and not a direct report on the operations of numbers.

In such type of a discourse, Ben-Yehuda et al points out that the numbers do not have the permanence of extra discursive objects. It implies that the discourse being produced of number words and symbols seem to be functioning as temporary symbolic entities, the existence of which was restricted to the highly personal computations processes of which they were a part. Then one wonders, what is the implication of such a discourse in teachers training colleges? This tells us of the overall quality and effectiveness of the discourse that student teachers are exposed to in teachers training colleges. However, this is not to say, when they start teaching they will display such type of a discourse, however, one can speculate of what type of arithmetical discourse would these teachers use when they start teaching in primary schools? In Malawi there is a lot of complaints regarding the performance of students in mathematics, could this be one of the reasons? Ball’s 2002 study shows that elimination of any reference to human agency from numerical discourse may be indicative of deeper changes in the discourse that eventually would lead to improvement in performance. In addition, literature indicates that the degree of objectification of a discourse correlates with one’s success as a participant in the discourse.

**Use of mediators**

The analysis of this study shows that throughout the lessons, both the teacher and the student teachers were mediating their computations with numerical symbols. For example as the teacher or the student teacher explains the procedure for multiplying two numbers or dividing two numbers, whatever expression or mathematical statement was said it would be followed by the numerical symbols being written on the chalk board. Literature shows that arithmetic symbols are a defining feature of literate arithmetic discourse. The analysis shows that both the teachers and student teachers preferred a single type of mediation. As they were solving the symbols were replaced by other symbols in a defined way. This manner is called symbolic syntactic (Ben-Yehuda et al (2012) p200). The author explains that this type of mediation does not require one to know the names and order of the digits and the rules for replacing digits with other digits. Since the analysis shows that there is no change of the computational order, it shows that there is no flexibility in terms of use of symbolic mediators.

**So what of the use of mediators**

The discourse focused on is not diverse. This syntactic mode of mediation is another symbol that the discourse is not objectified in the discourse.
Routines

Ben-Yehuda et al defines routine as a set of meta-rules that specify both when and how repetitive discursive action is employed. The how of the routine is the routine discursive procedure, that is, the set of rules that determine or constrain the participants’ discursive performance (p. 203). This shows that syntactic computations are implementations of deterministic procedures.

The analysis of this paper focused on the how, where the researcher looked at the participants’ ability to perform a given routine procedure in the specific situation in which the use of the procedure is being tested. This will be looked at from two categories.

Flexibility

This flexibility has also two categories, that is (i) whether a presentation of a certain procedure has been presented in more than one routine response and (ii) a person arriving at a numerical equivalence not only by a direct computation but also by derivation from another endorsed narrative.

In the lessons observed, there is no meditational flexibility; no transition was made from one meditational mode to another. They only used one meditational mode of symbolic syntactic. Of course there was no prompt from the teachers to do this. This still shows that the discourse produced is entirely devoid of objectification

Corrigibility

Using Ben-Yehuda et al framework, corrigibility is defined in three ways. Firstly is retracing that is going back to an earlier point in the discourse in search of possible mistakes. This retracing was not observed in any of the presentations. The teachers would ask the class to see if the procedure and the answer given is correct. Secondly is the issue of mediation switching for example from symbolic objectified to symbolic syntactic. This is used if one wants to correct mistakes in the process of retracing. This was completely absent in the presentations. Lastly is proficiency which refers to the proficiency of the student teachers in meditational mode.

Summary and conclusions

Objectification as already indicated refers to when the number words being produced by the participants and their corresponding symbols are used as if they signify “mind-independent” entities (p.215 emphasis added). The findings of this study reveal that in the discourse produced, there was no evidence of the objectification of the discourse. Starting with the way numbers words were used, the operational words and the meditational modes, there was no trace of objectification. What was more evident is the operation and personal type of discourse. This implies that in teacher training colleges, there is no space given for the student teachers to be flexible and transitions from one mode to another. Even though the literate discourse was used by both the teacher and the student teachers, there is no wider applicability of the literate procedures that were displayed by the student teachers.

The results mean that the discourse produced in college mathematics classrooms is strictly defined set of rules that cannot be changed even at a personal level. The focus of the teachers and the student teachers discourse is to ensure that they follow the mathematical set of rule without fail, thus ensuring their membership in a community of mathematicians. This type of approach does not give space to the student teachers to explore the “universe of extradiscursive entities” (p. 216). The student teachers discourse is strictly observing the rules of
the discourse. The idea behind is that the accepted discursive rule must be followed without question. In other words the discourse is completely authoritative (Bakhtin, 1981). Thus the discourse being produced by both the teacher educators and student teachers is one of rituals rather than explorations.

Final words

This analysis raises the following questions: Considering the mode of acting in this particular college mathematics classroom, would this type of acting lead the student teachers to a full-developed diverse and consolidated computational discourse? What should be done in teachers training colleges so that student teachers are exposed to a full-developed diverse and consolidated discourse? What are the implications of these results when the student teachers start teaching? Does exposure of the student teachers to an objectification discourse of arithmetic a resource for a quality teaching of the discourse in schools? Ben-Yahuda et al (2012) study has shown that there is a positive interdependence between objectification and the effectiveness of arithmetical discourse. One of the benefits of being able to objectify the discourse is that whenever one is stack with a certain route to a solution, other parts are still available to finding the solution. Furthermore objectification goes hand in hand with the participant’s ability to view endorsed narratives about numbers as helpful descriptions of an extra-discursive reality. Those who are able to use objectified discourse on their own accord are able to use arithmetical discourse on their own accord without being helped by others.

References

A Modeling and Models Approach: Improving Primary Mathematics Learner Performance on Multiplication.

Emmanuel Dlamini1, Hamsa Venkat2 & Mike Askew3
1Wits School of Education, University of the Witwatersrand, South Africa
2Wits School of Education, University of the Witwatersrand, South Africa & Jönköping University, Sweden
3Wits School of Education, University of the Witwatersrand, South Africa & Monash University, Melbourne
e.dlams@yahoo.com; hamsa.venkatakrishnan@wits.ac.za; mike.askew@monash.edu

This paper is located within research in the South African context that aims to improve primary mathematics learners’ performance in mathematics. Our specific focus is on improving learners’ understanding of, and attainment in, problems that can be modeled multiplicatively. We report on a six-lesson intervention that used context-based problems to encourage learners to model problem situations before proceeding to calculate an answer. Overall performance improved substantially between the pre- and post-tests. More detailed analysis revealed that this increase in successful performance was largely due to learners moving away from using the traditional column algorithm model and strategy, to other models and methods for finding solutions. Our findings suggest that a carefully structured content and pedagogic sequence focused on multiplication can, over a relatively short duration, help learners to better understand a range of word problems requiring multiplicative reasoning and solve word problems independently.

Introduction

Learner performance in mathematics continues to be weak at all levels in South Africa (DBE, ANA and Matric performance reports). Poor performance in the Intermediate Grades (grades 4-6) has been highlighted across a range of national, regional and international tests (see Fleisch, 2008 for an overview). Within this low performance, Murray (2003) has noted that South African teachers perceive word problems to be particularly difficult for learners, because they compound mathematical demands with language demands in a context where the majority of children are learning mathematics in English as a second or third (or more) language.

While there is evidence of what has been described as the ‘suspension’ of sense making internationally (Verschaffel, Greer & de Corte, 2000), an alternative view of word problems suggests their usefulness as contexts for supporting reasoning about the nature of arithmetical relations in the problem situation. Thus, a trend in mathematics in many parts of the world, including South Africa, is to put more emphasis on the relation or link between mathematics and the environment through the use of contextual situations (Verhage and De Lange, 1996). In order to address the suspension of sense-making, moves towards less stereotypical, and more realistic word problems are advocated by these authors, as well as changes in the ‘didactical contact’ (p176) through incorporating attention to problem solving, encouraging collaborative working and whole class discussion and reflection and encouraging shifts in the classroom culture towards ‘normalizing’ modeling activity.

These recommendations were followed in an intervention-based study focused on exploring learner performance on multiplication problems. The context was a suburban Johannesburg
Grade 6 class in a government school (where the first author is the mathematics teacher), serving a historically disadvantaged learner population. In this case study, a pre-test – intervention – post-test design was devised and implemented over a six-week period between September – November 2013. The intervention was based on Askew’s (2005) ‘Big Book of Word Problems – Year 5 and 6’ resource and pedagogic model, with particular focus in the study on the models and strategies used by learners as they solved a range of multiplication word problem types. This design allowed us to track shifts with regard to previous (pre-test) models and strategies and newly emergent models that learners were able to produce during the course of the intervention lessons and in the post-test. The post-test (a repeat of the pre-test) responses reflected the kind of models and strategies that learners were able to produce after participating in the intervention lessons. Our analysis of data was based on detailing of multiplication situations, models and strategies that are outlined in the literature. It took the form of coding both models and strategies, organizing these codes into categories that were then analyzed, in order to determine the relationships and patterns produced. In this paper, we report the outcomes of the intervention study, with particular emphasis on the pre- and post-test results. More extensive detail on the patterns of shifts in model use across the intervention lessons can be found in Dlamini’s (2014) Masters Study report – which is the broader study that these results are drawn from.

We begin this paper with an outline of the notions of modeling and models, drawn from the Dutch work on Realistic Mathematics Education (RME) that provided the theoretical underpinning of the intervention. Literature on modeling and models relating to multiplication situations are then detailed. Following an outline of the context of the study, and the content structure and pedagogic model used in the intervention lessons, we describe the data that were collected within the study. An analysis of the results is then provided, viewed through the lens of models for multiplication. We conclude the paper with a consideration of what this analysis suggests for further work in Intermediate Phase mathematics in South Africa.

**Modeling and models: RME theory**

The RME approach views mathematics as a human activity (Gravemeijer, 1994). The theoretical underpinnings of RME focus mainly on ‘mathematizing’ or organizing subject matter from reality or realistic contexts through modeling the relations between quantities/aspects of the situation to produce a model. This approach stands opposed to the more traditional approach in which models that present mathematics as a ready-made system are simply given to learners, without them having to go through mathematizing activity to produce them. Through the process of mathematization, learners are expected to learn through reinventing mathematical insights, relationships and procedures.

RME distinguishes between the processes of horizontal and vertical mathematization. According to Gravemeijer (1994), horizontal mathematization refers to processes in which learners describe represent contextual problems, including instances of solving a given contextual problem within this process. Vertical mathematization refers to processes in which the learner’s informal representation or model is used to solve a given problem using what he/she deems a suitable mathematical algorithm or procedure. In vertical mathematization activity, the problem context recedes: instead a model comes to stand in its place. RME advocates for both processes, and thus, their approach is characterized by problem situations that are realistic or imaginable, and learners are guided to develop their own informal mathematical models and strategies into more formal and efficient models over time. In pedagogical terms, this involves working with the problem at different levels as shown in
Figure 1 below. In the broader study, learners’ representations of problem situations were regarded as models (horizontal mathematization) and their processes of calculation using models, sometimes involving algorithms, were regarded as strategies (vertical mathematization) (Gravemeijer, 1994).

In this study, we used Askew’s (2005) multiplication focused word problems as situations that would give learners an opportunity to organize relations and set up models (horizontal mathematization) that could be used to solve problems through purposive selection of a strategy, rather than guessing of a strategy. Within classrooms, and built into the instructional design, is the expectation that different models and strategies will emerge as learners engage with contextual problems. The move overall can be described in terms of a shift from situational worlds to mathematical worlds (Van den Heuvel-Panhuizen, 2003). Figure 1 summarizes this framework thus:

![Diagram](image)

**Figure 1.** From Gravemeijer, 1994: Horizontal mathematization ..... Vertical mathematization ↑

In instructional terms, RME argues for three principles: guided reinvention, didactical phenomenology and emergent models. Guided reinvention refers to processes whereby learners are given the “guided” opportunity to “re-invent” mathematics by doing it (Gravemeijer, 2004; Van den Heuvel-Panhuizen, 2001). Didactical phenomenology is the principle in which problem contexts, in conjunction with mathematization, are used to lead learners to discover mathematical structures and concepts. According to RME theory, concept formation may be rooted in learners’ mathematization of their informal mathematical activities that develop into formal mathematics. Model formation plays a key role in RME because modeling is viewed as way of organizing within which symbols emerge as well as the model itself. According to Gravemeijer and Stephan (2002), the concept of “emergent models” characterizes the way in which models emerge within the RME instructional design. These are models developed by learners in the process of solving problems. Secondly, it characterizes the ways in which models gradually evolve over time to become models for more formal mathematical reasoning.

The role of whole class and group discussion as applied to interpretation and solution methods has also been highlighted by Gravemeijer and Terwel (2000). They argued that discussions could encourage shifts in models and strategies towards more efficient forms. In the context of RME it is anticipated that each problem will give rise to different solution methods. Whole class discussion allows for comparisons of models and strategies, providing opportunities for all learners to see more efficient (and usually more formal models/strategies) contrasted with less efficient models and strategies.
Models and strategies for multiplication

In this section, we use literature to identify models that are widely used to represent multiplication situations.

Doubling: Bigger products can be obtained through doubling either the multiplier or the multiplicand. A common procedure for middle years’ learners is to add pairs and then pairs of pairs until all the numbers have been added up (Caliandro, 2000).

Array models: Array models provide visual representations of grouped count situations. Learners may progress from multiplication with single digit numbers to multiplication with 2 digit numbers. This model lends itself in turn to grouped counting strategies for calculating totals working either with a column or a row as a group. By viewing each column as a group or each row as a group, the array model gives learners an opportunity to realise that commutativity holds and also allows them to decide which ordered version to use in their calculation strategy.

Area model: This model is based on place value quantity decomposition of the numbers being multiplied and can be very effective for dealing with two-digit and three-digit numbers. Haylock (2010) argues that the use of area to interpret multiplication makes it easier for learners to understand multiplication of two-digit numbers. It enables learners to compute multiplication through handling computations involving single digit numbers and multiplication by multiples of 10, 100, etc.

Horizontal distributive models: These models, like area models, use place value decomposition of large numbers into smaller multiplication parts that can be combined through addition. Distributive models make it possible to solve large numbers through mental calculations.

Completing model: Awkward numbers can be made easier to work with in multiplication by rounding and then compensating for the difference later. This approach can be used in the context of the array model (Barmby, Harries, Higgins & Suggate (2008)) or grid-based area models (Askew, 2012).

Column model: Column representation has been the traditional way of presenting long multiplication when two or more digits are being multiplied. The model is associated with a range of partial product calculations and is based on the distributive law for multiplication. Column representation can be approached from right-to-left (R-L) or from left-to-right (L-R). According to Ma (1999), the column model is associated with a range of learner errors in multidigit multiplication, associated with incorrect partial product calculation strategies and placement of numbers on subsequent lines.

Across these models, underlying fluencies include efficient mental calculation up to 10 x 10, skip counting, splitting the multipliers into simpler constituent parts, repeated addition and multiples of 2, 5 and 10 (Cathcart et al, 2000). Area models are also supported by awareness of place value decomposition of numbers and efficient multiplication by multiples of 10, 100, etc.

Research design and data sources

The class in which the study was set consisted of 33 Grade 6 learners. A pencil and paper pre-test was administrated in order to establish the kind of models for solving multiplication word problems that these learners were using prior to the six-lesson intervention. The test consisted of ten problems. Six of these problems could be represented as multiplicative
situations; the remaining four items were ‘buffer’ items involving other operations and/or ‘straight’ multiplication calculations in order to avoid ‘cueing’ learners into multiplication calculation strategies. 6 questions were word problems drawn from Askew’s (2005) problem sets, while the 4 remaining multiplication problems were drawn from previous years’ Annual National Assessments (ANAs). The majority of Askew’s (2005) multiplication word problems included additional superfluous information in relation to the question set. One example of this is question 1 on the pre- and post-test:

Coco ordered 8 boxes of neck ties. Each tie had 25 spots on it. Each box contained 16 ties. Coco unpacked the ties and tried them all on. How many ties did Coco try on?

While making the test and intervention lesson problems more difficult, this inclusion of additional unnecessary information was important in relation to more authentic mathematical problem-solving where the identification and selection of critical information has been identified as a key marker (Silver & Thompson, 1984).

Six intervention lessons were carried out over the six weeks following the pre-test administration. Lessons ranged across focus on multiplication as repeated addition (situations where several groups all the same in size need to be added together), multiplication as rate (situations where there is an implicit ratio, where, explicitly or implicitly, there is a “per” in the context) and multiplication as scaling (situations where a continuous quantity is increased in size by a scaling factor). In the latter three lessons, problems based on more than one of these root situations were included.

Each intervention lesson consisted of three sections: In phase 1 learners worked in pairs to solve three problems. In between each problem a follow up discussion captured the types of models used. In phase 2 all three problems were compared to highlight similarities and differences in the root situation structure. In phase 3 follow up tasks with similar structure to the introductory problems were completed individually. These sections and the word problems used within them reflected both RME theory and the positions advocated in the word problems literature. The post-test, administered after the intervention lessons, consisted of a repeat sitting of the pre-test.

Findings and analysis

In this section, we present findings from the study related specifically to the models used within learner responses with particular focus on the results of the pre- and post-tests.

Results reflected shifts in learner performance between the pre-test and the post-test assessment across the six-week intervention schedule. In terms of the total number of possible responses on the tests (n = 198 based on 33 learners x 6 word problem items), pre-test performance showed 127 correct answers (64% correct). Post-test performance indicated that there were 164 correct answers (83%). This suggested that learners benefitted from the intervention lessons, with substantial gains on all items apart from item 9. These data are summarized in Figures 2.1 and 2.2 which show, respectively, the pre- and post-test results ordered in two directions – vertically from the learner achieving the highest score to the learner achieving the lowest score and horizontally from the test item with the highest number of correct responses to the test item with the least correct responses.
This overview performance gain pointed to the usefulness of looking at the detail of model use underlying improved overall performance. Only three models were observed in the pre-test: column, area and doubling. Results indicated that the column model was predominantly favoured by most of the learners in the sample – used in 122 (62%) of the 198 possible instances in the pre-test. In comparison, area and doubling models were only seen in 19% and 11% of all instances respectively. No model use was indicated in the remaining 10% of responses.

Of particular interest was the reflection of research findings related to problems associated with column model use: out of the 71 incorrect answers seen in the pre-test, 49 of them (69%) were associated with the column model. 27 of these errors were associated with errors in translating from the given word problem situation to the column model, indicated by selection of incorrect combinations of figures from the word problem for the required quantity. The remaining errors were associated with the calculation steps (22 of the 49 column model errors).

Our analysis of the models used in the post-test reflected the use of a range of ‘new’ models (distributive, number line and completion models), as well as a pronounced shift away from the column model (used now in 18% of all 198 instances in comparison to the previous 62% usage). This indicated a substantial voluntary shift across the six-week period of intervention activities. The area model was used more widely by learners, moving from being seen in 18% to 29% of responses, and doubling models moving from use in 11% to 15% of responses. New models also emerged in the post-test following the intervention. These were the distributive model (seen in 24% of responses), number line models (used in 5% of responses) and completing-based models (used in 9% of responses). The relative popularity of the distributive model could, partially at least, be attributed to Annual National Assessment.
(ANA) questions where the memoranda required learners to use this model in the exemplar papers that were distributed to schools early in the year and in the final examination papers. Of interest within this was that the intervention appeared to have given learners sufficient confidence to use this model much more widely and voluntarily, in a way that they had not chosen to do in the pre-test (which was also administered after the ANA process in 2013).

Limited gains on Question 9 suggested also though, that extending some of these models to multiplication involving a three-digit number remained problematic for many learners, pointing to the need for further attention to such expansions within teaching. Problems with three and four-digit number multiplication were also seen on the two numerical format multiplication problems in the pre- and post-tests.

Discussion

Qualitative analysis of learner work thus indicates substantial shifts in both the use of models and success rates between the pre- and post-tests. Initial indications of a limited number of models (three) were extended to the use of a broader range of models within problem-solving. Overall, there are strong indications that modeling as an approach to multiplication within the RME framework has potential, considering the brief and concentrated nature of the intervention. The voluntary shift from column model to other models supports Caliandro’s (2000) assertion that learners can invent their own methods for solving multi-digit multiplication problems. This also indicates that learners themselves are in a position to realize the limits of the column model as a result of exploring other models.

A further comment relates to underlying fluencies. Teachers frequently comment that learners are hampered by limited fluencies, especially in relation to number facts and times tables. Some attention was given to practising multiplication facts and times tables within the intervention lessons. As the intervention lessons progressed, learners began to display improved understandings of multiplication and numerical relationships in the presence of situation-driven problems. This finding suggests that it is possible to increase success and accuracy in multiplication in the context of an intervention focused on modeling word problems with some attention to fluencies within this work (Anghileri, 2007).

This short study indicates that a pedagogic approach based on the use of situations and models can have a positive impact on performance and processes in solving word problems in multiplication. We are currently running a repeat trial of this intervention with a Grade 5 class at the same school in order to understand the extent to which these findings can be seen as ‘robust’. Given the lack of generalizability in case studies, we share the findings in order to encourage others to conduct similar small-scale trials in their own classrooms, and contribute to the development of a rigorous evidence base from which we can develop materials and teacher development activities that may be of broader use.

References


**Acknowledgements**

This paper forms part of the work in progress within the Wits SA Numeracy Chair project, entitled the Wits Maths Connect – Primary project. It is generously funded by the First Rand Foundation, Anglo American, Rand Merchant Bank, the Department of Science and Technology and is administered by the NRF-National Research Foundation.
Identifying Stages of Numeracy Proficiency to Enable Remediation of Foundational Knowledge Using the Learning Framework in Number

Mellony Graven¹, Debbie Stott¹, Zanele Mofu² & Siviwe Ndongeni³
¹South African Numeracy Chair Project, Rhodes University, South Africa
²Foundation Phase Mathematics Curriculum Planner, Eastern Cape Province, South Africa
³Intermediate Phase Educator, Eastern Cape Province, South Africa
¹m.graven@ru.ac.za; d.stott@ru.ac.za

The Annual National Assessments (ANAs) implemented in all South African government schools since 2011 in primary Grades 1-6 and Grade 9 point consistently across the three years for which results are available to learners operating far below their grade level in numeracy and mathematics. While a key aim of the ANAs is to provide system wide and teacher specific information on how to pin point several challenges the results are so poor for the majority of learners that the assessments fail to identify for teachers the numeracy development level at which their learners are at and thus fail to provide useful information for informing remediation interventions. This paper reports on each of our use of a numeracy assessment and recovery framework from the Maths Recovery Programme (by Wright and colleagues) as a tool for assessing learner levels of numeracy proficiency across learners in four foundation and intermediate phase after school mathematics clubs in the Eastern Cape. The findings across these studies point to the usefulness of this tool for planning subsequent interventions. The paper illuminates, through examples of data gathered across each of our research projects, the usefulness of identifying stages of numeracy development across their different research foci.

Introduction

Mathematics Education in South Africa is widely acknowledged to be ‘in crisis’ (for example Fleisch, 2008) and increasingly attention is diverted from only addressing the problem in the Further Education and Training band (FET) to addressing it in the early foundation years of learning. The Foundations for Learning Campaign (Department of Education, 2008) was introduced by the DBE in 2008 in order to bring a specific focus to improving reading, writing and numeracy in South African learners. One feature of this campaign has been the introduction of systemic assessments in the form of the Annual National Assessments (ANAs) written in Numeracy/ Mathematics and Language in Grades 1-6 and Grade 9 in all government schools.

The results of these assessments over the past three years confirm that the majority of learners do not have basic numeracy skills and that with each progressive year of schooling more and more learners lag behind meeting the basic requirements for their grade level (Schollar, 2008). The data given below constructed from the 2013 ANA report (Department of Basic Education, 2013) shows these results:
### Table 1: 2012 and 2013 National ANA results for grade 3 and 4

<table>
<thead>
<tr>
<th>Grade 3</th>
<th>2012 learners achieving less than 50%</th>
<th>2012 Learners average</th>
<th>2013 learners achieving less than 50%</th>
<th>2013 Learners average</th>
</tr>
</thead>
<tbody>
<tr>
<td>National</td>
<td>36.3%</td>
<td>41.2%</td>
<td>59.1%</td>
<td>53.1%</td>
</tr>
<tr>
<td>Grade 4</td>
<td>26.3%</td>
<td>37%</td>
<td>27.1%</td>
<td>36.8%</td>
</tr>
</tbody>
</table>

We can thus conclude from the above table that the majority of South African Grade 3 learners in 2012 had not developed foundational number sense before entering the intermediate phase (IP) and that while the figures had improved somewhat in 2013, still almost half the learners had not achieved what the Department of Basic Education terms ‘acceptable achievement >/50%’ or what we term basic foundational number sense required for enabling progressive learning in the intermediate phase. We note also in the table the large drop in results from Grade 3 to Grade 4 and argue that this is likely the result of learners having to learn intermediate phase content without having the requisite foundational mathematical knowledge of the foundation phase.

Given that the above data points to the majority of intermediate phase learners not having the grade level competence of the grade in which they are studying points to a problem with the opportunity to learn and the validity of the assessments that they participate in. So for example a Grade 4 learner asked to solve 243 x 59 in class or in an ANA cannot participate if they are still at the level of drawing three groups of 9 in order to find 3 x 9. In this sense their performance on this ANA question would tell the teacher little about the level of multiplicative reasoning that the learner does have and whether remediation should begin with focusing on grade 1, 2 or 3 work.

A wide range of research points to the need for coherence and progression in the teaching of mathematics (Askew, Venkat, & Mathews, 2012; Schollar, 2008). However teachers are unlikely to identify useful resources or generate resources with carefully inlaid progression without a solid understanding of the level at which learners are operating and the various levels through which learners must progress in order for foundational numeracy proficiency to be sufficiently in place in order to progressively progress through the mathematics required in the IP grades.

In this respect, across each of our research projects, we have found the work of Wright, Stafford, Stanger and Martland (2006) on delineating levels of mathematical progress in their early Learning Framework in Number (LFIN) to be particularly useful. We have used this framework not only for our analysis of learner levels of mathematical understanding in order to design learning activities but also for teacher development. Wright (2013) has argued that the interview tool from their mathematics assessment and recovery programme is useful for teacher development and understanding the developmental nature of numeracy learning. Wright et al.’s (2006) Maths Recovery (MR) programme is gaining popularity both internationally (it has been used in Australia, New Zealand, UK, USA, Canada) and in South Africa (see for example Weitz (2012), Mofu, (2013), Ndongeni (2013) and Stott (2014)) and has thus been tested across multiple contexts.

In this paper we discuss the ways in which our four research projects, in the context of primary after school mathematics clubs, drew usefully on Wright et al.’s framework in order to illuminate the usefulness of this framework as both an analytical and a developmental tool for informing teaching practice.
The empirical field of our research

The South African Numeracy Chair Project (SANCP) is tasked with researching sustainable ways forward to the many challenges faced in primary mathematics education in South Africa. As part of this project one development initiative that we piloted in 2011 and rolled out in 2012, was that of after school mathematics clubs. Within the SANC project, the clubs serve two purposes: firstly, they are a place where we can directly influence what happens with learners and secondly, they provide us with an empirical research field in which we can interact directly with the learners and thus be insiders to the learning process.

These clubs are conceptualised as informal learning spaces focused on developing a supportive learning community where learners can develop their mathematical proficiency, make sense of their mathematics and where they can engage and participate actively in mathematical activities. Individual, pair and small group interactions with mentors are the dominant practices with few whole class interactions. The clubs were intentionally designed to contrast more formal aspects observed in the classrooms of the SANC project participating schools (Graven & Stott, 2012; Graven, 2011). Of note is the promotion of learner-centred practices in clubs are also promoted in the official curriculum documents (Department of Basic Education, 2011) and which Hoadley (2012) notes are absent in South African classrooms.

All four authors are part of the SANCP. All ran a club in 2012 / 2013 and conducted research in their clubs drawing on Wright, Martland and Stafford’s (2006) assessment interview instrument in order to assess learner levels of conceptual understanding as part of their broader club research. Details of the research are available in their theses and other publications (see for example (Graven, 2012; Mofu, 2013; Ndongeni, 2013; Stott, 2014; Weitz, 2012).

Theoretical framework, methodology and analytic tools

Across all of our research we have taken a broad socio-cultural perspective in relation to interpreting learner understanding and progression. This assumes that learning is an active construction of knowledge through social interactions with others.

Wright et al. (2006) state that they are “strongly constructivist” (p. 7). Their work is based on the principles that learning mathematics is an active process, each child constructs their own mathematical knowledge and that they develop mathematical concepts as they engage in sense-making, mathematical activity. Their MR programme is based on sense making and mathematical activity and normally takes place alongside a teacher or other adult. In this way learners are not working on their own discovering knowledge per se but are assisted by a more knowledgeable other. This view coheres with those taken by each author in their individual studies.

The Learning Framework in Number (LFIN) developed by Wright and his colleagues (2006) provided us with a useful way of tracking and assessing mathematical progress over time. Wright et al. (2006) described the LFIN as providing a “blueprint for the assessment and indicates likely paths for children’s learning” (p.7). This framework has been used to research and document progress in number learning of five to eight year old students in the first three years of schooling. As an intervention programme it involves intensive one-to-one teaching of low-attaining students but the programme has also been used with students of all levels of attainment (Bobis et al., 2005).
The four research studies detailed here were qualitative and drew on the case study research design. The data gathering method used for investigating the aspect of our research reported on here were structured interviews, particularly those used in the MR programme (Wright, Martland, & Stafford, 2006). It is beyond the scope of this paper to include the entire interview script but one item from the interview is given in Figure 6 below as an example:

<table>
<thead>
<tr>
<th>If I tell you that eight times seven is 56</th>
<th>Show this card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can you use these numbers and signs to make a division sentence?</td>
<td>8 x 7 = 56</td>
</tr>
<tr>
<td>Can you make another division sentence?</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6. Sample interview task (Commutativity and inverse relationship in multiplication and division) (Wright, Martland, & Stafford, 2006, p. 182)

Each author used one, some or all of the LFIN aspects as an analysis tool for their study. In this section we detail three of the five LFIN aspects and the associated levels or stages, so as to illustrate how we determined where to position learners on the LFIN using data collected from the interviews. We specifically worked with a version of the LFIN that combines elements from Wright et al.’s 2006 and 2012 works. The key aspects of the LFIN are:

- Structuring numbers 1 to 20
- Number words and numerals (including forward and backward sequences)
- Conceptual place value knowledge (ability to reason in terms of tens and ones)
- Early arithmetic strategies (strategies for counting and solving simple addition and subtraction tasks)
- Early multiplication and division
  (Wright, Ellemor-Collins, & Tabor, 2012; Wright, Martland, & Stafford, 2006)

Each of the key aspects of the LFIN are elaborated into a progression of up to six levels or stages with each model describing the characteristics of the levels or stages (Wright, Martland, Stafford, et al., 2006). These are detailed below for Conceptual Place Value, Early Arithmetic Strategies and Early Multiplication and Division.
Table 2. Conceptual place value

<table>
<thead>
<tr>
<th>Level Number</th>
<th>Level Descriptor</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial concepts of 10 (ten as a count)</td>
<td>Not able to see ten as a unit composed of ten ones. The child solves tens and ones tasks using a counting-on or counting-back strategy. One 10 and 10 ones do not exist for the learner at the same time.</td>
</tr>
<tr>
<td>2</td>
<td>Intermediate concepts of 10 (ten as a unit)</td>
<td>Able to see ten as a unit composed of ten ones. The child uses incrementing and decrementing by tens, rather than counting-on-by-one to solve uncovering board task. The child cannot solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences.</td>
</tr>
<tr>
<td>3</td>
<td>Facile concepts of 10 (tens and ones)</td>
<td>Tens and ones are flexibly regrouped. Ten is a unit that can be repeatedly constructed in place of 10 individual ones. Child is able to solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences by adding and/or subtracting units of tens and ones.</td>
</tr>
</tbody>
</table>

Table 3. Early arithmetic strategies

<table>
<thead>
<tr>
<th>Stage Number</th>
<th>Stage Descriptor</th>
<th>Characteristics (representing increasing levels of sophistication)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Emergent counting</td>
<td>Cannot count visible items. The child might not know the number words or might not coordinate the number words with the items.</td>
</tr>
<tr>
<td>1</td>
<td>Perceptual counting</td>
<td>Can count only visible items starting from 1. Including seeing, hearing and feeling.</td>
</tr>
<tr>
<td>2</td>
<td>Figurative counting</td>
<td>Can count concealed items but the learner will ‘count all’ rather than ‘count on’.</td>
</tr>
<tr>
<td>3</td>
<td>Initial number sequence</td>
<td>Initial number sequence. The child can count on rather than counting from one, to solve + or missing addends. May use the counting down to solve removed items. (count-back-from).</td>
</tr>
<tr>
<td>4</td>
<td>Intermediate number sequence</td>
<td>Count-down-to to solve missing subtrahend (e.g. 17-3 as 16, 15 and 14 as an answer. The child is able to use a more efficient way to count down-from and count down-to strategies (count-back-to).</td>
</tr>
<tr>
<td>5</td>
<td>Facile number sequence</td>
<td>Uses of range of non-count-by one strategies. These strategies such as compensation, using a known result, adding to 10. Commutativity, subtraction as the inverse of addition, awareness of the 10 in a teen.</td>
</tr>
</tbody>
</table>

Table 4. Early multiplication and division strategies

<table>
<thead>
<tr>
<th>Level Number</th>
<th>Level Descriptor</th>
<th>Characteristics (representing increasing levels of sophistication)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Initial grouping and perceptual counting (Forming equal groups)</td>
<td>Able to model or share by dealing in equal groups but not able to see the group as composite units; count each item by ones.</td>
</tr>
<tr>
<td>1</td>
<td>Intermediate composite units (Perceptual multiples)</td>
<td>Able to model equal groups and counts using rhythmic, skip or double counting; counts by ones the number of equal groups and the number of items in each group at the same time only if the items are visible.</td>
</tr>
</tbody>
</table>
Abstract composite units (Figurative units) | Able to model and counts **without visible items** i.e. the learner can calculate composites when they are screened, where they are no longer rely on counting by ones. The child may not see the overall pattern of composites such as “3, 4 times”.

Repeated addition and subtraction | Co-ordinates composite units in **repeated addition and subtraction**. Uses a composite unit a specific number of times as a unit e.g. 3 + 3 + 3 + 3; may not fully co-ordinate two composite units.

Multiplication and division as operations | **Two composite units are coordinated abstractly** e.g. “3 groups of 4 makes 12”; “3 by 4” as an array

Known multiplication and division facts strategies | **Recalls or derives easily, known multiplication and division facts**; flexibly uses multiplication and division as an inverse relationship, is able to explain and represent the composite structure in a range of contexts.

---

**Our findings**

In the next section of the paper we share findings from our four research studies undertaken over the last 3 years. We share the way in which the LFIN enabled our research and our analysis as well as how this framework enabled the developmental aspect of planning for future club activities and teacher development.

**Analysing learner developmental levels for design of after school maths club activities** (see Stott, 2014)

In her doctoral study, Stott investigated how Grade 3 learners’ mathematical proficiency progressed (or not) whilst participating in two after school maths clubs over the course of a year and offered insight into how mathematical proficiency may develop in Grade 3 South African learners. Stott used the LFIN as an analytic tool to track progress between March and November 2012 for 17 club learners in all five LFIN aspects. A key contribution of her study was the extension of the LFIN to obtain quantifiable data in the form of scores, so as to analyse progression of the club cohort of learners in addition to the progress of individual learners.

Bob Wright (2003) has specifically stated that the data derived from the one-to-one MR assessment interview “does not result in a score” (p.8), the interview data is always used to profile the individual child's stage of early number learning onto the LFIN using stages and levels. However, Stott argued that such scoring could be useful. Working as she does in many clubs (subsequent to her research clubs), it is useful for her to compare different clubs to each other, thus she generated quantifiable data which she called ‘Mathematical Proficiency (MP) Interview Scores’. By working with percentages, she was able to usefully aggregate these scores in order to make comparisons across more than one club using tables and graphs. These types of comparisons across the whole club or sets of clubs are not easily noted from the aspect stages or levels detailed within the LFIN itself, as each set of stages or levels is profoundly different and one would not be comparing like with like (see Tables 2, 3 and 4 for examples).

In her research clubs she tried to balance the needs and progress of the whole group with those of the individual learners. After conducting the first series of interviews and generating the scores Stott was able to see where the club learners had achieved high scores and low scores and used this information to plan activities for the whole club that addressed areas of weakness.
The findings from Stott’s study suggested that the learners assessed in both clubs made progress to varying degrees as evidenced by the Mathematical Proficiency interview results. The graph in Figure 7 below shows how Stott used the percentage scores generated from the interviews to draw comparisons between her research clubs. The graph shows the overall club percentage change figures for each LFIN aspect for both case study clubs. Of interest is that the scoring allows one to see the similarity in improvements across LFIN aspects across the two clubs.

![Figure 7. MP interviews: Club comparison - overall % change for each LFIN aspect](image)

**Analysing individual learner numeracy levels and relating this to the opportunity to learn**

Here Graven shares how the analysis of two learner’s interview responses and assessed levels of numeracy proficiency influenced their opportunity to learn and participate in subsequent club activities. Analysing the Wright et al. (2006) interviews conducted in February 2012 and again in November 2012 enabled Graven to note learner numeracy progressions over the year. Graven noted that Jade had progressed from her dependence on a 1-to-1 counting all strategy that dominated across questions in the first interview to counting and using more efficient counting on strategies in the second interview. Lebo had also progressed from occasionally needing to refer to concrete objects to knowing and using several number facts for enabling efficient solutions. Thus in terms of the LFIN, Jade progressed from a level 2 borderline 3 in early arithmetic strategies to a level 3 later in the year, whilst Lebo progressed from a level 3 to level 5 (see Table 3 above for detail of these levels). In conceptual place value, Jade remained at level 1 in both interviews but there was some evidence of her developing level 2 knowledge in one of the items in the November interview but this was not carried over to subsequent assessment items. Lebo progressed from level 2 to 3 during the year. This analysis of early arithmetic strategies and conceptual place value using the LFIN enabled Graven to notice how in several club activities, Jade was unable to participate fully in the way that Lebo did. An example follows.
The differential levels of numeracy progression meant that each learner brought different capabilities to the activities set by Graven. So for example in one session Graven asked learners to generate spider diagrams where they generate a set of sums (as the legs) that provide the answer of the number in the centre circle (the spider’s body). For this activity Graven began with the number 10 in the circle and encouraged learners to generate sums that make 10. Jade participated actively in this and generated sums like 9+1 using her fingers. Lebo quickly generated several sums, without fingers, including subtraction sums. For the next activity Graven wrote 36 + 25 in the spider’s body and, using place value cards, discussed how they could find the answer. Lebo was able to participate here and related the adding of the units and the tens to the place value cards. Jade on the other hand tended to look out the window as the place value discussion did not connect with her finger method of calculation.

Graven’s reflection on this episode, based on her knowledge of Jade’s level of numeracy proficiency at the start of the club (as Level 1 for conceptual place value and Level 2/3 for early arithmetical strategies) resulted in Jade being unable to contribute meaningfully to a conversation on place value which Lebo was able to contribute to. Furthermore following the place value discussion Lebo was able to participate fully in the activity that followed quickly generating multiple sums through manipulating numbers efficiently (e.g. 20 + 40 + 1). He was thus able to make the most of his opportunity to learn through the activity and generated new ideas and extended his thinking – all of which was enabled by his fluency (Lebo was Level 3 for early arithmetic strategies and at Level 2 for conceptual place value at the start of the club) with numbers. Graven’s realisation here was that more individualised mediation was needed when providing an activity to a group of learners who are at different levels of numeracy proficiency. She realised that failure to do so could exclude learners at lower proficiency levels from participating in discussion and activities where higher levels of learning are introduced (and sustained by learners operating at higher levels of proficiency). This thus led to the realisation that certain club activities provided learners differential access to the opportunity to learn. This reflection is informing collaborative research currently underway with Heyd-Metzuyanim focused on exploring the relationship between forms of numeracy participation and the opportunity to learn.

An investigation of a Mathematics Recovery Programme for multiplicative reasoning
(see Mofu, 2013)

This part of the paper focuses on a Masters study undertaken by Mofu in 2013. The aim of her study was to inform mathematics teaching in her own school and to find ways to support primary school teachers at large in developing the strategies to teach and remediate multiplication reasoning. Mofu’s experience in the classroom confirmed that learners experienced difficulties with multiplication. She observed that when working with multiplication, her grade 5 learners were still counting visible objects in ones. Some learners, when performing multiplication tasks, draw circles or small lines for counting and some just added the numbers. Thus, in addressing this problem her study examined what level of multiplicative reasoning was displayed by the learners in the case study group and how effective the use of the Mathematics Recovery programme was in the South African context when used to remediate a group of learners.

Using a qualitative case study approach, Mofu collected video recorded one-to-one oral interviews with the learners. A sample of six Grade 4 learners were purposively selected using a basic written assessment instrument to a class of Grade 4’s which specifically looked at assessing their knowledge and understanding of multiplication. From the scored results
Mofu selected: 2 top scoring learners, 2 middle scoring learners and the 2 bottom scoring learners. These learners were invited to participate in an after school intervention programme aimed at supporting and remediating multiplicative reasoning. Mofu used the LFIN to profile the learners using pre and post intervention interview data and to determine their levels of multiplicative reasoning.

Learner progress in LFIN levels data was analysed using guidelines provided by Wright, Martland and Stafford (2006) as shown in Table 4 above. Table 5 below gives an overall picture of how the learners in her study progressed in terms of the LFIN levels from the pre (March 2013) to the post (April 2013) assessment.

**Table 5.** Learners overall progress in LFIN levels over time from pre to post assessment (Mofu, 2013 p. 49)

<table>
<thead>
<tr>
<th></th>
<th>Learner A</th>
<th>Learner B</th>
<th>Learner C</th>
<th>Learner D</th>
<th>Learner E</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEVELS</td>
<td>PRE</td>
<td>POST</td>
<td>PRE</td>
<td>POST</td>
<td>PRE</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Given the relatively short intervention in this study (4 sessions over 5 weeks), progress made from level one to another level was one of the most important results for Mofu. Her data showed that in the pre assessment, learners were counting in ones (positioning them at level 1) and relying on using constrained methods to solve multiplication tasks. After the intervention, the post assessment showed that constrained methods disappeared and learners were able to count in equal groups and use more efficient and fluent methods to solve the multiplication tasks. The rate of progression in Mofu’s study was far greater than she expected; all learners progressed at least one level. Of note is that Learner C progressed from level 1 to level 3 in the short time, which represents a significant shift in her multiplicative reasoning.

Of interest is that Mofu drew on the efficiency spectrum for procedural fluency developed by Graven and Stott (2013). Their efficiency spectrum for procedural fluency ranged from restricted / constrained procedural fluency towards elaborated and fully flexible fluency. The strategies used by the learners in Mofu’s case study confirmed the notions of efficiency and fluency she had coded and analysed in the oral interview and showed an overlap of learner strategies. The learners displayed a range of responses from restricted / constrained procedural fluency towards elaborated and fully flexible fluency. This resonated with her sense that learner’s multiplicative proficiency or fluency needed to captured in its own right. Thus Mofu adapted the Graven and Stott (2013) spectrum for procedural fluency into multiplicative spectrums to help understand learner progress. Figure 8 below shows Mofu’s adapted spectrum of multiplicative proficiency.

**Figure 8.** Spectrum of multiplicative proficiency for constrained, fluent and flexible fluency (Adapted from Graven & Stott, 2013)
In order to analyse multiplicative proficiency Mofu quantified the qualitative data to track possible progress using the spectrum discussed above. The progress of the learner was evident when they moved to the middle or upper end of the spectrum, which indicated increased fluency, flexibility and efficiency in multiplicative thinking. Figure 9 shows the positions of each learner according to the methods each used on the spectrum for the pre and post assessments starting with constrained (I-Inefficient) on the left, fluent (IE) methods in the middle and flexible fluency (E-Efficient) on the right. The values are the number of tasks where the learners showed the usage of different methods. So for example, Learner A progressed form using mostly constrained methods in the pre interview (in 5 questions) to more flexible methods in the post interview (in 5 questions)

<table>
<thead>
<tr>
<th></th>
<th>I Constrained</th>
<th>IE Fluent</th>
<th>E Flexible fluency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LEARNER A</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRE</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>POST</td>
<td>2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td><strong>LEARNER B</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRE</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>POST</td>
<td>2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td><strong>LEARNER C</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRE</td>
<td>6</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>POST</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td><strong>LEARNER D</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRE</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>POST</td>
<td>0</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td><strong>LEARNER E</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRE</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>POST</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

**Figure 9.** Summary of spectrum methods for all learners across 7 tasks (Mofu, 2013 p. 57)

Mofu found that the use of the Maths Recovery (MR) Programme made it possible for the learners in her case study to progress in terms of multiplicative reasoning. The MR programme highlighted that, as teachers we need to understand the levels that the learners are operating at so as to assist them in their learning trajectory. During the intervention, Mofu gave the learners guided support, helping learners to think about multiplication and division, encouraging them to use their own strategies and make mistakes. Learners were encouraged to enter into discussion and engage in activities that involved active learning, problem solving and critical thinking were considered in the teaching strategies in the MR programme in keeping with the social constructivist notion that learning takes place in a social context before it is internalised.

A key aim of Mofu’s research was to explore the extent to which the MR programme could be used to support learners in developing multiplicative reasoning and proficiency. As a teacher she learnt the importance of providing learning tasks that allow collaboration with
peers, having access to concrete materials like arrays for multiplication and division. She found that the MR programme offered rich learning activities for teachers to use in interventions. Mofu also saw the usefulness of learning as an educator from the interview and see it as a useful developmental tool. Wright (2013) himself urged teachers and teacher educators to find ways to “incrementally trial and implement” (p.38) MR programme approaches. He stated that this professional learning is a “pathway to profoundly strengthening children’s learning of basic arithmetic” (p.38) and that this can lead to young children achieving at significantly higher levels.

Mofu found that the key disadvantage of the LFIN was that it was labour intensive and time consuming to administer for more than a few learners. The assessment interviews took approximately one and a half hours for each learner. Additional time was spent coding and allocating learners to LFIN levels. Thus while Mofu would recommend that teachers conduct the interviews with a range of their learners in order to gain in-depth insight into learner levels and difficulties in multiplicative reasoning, it is not feasible to assess all learners in this way. However the implementation of the multiplication part of the MR programme to a group of learners holds potential for work in class. In her new role of Foundation Phase Mathematics Curriculum Planner, in the Eastern Cape, Mofu has subsequently conducted many fruitful workshops in this regard with Foundation Phase teachers in the Eastern Cape.

**Exploring relationships between levels of numeracy reasoning, conceptual understanding and productive disposition (see Ndongeni, 2013)**

This part of the paper reports on the findings of the fourth author’s Masters research that focused on the relationship between ‘conceptual understanding’ and ‘productive dispositions’ (Kilpatrick et al., 2001) in the context of multiplication. Having noticed over a period of years that the Grade 7’s in her school still relied on unitary counting and written tallies when dealing with multiplication and division problems this pointed to an important area of research. Ensor, Hoadley, Jacklin et al. (2009) and Schollar (2008) have argued that there is a lack of shift from concrete counting-based to more abstract calculation-based strategies and this seemed evident also in the case of multiplicative reasoning.

The study drew on the LFIN to establish learner levels of conceptual understanding in multiplication. Wright et al. (2006) argue that the topics of multiplication and division build on the students’ knowledge of addition and subtraction, and also multiplication and division provide foundational knowledge for topics such as fractions, ratios, proportion and percentage, all of which are core and essential areas of mathematical learning typically addressed in the primary or elementary grades. Notions of conceptual understanding and productive dispositions were theoretically informed by Kilpatrick, Swafford, and Findell’s (2001) five-stranded framework of mathematical proficiency. These strands are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. The fifth strand, productive disposition, is defined as “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (p. 131). This strand however is largely under researched (Graven, 2012).

In the study a purposively selected sample of six Grade 4 learners was used: two high, two average, and two low performers as indicated by performance on an initial basic assessment of multiplication. Individual interviews were conducted using the Wright et al (2006) instrument for exploring the nature of students’ conceptual understanding of multiplication. For learner dispositions, an instrument adapted from Graven’s (2012) productive disposition instrument was used and all these interviews were transcribed. The questions asked were
structured in order to elicit the presence or absence of indicators of productive disposition in the context of multiplication.

Below is the summary by levels in the progression of multiplication over time with descriptors of learner responses that serve as indicators of each level.

**Andile - Level 1: Forming Equal Groups or Initial Grouping**
The child did no see groups as composite units and thus counted items mainly in ones instead of multiples. Thus he mainly used perceptual counting and sharing.

**Viwe - Borderline between Level 1 and 2**
The child used counting in 1’s and in some instances in 2’s but he still used perceptual counting because he was reliant on seeing items.

**Nako - Level 2: Perceptual Counting in Multiples**
The child used multiplicative counting strategies to count visible items arranged in equal groups but had difficulty in solving items where groups were screened.

**Anda - Level 3: Figurative Composite Grouping**
The child used multiplicative counting strategy to count items arranged in equal groups where individual items are not visible. So she was not dependent upon direct sensory experience. She did not use the composite unit a specified number of times.

**Lulu - Borderline between 4 and 5**
The child was able to use the composite unit a number of times and was not dependent upon direct sensory experience. She fell short of the next level because she did no see the inverse relationship of multiplication and division.

**Sindy - Level 5: Multiplication and Known multiplication and division facts strategies**
The child was able to immediately recall or quickly derive many of the basic facts for ‘division as operations’ She was also able to see and the inverse relationship of multiplication and division.

The analysis of learner levels of conceptual understanding using the LFIN and Kilpatrick et al.’s (2001) indicators of a productive disposition enabled the construction of individual learner stories that foregrounded the relationship between these. The frequency of Kilpatrick et al.’s (2001) key indicators of a productive disposition present in learner responses is provided in Figure 1 below:
Of interest is the low presence of sense-making for learners (Andile and Viwe) matched with the lowest level of multiplicative reasoning in the LFIN (level 1) while seeing oneself as an effective learner and doer of mathematics and belief in steady effort did not seem to be clearly related to learner levels of conceptual understanding. So for example even while Andile was at Level 1 he said he saw himself as a strong mathematical learner. The study was limited in that it only had a small sample of learners but pointed to further research and the usefulness of the LFIN for assessing learner levels of conceptual understanding.

**Concluding Remarks**

This paper reported on each of our use of Wright et al.’s (2006) numeracy assessment and recovery framework as a tool for assessing learner levels of numeracy proficiency (and progress) across four research studies focused on learners in primary after school mathematics clubs in the Eastern Cape. The findings across these studies point to the usefulness of this tool for assessing learner levels of understanding and for planning subsequent interventions. In the paper we have made the case that while the ANAs are intended to provide teachers with useful information for planning future teaching they do not provide the teachers with opportunities to assess learner levels as the vast majority of learners are performing way below the grade level for which the ANAs are set and are thus unable to participate in several of the questions. Our paper has illuminated, through examples of data gathered across each of our research projects, the usefulness of identifying stages of numeracy development across their different research foci.

**References**


Acknowledgements
The work of the SA Numeracy Chair, Rhodes University is supported by the FirstRand Foundation (with the RMB), Anglo American Chairman’s fund, the Department of Science and Technology and the National Research Foundation.
An Exploration of Learners’ Learning of Mathematics by Using Selected (VITALmaths) Video Clips: A Case Study

Thomas Haywood¹ & Marc Schäfer²
¹Rhodes University Mathematics Education Project (RUMEP), Grahamstown, RSA.
²Education Department Rhodes University, Grahamstown
¹t.haywood@ru.ac.za ²m.schafer@ru.ac.za

Introduction
This paper, which is part of an on-going MEd study explores how 11 selected grade-10 learners from a secondary school in the Northern Cape Province use a number of the Visual Technology for the Autonomous Learning of Mathematics (VITALmaths) video clips to learn certain mathematical concepts autonomously in out-of-school contexts.

Research Questions

Main Question
How do selected grade-10 mathematics learners experience the autonomous use of selected VITALmaths video clips, which incorporated animated manipulatives, in their learning of mathematics?

Sub Questions
• Does the use of the video clips in conjunction with specially prepared worksheets specifically encourage: a) the use of manipulatives in their learning of mathematics, and b) the growth of a discourse-for-oneself?
• Does the use of the video clips enhance the learning of the Pythagorean theorem and the addition and subtraction of fractions?

Background
According to Census South Africa 2011 the South African population stands at 51.77 million people. Although the Northern Cape Province (NCP) is the largest province, it has the country’s smallest provincial population of 1.15 million people.

The John Taolo Gaetsewe District Municipality, the geographical location of our research, one of five district municipalities in the NCP, consists of 15 towns and villages (Northern Cape Department of Cooperative Governance and Traditional affairs, 2012). The majority of the people in this district municipality live in rural areas that have backlogs with regard to basic infrastructure (Northern Cape Department of Cooperative Governance and Traditional affairs, 2012).

Herselman (2003) writes that many of South Africa’s rural areas exist below subsistence levels and remain impoverished because they have little access to basic infrastructure essentials such as water, proper sanitation and learning resources for economic growth and development. This was evident during the Rhodes University Mathematics Education Project (RUMEP)’s visit to schools in the John Taolo Gaetsewe District Municipality areas of the NCP. Thirty-three teachers from the NCP registered for a B.Ed. (In-service) in mathematics course through RUMEP. Although the NCP’s pass rate increased from 68.8% in 2011 to 74.6% in 2012 (Gernetzky & Magubane, 2013), Northern Cape Department of Education
officials from both rural and urban schools have identified mathematics as a problem subject during interviews with RUMEP.

According to Taylor (2008) it is self-evident that what children learn is heavily dependent on what teachers know and do in their classrooms. This is particularly true for children who receive little support for schoolwork from their homes and little intellectual stimulation in their broader social environments (Taylor, 2008). This is particularly evident in poor areas. Taylor (2008) asserts that two factors are commonly associated with improved performance in schools, that is, reading and homework. Mulkeen (2005) writes that homes in rural areas are often ill-equipped to meet the educational demands of learners due to the lack of basic facilities, like electricity. Furthermore, parents in rural areas are less likely to be educated themselves and thus might have less ability to support their children with education at home (Mulkeen, 2005). We thus concur with Elmore and Fuhrman (2001) who write that in order to improve school performance, especially in rural areas, we must do different things and not do the same things differently. One way of doing things differently and potentially lessen the above difficulties are to introduce learners to technology in and out of school.

Mobile technology

Isaac (2002) writes that this is the digital age and technology allows learners to engage positively in subjects that they were not confident about. Although there has been an increase in the number of people with access to computers from 2007 to 2011 in South Africa, only 25.9% of households in the NCP have access to computers (Statistics South Africa, 2012). Mobile phone usage, however, in the NCP has increased from 24.5% of households in 2007 to 81.1% in 2011 (Statistics South Africa, 2012). Aljohani, Davis and Tiropatis (2011) write that the swift growth in mobile communication technologies has allowed people to no longer be restricted in their interactions and communication through geographical positioning. They are also able to access and download endless varieties of data on mobile devices from any location (Hyde, 2011). The use of mobile phones thus may provide an alternative for engaging in mathematical concepts and ideas, especially in out of normal school hour learning in the rural areas of the NCP.

Lenhart, Ling, Campbell & Purcell (2010) write that 24% of learners attend schools in South Africa where mobile phones are banned, while 62% were permitted to have their mobile phones at school but were not allowed to use them in their classrooms. Ormiston (2013) argues that regardless of the school’s mobile phone policy, the reality is that all students carry mobile phones with them, so why not use these tools for “good rather than evil”. Koebler (2011) asserts that schools are supposed to prepare learners for real life and in real life people use mobile phones. We concur with Koebler (2011) who further argues that it makes sense to use mobile phones for teaching and learning especially where there is a lack of the latest technology. Another reason to rethink the mobile phone debate is that mobile phone usage can be extended beyond the walls of the school or the confines of the classroom period and promote autonomous learning (Ormiston, 2013).

Autonomous Learning

One of the major problems in learners’ achievements in mathematics is the difficulty they experience to perform tasks involving higher level thinking skills which are developed through autonomous learning behaviours (Karp, 1991). Thus, to engage meaningfully in high level mathematical tasks, one should be able to work independently (Karp, 1991). According to Thanasaoulas (2000) an autonomous learner takes a pro-active role in his/her own learning process, generating ideas and availing himself/herself of learning opportunities rather than
simply reacting to various stimuli presented by the teacher. Autonomy, however, is not a product ready-made for use or merely a personal quality or trait (Thanasoulas, 2000). Learners have to groom and develop skills towards autonomy. This requires the guidance and support of the teacher (Thanasoulas, 2000).

Wood’s (2008) definition of autonomous learning focuses on the learner’s desire to understand experiences including both physical experiences and interactions with others. Autonomous learning is a constellation of mathematizing and identifying activities, reflecting curiosity about how things are and what others think and say about what seems to be true (Wood, 2008). Wood’s (2008) purposeful use of curiosity captures the ways in which autonomous learners compare their thinking with their observations of experiences.

Ben-Svi and Sfard (2007) cited in Wood (2008) explain autonomy in terms of the learners’ ability to explore the discourse of others and to make the discourse-for-others into a discourse-for-oneself. A discourse-for-oneself is that which a learner would spontaneously turn to whenever it may assist the learner in solving his/her own problems. For example a teacher might explain to the learner that in the theorem of Pythagoras, the sum of the squares of the lengths of the sides of a right triangle is equal to the square of the length of the hypotenuse. In order to make this into a discourse-for-oneself, the learner will investigate this discourse by constructing squares of the same length of the sides on each side of the right angled triangle. By cutting out the two squares, which fit onto the sides of the right angled triangle and altering them to fit onto the square on the hypotenuse, the learner examines the discourse and incorporates the discourse into his/her own thinking. The discourse thus becomes a discourse for the learner (a discourse-for-oneself). As the learner transforms a discourse into a discourse-for-oneself, he/she becomes able to use the discourse to solve problems that involves the discourse (Wood, 2008). The discourse thus is not merely recited to get the approval of other people such as a teacher, but because of the learner’s ownership of the meaning of the discourse, the learner is able to use the discourse as tool to solve problems (Wood, 2008). However, ownership of meaning acknowledges the social nature of a discourse. If a learner’s use of a discourse is not consistent with the way the discourse is used by others, then the learner does not own the meaning in a way that it is valued within the community (Ben-Svi and Sfard, 2007 cited in Wood, 2008). According to Wood (2008) there are four distinct features for discourses: word use, visual mediators, endorsed narratives and discursive routines. If a discourse is loyally adopted it means that the learner’s use of the discourse is consistent with others’ use of the discourse across the four above mentioned features (Wood, 2008). Sfard (2008) writes that the use of specific words or expressions in certain ways indicates that we have a mathematical discourse.

According to Berger (2013) visual mediators are visible objects which the autonomous learner uses to show his/her thinking or communication about the mathematical discourse. The use of visual mediators constitutes the autonomous learner’s thinking about the specific mathematical discourse (Berger, 2013).

“Endorsed narratives are any text, spoken or written, that is framed as a description of objects and that is subject to endorsement or rejection, that is, to being labelled as true or false” (Sfard, 2007, p 574). According to Sfard (2008) cited by Wood (2008), the substantiation of a mathematical discourses is directed by meta-rules, which are in turn guided by mathematical proof.

Just as with endorsed narratives, discursive routines are guided by certain rules (Berger, 2013). These rules may include rules about the objects in the discourse (object-level rules) or rules that constitutes an acceptable mathematical proof (meta-rules) (Berger, 2013). For
example, if a learner needs to describe the Pythagorean theorem and the learner describes it as the sum of two sides of a right-angled triangle is equal to the hypotenuse side, it will be rejected because it is not consistent with the meta-rules that determines the truth of the statement.

Although autonomy does not necessarily mean that the learner does not interact with other people at all, it still requires from the learner to engage in their own examination of a discourse even if that examination is supported or initiated by another (Wood, 2008). Autonomous learners thus need to demonstrate the ability to use the meta-rules of a discourse to substantiate its narratives by accessing the support from others (Wood, 2008). Through this interaction with the more proficient the learner is able to better evaluate the narratives he/she produced in terms of the mathematical discourse and become more autonomous in his/her ability in communicating about and using a mathematical discourse (Wood, 2008).

Linneweber-Lammerskitten, Schäfer & Samson (2010) write that videos and video clips as a medium of learning and teaching are well suited for autonomous learning in many mathematics classrooms. They argue however that many of the available videos are too long and often underpinned by predetermined specific outcomes and narrow pedagogical imperatives. The VITALmaths database of video clips, which consists of very short video clips (1-3 minutes long) developed by students and researchers at the School of Teacher Education at the University of Applied Sciences North-Western Switzerland and Rhodes University in South Africa, has been established as an alternative (Linneweber-Lammerskitten, Schäfer & Samson, 2010). The video clips can be downloaded from www.ru.ac.za/VITALmaths on mobile phones. The use of mobile phones may thus provide an alternative for engaging in mathematical concepts and ideas, especially in out of normal school hour learning context, especially in rural areas.

For this interpretive research project we explore the subjective understandings and interpretations, ie the experiences, of selected grade-10 learners when they used a number of VITALmaths video clips in their free time. The particular focus was how the use of physical manipulatives in the clips enhanced their autonomous learning.

**Methodology**

The study was divided into four phases:

**Phase 1:** The aim of this phase was to explore what selected learners do with selected VITALmaths video clips in their free-time. We conducted a general awareness workshop with the participants to introduce them to some of the VITALmaths video clips found on the VITALmaths website. We did not use any of the video clips that were used in phase 2, 3 and 4. We engaged them with some of the clips to create an awareness amongst the group about doing mathematics with a mobile phone. The participating learners also completed a questionnaire on their own use of mobile phones. The questions in the questionnaire probed into how learners use mobile phones.

We then downloaded three selected VITALmaths video clips onto the mobile phones of the 11 learners from the selected school that participated in the research study. We asked them to go and explore these clips. We purposefully did not provide them with any prescribed guidelines or activities, as we wanted to obtain an initial sense of how the learners used the video clips autonomously. The learners returned after two days and shared their experiences in using the video clips. We asked them to describe their experiences and asked questions such as, did they show any of the video clips to their friends or family. We also asked them what they learnt from the video clips and whether they thought the video clips were a good
idea. We also probed them on what they thought about the manipulatives used in the video clips – were they appropriate, did they try out the mathematical activities themselves using the manipulatives? We also asked them whether they thought that they could use the video clips in their own study of mathematics. Our conversations were audio recorded and transcribed for analysis.

**Phase 2:** After the completion of phase 1, we engaged the participants more formally. We selected six video clips based on the Pythagorean Theorem and Fractions. We found during schools visits as part of the first author’s work for RUMEP, that learners struggle with the conceptual understanding of the Pythagorean Theorem and Fractions. These topics are consistently used in the teaching of mathematics, in particular the teaching and learning of trigonometry in grade-10. The participants wrote a pre-test based on the Pythagorean theorem and Fractions with the rest of their Grade-10 mathematics class. After writing the pre-test, we conducted a workshop using one of the VITALmaths video clips (not on Pythagoras’ theorem or Fractions) with the eleven participants to once again familiarize them with the use of the VITALmaths video clips. The three selected Pythagoras video clips were then downloaded on the mobile phones of the eleven participants. Mobile phones were provided to the participants. They were also provided with worksheets that scaffolded the prompts in the video clips based on Pythagoras’ theorem. Participants had two weeks to complete the mathematics exercises based on the three video clips.

**Phase 3:** After the two weeks we came together again as a group and a post-test on Pythagoras’ theorem similar to the pre-test was written. All eleven participants were asked to do a presentation on the work that they had done with the video clips. The eleven presentations were video recorded and transcribed for analysis. The eleven participants were then interviewed individually on their experiences in using the video clips and the associated manipulatives in their understanding of the Pythagorean Theorem. The interviews were audio recorded and transcribed for analysis. We then downloaded the three newly selected VITALmaths video clips, which were based on the addition and subtraction of fractions, on the mobile phones of the eleven participants. A similar process was then followed as with the Pythagorean clips.

**Phase 4:** The eleven participants then wrote a post-test based on both the Pythagoras’ theorem and the Fraction video clips with the rest of their Grade-10 mathematics class. The eleven participants were also interviewed individually on their experiences in using the video clips and manipulatives in their understanding of fractions. The interviews were audio recorded and transcribed for analysis.

**Initial research analysis and findings**

Our data analysis started immediately after Phase 1 of the research design and continued throughout the other phases. Although learners engaged in the Pythagorean theorem and the addition and subtraction of fractions, for the purpose of this paper we will report only on the analysis of the Pythagorean theorem.

The graph below shows a comparison between the pre- and post test results of the whole Grade-10 mathematics class. Only one of the learners in the Grade-10 class did not show an improvement from the pre-test to the post-test in the Pythagorean test. The results of the learners who participated in the research study are represented on the graph from participant 23 to 32. All these participants showed an increase in their results from the pre-test to the post test. Only two of the 11 participants had a test score of above 50% in the pre-test, while only two of the 11 participants had a test score below 50% in the post-test. Six learners had test
scores above 60% in the post test. Only participant 28 had a test score above 80%. The data analysis of two of the participating learners will be highlighted to show how the analysis was done to find features of autonomous learning in the participants’ engagement with the VITALmaths video clips. The two participants’ data is represented as learner 28 (Keageletse), who obtained the highest score in the post-test and learner 31 (Itshepiseng), who obtained the lowest score in the post-test.

The table below shows the analysis of Pythagorean pre-and post-tests by using a grading scale, from poor to excellent, for how the 11 participants faired in every question of the test. For example, in Question 1 of both tests: Seven of the participants’ answers to the question were poor in the pre-test, while none of the participants gave poor answers to the question in the post-test. Three of the participants gave fair answers to the question in the pre- and post-test. One of the participants gave good answers to the question in the pre-test, while three gave good answers to the question in the post-test. None of the learners gave excellent answers to the question in the pre-test, while five gave excellent answers to the question in the post-test. A comparison of the grading from pre-test to post-test thus shows that the 11 participants’ understanding of the Pythagorean theorem improved from being mostly poor-to-fair in the pre-test to mostly good-to-excellent in the post-test.
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Total Number of participants</th>
<th>Grading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-test</td>
<td>Post-test</td>
</tr>
<tr>
<td>Question1</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Question 2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Question 3a</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Question 3b</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Question 4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Question 5</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Question 6</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
A qualitative analysis of the pre-test and post-test per question for Keageletse and Itshepiseng showed that Keageletse’s test scores improved, because she was able to move from a poor or fair conceptual understanding in the majority of the questions in the pre-test to a good or excellent conceptual understanding of the corresponding questions in the post-test. Itshepiseng, however, was stuck in poor or fair conceptual understanding in most of the questions in both the pre-test and the post test of the Pythagorean theorem. Below are the analysis of the two participants’ responses to the questions in the pre- and post test, based on the Pythagorean theorem, and the analysis of the presentations done by the two participants after their interaction with the Pythagorean video clip.

**Keageletse’s Pythagorean theorem pre-and post-test analysis**

In Question 1 Keageletse was unable to explain the meaning of area at all in the pre-test, while in the post-test she could partially explain the meaning of area by using the given information. In Question 2 she was unable to explain what the Pythagorean theorem entails in the pre-test, while she could partially explain the Pythagorean theorem in her own words. In Question 3b the participant was able to find the lengths of the two missing sides of the two right-angled triangles in both the pre-and post tests. In Question 4 she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle in the pre-test, in the post test she was able to choose the correct triangle and gave a good reason for her choice. In Question 5 Kaekeletshe chose the correct letter for the correct solution to the question in the pre-and post test. In Question 6 she was able to apply correctly the Pythagorean theorem to solve a problem involving a right-angled triangle in both the pre-and post test; and in Question 7 she attempted to demonstrate the Pythagorean theorem with the aid of manipulatives but did it incorrectly in the pre-test. In the post-test she used manipulatives to demonstrate the Pythagorean theorem correctly.

**Analysis of Keageletse’s presentation**

During the demonstration of the Pythagorean theorem, Keageletse showed the Pythagorean theorem by using different coloured cards for the squares that she stuck onto the three sides of her right-angled triangle. She was able to demonstrate both methods for proving the Pythagorean theorem using manipulatives. Although she mentioned the sides when she described the theorem of Pythagoras, she pointed to the squares that were stuck onto the sides of her right-angled triangle. In her response to the question on the worksheet on what she understood by the theorem of Pythagoras, she described the theorem as a method to find the unknown side if you are given the other two sides of a right-angled triangle. When she
described the method, she pointed to the sides of her model and naming the different sides the adjacent side, the opposite side and the hypotenuse. She described how to find the hypotenuse. When we further probed her on finding the adjacent side when given the hypotenuse and the opposite side of her right-angled triangle, she said that she would follow the same steps but as you proceed with the steps, you would minus the values. This shows that she has a conceptual understanding of what the Pythagorean theorem entails.

**Initial findings for Keageletse**

Keageletse’s use of words was consistent with the words used by others to describe the Pythagorean theorem in her post-test and presentation. She was unable to demonstrate or describe the Pythagorean theorem in the pre-test. In both her presentation and post-test she was able to use manipulatives correctly to show her thinking about the Pythagorean theorem. She was also able to use endorsed narratives about the Pythagorean theorem in her presentation and the post-test. However, the endorsed narratives she used in the pre-test were vague. Keageletse description of the Pythagorean theorem in both her presentation and post-test was consistent with the meta-rules that guide the discursive routines of this mathematical discourse. Keageletse thus demonstrated that, through her exploration of and interaction with the discourse-for-others, she was able to make the discourse-for-other into a discourse-for-herself. She thus demonstrated features of an autonomous learner after her engagement with the VITALmaths video clips.

**Itshepiseng’s Pythagorean theorem pre-and post-test analysis**

In Question 1 Itshepiseng gave a vague meaning of area in the pre-test, while in the post-test she was able to give a clear and concise meaning of area. In Question 2 she had a vague idea of what the Pythagorean theorem entails in the pre-test and the post-test. In Question 3a she was only able to find the area of one missing square in the pre- and post-test. In Question 3b the participant was able to find the lengths of the two missing sides of the two right-angled triangles in both the pre-and post tests. In Question 4 she chose the correct right-angled triangle but was unable to give a reason for choosing the specific triangle in the pre-test and post-test. In Question 5 Itshepiseng chose the incorrect letter for the correct solution to the question in the pre-and post test. In Question 6 she did not attempt the question at all in the pre-test, she was however able to solve the problem correctly by using the Pythagorean theorem in the post-test; and in Question 7 she attempted to demonstrate the Pythagorean theorem with the aid of manipulatives but did it incorrectly in the pre-test, in the post-test she used manipulatives to demonstrate the Pythagorean theorem correctly.

**Analysis of Itshepiseng’s presentation**

Although Itshepiseng was able to show both Alex’s and Ben’s method for proving the Pythagorean theorem, she was unsure what the theorem entails. She mentioned that she tried out what she saw in the video clips, but struggled to put the model together to demonstrate the theorem. When she was asked to describe the Pythagorean theorem, she described it as the relationship in Euclidean geometry. When probed further about the Euclidean relationship, she said that it forms a formula that you use to find C squared. She pointed to the hypotenuse of her right-angled triangle. When probed about the formula, she said that the formula is $A^2 + B^2 = C^2$ and that she got the formula from the books. She pointed to the different sides of her right-angled triangle when she mentioned $A^2$, $B^2$ and $C^2$. She could not explain what was meant by square.
**Initial Findings for Itshepiseg**

Itshepiseng’s use of words to describe the Pythagorean theorem was not consistent with the words used by others. She, for example, described the Pythagorean theorem as a relationship in Euclidean geometry. During her presentation, she later tried to explain the Pythagorean theorem by using the adjacent side, opposite side and hypotenuse. However, her description of the theorem was still vague. She also struggled to demonstrate her understanding of the Pythagorean theorem in the pre- and post tests. She was unable to use the endorsed narratives of the mathematical discourse for the Pythagorean theorem in her presentation, pre-test and post-test. Itshepiseng description of the Pythagorean theorem was not consistent with the meta-rules that guide the discursive routines of this mathematical discourse. She thus was not able to make the discourse-for-others into a discourse-for-herself. Although she was able to apply the Pythagorean theorem in solving a problem involving the theorem in the post-test, she did not show all the features of an autonomous learner after her engagement with the VITALmaths video clips.

**Conclusion**

After the initial analysis of the eleven participants’ tests and presentations, it was found that the use of the video clips with specially prepared worksheets encouraged the participants to use manipulatives in their learning of mathematics. All the participants used the cards that they were given, in their attempts to demonstrate the Pythagorean theorem. Ten of the eleven participants were able to demonstrate the Pythagorean theorem correctly by using one of the three methods that were explained in the video clips. Eight of the eleven participants showed features of autonomous learning by making a discourse-for-others into a discourse-for-oneself after their engagement with the video clips on the Pythagorean theorem. The participants’ engagement with the video clips thus enhanced their learning of the Pythagorean theorem.

**References**


Exploring Frameworks for Identifying Learning Dispositions: the Story of Saki

Diliza Hewana & Mellony Graven

South African Numeracy Chair Project, Rhodes University, Grahamstown, South Africa
m.graven@ru.ac.za

This paper investigates how one sampled learner’s mathematical learning dispositions evolved within the context of his participation in a weekly after school mathematics club over a one year period. The study is informed by Kilpatrick, Swafford & Findell’s (2001) definition of a productive disposition (the fifth strand of mathematical proficiency) and Carr & Claxton’s (2002) three indicators of key learning dispositions. This paper analyses the shifting nature of Saki’s responses to an instrument focused on learning dispositions and points to ways of extending dispositional definitions and frameworks. We use Saki’s learning story to illuminate the way in which restricted mathematical learning dispositions, particularly in terms of sense making and resourcefulness, can impede mathematical proficiency progress and thus require increased attention.

Introduction

This paper draws on a broader research project in which the first author, as part of the South African Numeracy Chair (SANC) project and as part of his masters research investigated the evolving learning dispositions of three learners participating in an after school mathematics club run by the second author. Data collection involved video recordings of club sessions, numerous individually administered learner interviews (including task based interviews) and written numeracy assessments. All learner interviews were conducted by the second author, as the club facilitator, audio recorded and later transcribed. Additionally the first author’s research was supervised by the second author and insights from the broader SANCP research into learner dispositions informs this research.

It is beyond the scope of this paper to elaborate on all three case study learners’ dispositions and so we have chosen to share the story of only one learner, Saki, in order to illuminate ways in which dispositional definitions and frameworks enable us to explore dispositions of learners as well as to unpack the limitations of these definitions.

In this paper we thus summarize and discuss the responses of Saki, to the productive disposition instrument (PD Instrument) developed by Graven (2012). In this paper we focus on and compare Saki’s written responses to this orally administered written questionnaire in 2012 and his oral interview responses in 2013. Our work is guided by a socio-cultural theory of learning where learning involves developing dispositions and ways of being (Gresalfi & Cobb, 2006). Additionally our work assumes that learning to learn can be actively and deliberately supported by developing productive learning habits (Claxton & Carr, 2004). Our analysis of Saki’s responses theoretically draws from Kilpatrick et al.’s (2001, p.131) definition of productive disposition as the fifth of five interrelated strands of mathematical proficiency, i.e. ‘the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner,’ and Carr and Claxton’s (2002) three dimensions of learning disposition (viz. resilience; reciprocity; playfulness).
Contextualizing the study

Across numerous national, regional and international studies (e.g. the Annual National Assessments (DBE, 2013); SACMEQ (see Spaull, 2011) and TIMSS (see Reddy, 2006) South African primary mathematics learners fare poorly. Fleisch’s (2008) book, Primary Education in Crisis: Why South African schoolchildren underachieve in reading and mathematics explores influencing factors on South African learner performance such as parental education, books in the home, language in the home, opportunity for ECD care, families without parents as well as the effects of child labour. He argues however that even while the quantum of poverty in SA is experienced in other countries, the dependency of the poor and their profound disempowerment is perhaps greater than for the “poverty stricken peasantry of our neighbours of the north” (p. 59). This perhaps is a factor in why our performance in international comparative mathematics assessments such as TIMSS (Reddy, 2006) has us performing below other developing countries with less wealth. Thus Graven (2014, 9) argues ‘more research is needed to examine ways in which dependent poverty and dependent passive learning dispositions might impact on mathematical learning’.

The results of the 2003 TIMSS study place SA 50th of 50 countries but furthermore foregrounded that had the largest variation in scores where the average scores of SA learners in African schools were almost half of that of historically white schools. Furthermore the average scores of these African schools has decreased from TIMSS 1999 to TIMSS 2003. Finding possible ways forward in the South African mathematics education crisis in ways that addresses the inequality and inequity of performance of our schools previously disadvantaged under apartheid is a key aim of the South African Numeracy Chair Project. This project began in 2011 and works with a large community of primary mathematics teachers and post-graduate researchers to explore ways forward to the many challenges faced. One innovation which was piloted in 2011 and introduced in 2012 is that of setting up after school mathematics clubs for Grade 3 and Grade 4 learners. This is discussed below as one such club formed the empirical field for the research on which this paper is based.

After school mathematics clubs

The after school weekly maths club that forms the empirical field for this study involved 6 learners participating in this club for about an hour. A central objective for the club was to develop learner sense-making in numeracy, ‘shifting learner dispositions from being passive learners to becoming active participators’ (Graven & Stott, 2012). Thus the clubs are intended to be a hub of increased sense-making where learners mathematically engage with content and with one another. This club is part of a broader after school club program which runs clubs across a range of schools and development centres in the broader Grahamstown area. The club program is expected to create an environment that is less structured than ‘traditional’ classrooms and that can offer opportunities and more affordances for the creation of active engagement, negotiation and participation for learners. Additionally the smaller numbers of learners in clubs (between 6-15 learners) enables increased individualized learner attention where activities can be directly tailored for where learners are in their numeracy development rather than dictated to by the grade level they are in at school. At the same time the clubs are not intended to overshadow in any way the normal and structured school curricula or program but clubs come in to ‘provide more freedom to focus on the deliberate construction of positive participatory mathematical identities, at the expense of covering the range of skills and knowledge required to ‘get through’ the curriculum’ (Graven & Stott, 2012, p. 96). The combination of the first author’s role as researcher and the second author’s
role as facilitator enabled a strong working relationship for investigating learner dispositions. Three key research questions provide the focus for this paper:

1. What is the nature of Saki’s mathematical learning disposition? How might this disposition evolve within the context of his participation in a weekly after school mathematics club over time?

2. What adaptations/elaborations of existing dispositional instruments are required to better access and assess learner dispositions?

3. What are the implications of this analysis for adapting/extending Kilpatrick, Swafford and Findell’s (2001) definition of productive disposition?

**Theoretical Framing**

The study is underpinned by a socio-cultural perspective of learning. An emergent body of literature (see Schoenfeld, 1992; Schoenfeld & Kilpatrick, 2008; Lerman, 2000) conceives of mathematics learning as an inherently social activity, an essentially constructive activity. The definition of mathematical/numeracy proficiency that guides our research is that of Kilpatrick et al.’s five interrelated strands, namely: procedural fluency, conceptual understanding, adaptive reasoning, strategic competence and productive disposition. Productive disposition, as they define it,

refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. (p.131).

In relation to the fifth strand Kilpatrick et al. (2001, p.131) note that developing a productive disposition requires ‘frequent opportunity to make sense of mathematics, to recognise the benefit of perseverance and to experience the rewards of sense making in mathematics.’ They argue that productive disposition develops when other strands develop. For example, as students build strategic competence in solving non-routine problems, their attitudes and beliefs about themselves as mathematics learners become more positive. The more mathematical concepts they understand, the more sensible mathematics becomes. This coheres with the South African Curriculum and Assessment Statements for Foundation and Intermediate Phase (DBE, 2011a, 2011b) which also connects sense making and conceptual understanding and says that mathematics is a creative part of human activity and that learners should develop a deep conceptual understanding in order to make sense of mathematics.

**Researching dispositions and identifying a gap in research literature**

Schoenfeld (1992, 348), mentions five aspects of cognition drawn from a range of literature important for mathematical problem solving, these include: ‘the knowledge base; problem solving strategies; monitoring and control, beliefs and effects, and practice’. While these can be linked to some of Kilpatrick et al.’s (2001, 131) strands of mathematical proficiency there is no mention of learning disposition although ‘beliefs and affects’ does connect with the part of Kipatricke et al.’s productive disposition definition that says ‘to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner’. Carr & Claxton (2002, pp. 9-10) drawing on Carr’s earlier work differentiate between capabilities and learning dispositions as follows:

Crudely, we might say that this real-life ‘learning power’ (Claxton, 1999a) consist of two interrelated facets: capabilities and dispositions. Capabilities are the skills, strategies and abilities which learning requires: what you might think of as the ‘toolkit’
of learning. To be a good learner you have to be able. But if such capabilities are necessary, they are not of themselves sufficient. One has to be disposed to learn, ready and willing to take learning opportunities, as well as able.

Despite increasing attention being paid to learning dispositions (in general) over the past two decades however Graven (2012) asserts that there is little mathematics education research that elaborates on the nature of the relationship between learners’ knowledge base (procedural fluency and conceptual understanding), problem solving strategies (adaptive reasoning and strategic competence) and learner dispositions. Such an understanding might be used to support the design of rich learning opportunities across the strands of proficiency. Thus, there is a gap in the maths education research on this strand (Graven, 2012; Graven, Hewana & Stott, 2013).

Gresalfi’s (2009, p.327) view is that ‘although this work has identified areas commendable of further enquiry, it has not stimulated the scrutiny of how classroom practice could boost learners learning practice and motivation.’ She further argues that ‘the literature does not help to explain why classroom practice does not impact all learners the same way or which aspects of classroom practice serve to support the development of various dispositions towards learning among learners who are members of the same classroom.’ Gresalfi and Cobb (2006, p.329) assert ‘thus learning is a process of developing dispositions; that is, ways of being in the world that involves ideas about perspectives on, and engagement with information that can be seen both in moments of interaction and in more enduring patterns over time.’ Thomas & Brown (2007, p.8) noted:

Dispositions involve ‘attitude or comportment’ toward the world, generated through a set of practices which can be seen to be interconnected in a general way…. dispositions are not descriptions of events or practices; they are the mechanisms that engender those events or practices. In short, dispositions capture not only to what one knows but how he or she knows it; and not only the skills one has acquired, but how those skills are leveraged.

Thus according to a range of literature developing a productive disposition requires frequent opportunity to make sense of mathematics, to recognise the benefit of perseverance and to experience the rewards of sense making in mathematics.

**Bringing together Kilpatrick et al.’s 2001 indicators of Productive Disposition with Carr & Claxton’s (2002) three dimensions of ‘Resilience, Playfulness and Reciprocity’**

Within the broader SANC project work on dispositions (e.g. Graven, Hewana & Stott, 2013) we have collaboratively worked towards combining the work of Carr and Claxton (2002) with the notion of productive dispositions and designed rubrics and observational grids that pull these together for the purposes of analysis. Below we explain the similarities and differences of Kilpatrick et al. (2001) and Carr and Claxton’s (2002) key dispositional indicators as these together provide the framework for analysis of Saki’s evolving disposition.
Table 2. Cross mapping dispositional indicators within definitions

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tendency to see sense in maths</td>
<td>Links to ‘resourcefulness’ – conceptual/explorative understanding</td>
</tr>
<tr>
<td>Perceive it as both useful and worthwhile</td>
<td>Not connected – no equivalent in Carr and Claxton’s three dimensions</td>
</tr>
<tr>
<td>Believe steady effort pays off</td>
<td>Links to resilience</td>
</tr>
<tr>
<td>See oneself as effective and doer of maths</td>
<td>Links to some extent to resourcefulness however the notion of self-efficacy is not directly addressed in Carr and Claxton</td>
</tr>
<tr>
<td>No indication of willingness to engage with others as an indicator of a productive disposition</td>
<td>Reciprocity – willingness to engage with others</td>
</tr>
</tbody>
</table>

From the above ‘reciprocity’ (the third dimension of Carr and Claxton) is notably absent from Kilpatrick et al.’s (2001) definition. Conversely there is no link to the notion of seeing mathematics as useful and worthwhile in Carr and Claxton’s three key learning dispositions - maybe because their suggested grid is not subject specific but developed from the early childhood learning context. However a positive affective relationship towards an area of learning like mathematics or even towards learning in general could be a useful fourth dimension or added indicator.

Resourcefulness links directly with sense making, conceptual understanding, adaptive reasoning, strategy (strategic competence etc.) and independence of learning in seeing that one can figure it out drawing on one’s own thinking.

While actively seeking help might be considered resourceful it is not placed here as it goes against this sense of resourcefulness in terms of one’s own ability. Additionally ‘enjoyment or passion’ - enthusiasm/creativity both are missing (Graven & Schafer, 2014). For the purposes of this research we have included this aspect as a dimension of strength to resilience, playfulness and reciprocity (when indicators are present).

Research Methodology

A qualitative research design with a case study approach was used for the broader study from which this paper emerges. A wide range of data collection tools informed the learner stories of the broader research. This range of data allowed for both triangulation of data and ‘thick description’ (Cohen, Manion & Morrison, 2005). However for the purposes of this paper we focus only on data gathered from one instrument used first as an orally administered but written response questionnaire and secondly as an interview.

The questionnaire involved several “complete the sentence” items. One involved locating oneself on a spectrum of learner performance (from 1-9), others involved describing Mpho and Sam who were explained to be weak and strong at maths respectively. A final question asked learners about what they do if they do not know an answer. Thus the five items discussed here include:
1. Maths is….
2. Effective scale (Range 1-9)
3. Describe an effective learner: Sam is…
4. Do you love maths or are you scared of maths?
5. What do you do if you don’t know an answer in a maths class?

The instrument is taken from Graven (2012). Interviews were fully transcribed. Permission was obtained from both the centre where the club took place as well as from parents of learners. Learners were given the right to withdraw at any time and it was made clear to learners and parents that participation was entirely voluntary and that learners could withdraw at any time. Thematic content analysis was conducted on learner responses.

Findings and Discussions

When categorising learner responses to the instrument the following legend was used for indicators: Kilpatrick et al. (2001) = (K); Carr & Claxton (2002) = (C&C); Emergent Categories = (E).

Table 6. Summary of Saki’s responses to the learning disposition instrument.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Questionnaire item</th>
<th>May 2012</th>
<th>May 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective learner and doer of mathematics (K)</td>
<td>Scale 1-9 (Q2)</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Seeing mathematics as useful and worthwhile (K)</td>
<td>Maths is: (Q1)</td>
<td>Die beste (the best)</td>
<td>goed om te leer (good to learn) want dit help (because it helps it makes you clever)</td>
</tr>
<tr>
<td>Sense making (K) resourcefulness (E) (which includes what Carr &amp; Claxton call playfulness (C&amp;C))</td>
<td>Maths is: (Q1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>What do you do if you don't know the answer in maths class? (Q6)</td>
<td>Ek vra die juffrou om te help (I ask the teacher to help)</td>
<td>vra die juffrou, (ask the teacher) tel op my hande (count on my hands) , tel op die telkaart (count on the counting card)</td>
</tr>
<tr>
<td>Steady effort (K) resilience (C&amp;C)</td>
<td>Describe an effective learner of mathematics (Q4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>What do you do if you don't know the answer in maths class? (Q6)</td>
<td>Ek vra die juffrou om te help (I ask the teacher to help)</td>
<td>tel op my hande (count on my hands) , tel op die telkaart (count on the counting card)</td>
</tr>
<tr>
<td>Reciprocity (C&amp;C)</td>
<td>No question directly related to reciprocity although some other learners indicated aspects of this when answering what they did when they did not know an answer. For example: ‘I discuss it with my friend.’</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compliant behaviour (E)</td>
<td>Describe an effective learner of mathematics (Q4)</td>
<td></td>
<td>Juister na die juvrou (listens to the teacher) hy doen goed want hy wen</td>
</tr>
</tbody>
</table>
From the above table we note the absences as much as the presences. Saki across both years clearly sees himself as an ‘effective learner and doer of mathematics’ (allocates himself at the top of the performance spectrum, at 9) and sees mathematics as worthwhile (‘the best’ and ‘it makes you clever’). Additionally he ‘loves maths’ – although this is because he loves counting – ‘I love to count’. Of interest, a love of mathematics is not included within Kilpatrick et al.’s (2001) definition and nor is a love of learning noted as a key dispositional dimension for Carr and Claxton (2002).

We see some shift in terms of his teacher dependence when he doesn’t know an answer (in 2012) towards still asking the teacher but also having counting on fingers or on a 100 counting chart available to him as an alternative strategy (in 2013). In terms of his description of an effective learner of Mathematics (i.e. Sam is…) he moves from being unwilling or unable to describe him in 2012 toward describing him as a compliant learner who ‘listens to the teacher’. While few teachers would argue against learner’s compliance and listening, our new curriculum puts emphasis on sense making. Additionally individual learner agency, steady effort, resilience and so on are noticeably absent from Saki’s shift. Additionally while Saki sees himself as good at mathematics his performance in other assessments does not match this self-evaluation. Thus perhaps it needs to be noted that various aspects of Kilpatrick et al.’s (2001) productive disposition should all be present for optimal learning. Thus in the absence of sense making confidence in one’s mathematical ability can be problematic. For Saki since he eventually arrives at answers correctly through slow one-to-one finger counting for calculation (even in Grade 4) he sees himself as good at maths and enjoys it. He puts in ‘steady effort’ (using one-to-one finger counting), is compliant and listens to the teacher. However these two aspects of his disposition do not enable mathematical proficiency as his sense of mathematics as being worthwhile needs to be
connected to the other strands of conceptual understanding, strategic competence, procedural fluency and adaptive reasoning. In this respect Saki’s disposition is restricted.

In response to the latter two research questions, Saki’s data points to some limitations of the instrument itself, which are explored elsewhere (e.g. Graven, Hewana & Stott, 2013; Graven & Heyd-Metzuyanim, 2014). Particularly that the instrument focuses on what learners say about mathematics or being good/bad at mathematics (or what they say they do – as in the last question) rather than what they actually do (which is better gleaned from observation) is a key limitation. Indeed we gathered much richer data from observing Saki across club sessions.

Saki’s data also points to an absence of the notion of learner independence as an important dispositional trait. Compliance coupled with learner agency and an ability to make progress even when a teacher is not present perhaps need to be considered. Compliance and agency are not either-or but perhaps need to be considered and developed together. Observational data pointed to Saki’s progress in mathematical proficiency being held back by his poor conceptual understanding and low levels of procedural fluency even while he displayed hard work and steady effort. He also showed a strong positive attitude towards the subject saying that he loved it. He was almost always first to arrive in the club and across all club learners had completed the highest number of homework pages each week. However his effort (and his love of maths) tended to foreground the method of one to one counting (with his fingers) often irrespective of how small or large the numbers in the calculation. As a result his performance progress was however slower than others in the club. His absence of sense-making – the aspect of Kilpatrick et al’s (2001) definition of productive disposition - is thus essential for progress and this sense making clearly links to conceptual understanding as this is defined in sense making terms. Saki and the other case study learners, while not generalisable, have provided us with powerful illuminatory vignettes which carry the theoretical insights of this study forward into our future teaching and work in mathematics education.

**Concluding remarks**

It has been beyond the scope of this paper to explore evolving dispositions in detail or to review the data obtained from more than one learner. However we hope that we have illuminated both the strengths and limitations of both our instrument used to gather dispositions as well as the definitional frameworks of Kilpatrick et al. (2001) and Carr and Claxton (2002). A much larger study would enable elaboration of a much wider range of indicators perhaps with some specificity to South African learners.

**References**


Department of Basic Education (2011a) (DBE) *South African Curriculum and Assessment Statements for Foundation Phase*. Pretoria: Department of Education.


Fleisch, B. (2008). *Primary Education in Crisis: Why South African schoolchildren under achieve in*


**Acknowledgements**

The work of the SA Numeracy Chair, Rhodes University is supported by the FirstRand Foundation (with the RMB), Anglo American Chairman’s fund, the Department of Science and Technology and the National Research Foundation. We thank the broader team of researchers within the Project, Varonique Sias and Debbie Stott for their support in the broader club project on which this paper is based.
Mathematical Literacy: Are We Making Any Headway?

Mark Jacobs¹ & Duncan Mhakure²

¹Faculty Management, Faculty of Engineering, Cape Peninsula University of Technology  ²Numeracy Centre, Centre for Higher Education Department, University of Cape Town, South Africa.

¹JacobsMS@cput.ac.za  ²duncan.mhakure@uct.ac.za

Student enrolment trends in Mathematical Literacy and Mathematics since the inception of the NSC Mathematical Literacy is a focus of this paper. We found that enrolments for Mathematics largely lagged behind those for Mathematical Literacy. While the statistics was not fixed, we found a greater emphasis on trying to prove the uptake and success in Mathematics than Mathematical Literacy. We also explored and developed the critique of Mathematical Literacy assessment standards and compared those standards against an external education authority that was the reference for a benchmarking process by the Department Of Basic Education in 2013. We determined that the South African programme compared unfavourably against that of New South Wales. We also showed that the external body included algebraic modelling and financial mathematics which seemed to align better with our Mathematics curriculum than Mathematical Literacy.

Lastly, we analysed the environment of Mathematical Literacy tasks and the cognitive loading of those tasks, and proposed, by way of illustration, ways in which similar tasks could be improved. These illustrative tasks we called authentic tasks.

Our conclusion is that Mathematical Literacy in its present form offers a weakened version of a mathematically based subject for “making sense” in the world and should be improved.

Introduction

Since the introduction of Mathematical Literacy (ML) in the Further Education and Training (FET) curriculum in 2006, all Further Education and Training (FET) students were required to take either Mathematics or ML as a subject. The Mathematics/ML divide replaced the system whereby students could chose to do Mathematics or not, and if they chose to do Mathematics, whether they could do Mathematics on the so-called Higher Grade or Standard Grade. This move was listed in government policy documents as an attempt to provide every student taking ML with the mathematical tools needed to make sense of, participate in and contribute to the twenty-first century world, a world characterised by numbers, numerically based arguments and data representation. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology to name but a few (NCS ML assessment guides, p.13).

This definition concurs with, among others, Schleicher (1999, p.39):

Mathematical Literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well informed judgements, and to engage in mathematics in ways that meets the needs of that individual’s current and future life as a constructive, concerned and reflective citizen.

The emphasis in the definitions is on “making sense” of the world through the use of mathematical tools, and included in the list of tools are skills such as “ability to reason” and “solve problems”, two key features which act, to some extent as markers for the quality of
the ML that is delivered. The use of “world” is understood to mean the world outside the classroom (although the latter provides the setting where ML takes place). To match up these two settings, researchers have suggested the use of authentic contexts to act as a bridge (Madison, 2014).

Mathematical literacy is not the same as mathematics, nor is it an alternative to mathematics – ML and Mathematics are two different subjects at the FET phase. In the words of Steen “Mathematics is abstract and Platonic, offering absolute truths about relations among ideal objects” (Steen, 2001, p.1). Madison (2004, p.12) posits that the reason why these two subjects were separated could well been that those advocating for the teaching of mathematics feared that “teaching contextualised mathematics will water down the mathematics, that fewer students will learn the formal mathematics for science and engineering”. Is this the rationale for the National Curriculum Statement Mathematical Literacy? The trend whereby students are divided along academic grounds according to whether they take ML or Mathematics is clearly observed in South Africa, where more students are opting to study ML than Mathematics at the FET phase, are as a resulted counted out of the major science, engineering, certain business and medical fields as possible careers.

The divide between ML and mathematics, understood in the traditional sense as composed of content such as algebra, geometry, trigonometry and calculus, favours a strong academic framing for mathematics and weaker academic framing for ML. This divide has implications such as access to certain fields of study at post-secondary schooling but also has the uncomfortable effect of dividing students along academic status lines in school (to all involved). ML does not enjoy the headline attraction of mathematics; in contrast it is marginalised. At best it is seen as a “safer” option to attain credits to enable students to success at the very significant high stakes examinations, notwithstanding its limited options beyond schooling.

This study

The challenge for ML resides in the question: Do the current ML teaching and assessments practices do justice in meeting the goals of ML as described by the NCS? If not, should the stakeholders in mathematics education be concerned? Although much of this study is concerned with the migration of students from Mathematics to ML, a word of caution is necessary here: ML can and must be seen as a subject in its own right. Consequently, critiquing ML has relevance outside of the debate of how and why students have ended up taking that subject. This study considers the implications from both angles but favours and emphasises ML as the subject of study. We seek to demonstrate that more students at the FET phase are opting to study ML, partly because it is perceived to be an easier option to get a mathematical subject pass. Many do the subject because they have no need to utilise mathematics subsequent to school and are not attracted by the promise of the worthwhileness or beauty of mathematics. In addition, the study also seeks to show that the teaching and assessment of ML uses non–authentic tasks which demand very low level cognitive skills, which is contrary to what the NCS states. Given also that students who study ML cannot enrol in quantitative disciplines such as science, technology, engineering, and mathematics (STEM) we are left wondering what are the future implications of so many learners taking ML in the mathematics education landscape in South Africa are designed to be? This will be addressed at the end.

To consider these issues we therefore, in this study, i) survey the trends in student enrolments for ML and Mathematics, both at a national, provincial and local (school) level (using the Western Cape as an example), ii) revisit the assessment standards against other
established education bodies, and, iii) consider the notion of authentic environments, the place it has in a ML classroom and ML assessments, against the current reality, as reflected in NSC ML assessments. We analyse statements made (or not made) by academics and government officials in relation to these trends and ask some probing questions as a consequence. In particular, we posit whether ML in its present form can or should continue.

**Enrolments in Mathematics and Mathematical Literacy**

At national level, the trend has been for ML to increase and Mathematics to decrease (Table 1). Although Mathematics shows an increase (from 2012 to 2013 it was 15 469) it is the comparatively more dramatic increase of ML that is the concern of this paper (33 715). More so, 85 126 more students were enrolled for ML compared to Mathematics. While we must applaud the fact that in the new curriculum everyone is engaged in a mathematically based subject, there is some evidence that the quality of the ML assessment is of a reduced quality and the implications of this has far reaching consequences, especially for the candidates.

**Table 1. Enrolments of students in ML and Mathematics between 2009 and 2013**

<table>
<thead>
<tr>
<th></th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematica 1 Literacy</td>
<td>284 174</td>
<td>288 370</td>
<td>281 613</td>
<td>297 074</td>
<td>330 789</td>
</tr>
<tr>
<td>Mathematics</td>
<td>296 164</td>
<td>270 598</td>
<td>229 371</td>
<td>230 194</td>
<td>245 663</td>
</tr>
</tbody>
</table>

*Source: Technical report 2013 Department of Basic Education, p.21*

At Western Cape provincial level, the strategies for Mathematics and Science document is illustrative in a number of ways. If Mathematics is the foil to ML then intentions about Mathematics need to be read both in its own right and for signs about the provincial or national intentions for ML. As we see stated in this document the province is intent on improving both the enrolment figures for Mathematics as well as the results in the examinations, especially for grade 12. Is there a parallel strategy for ML? Although there is acknowledgement that the behaviour of one may directly influence that of the other (enrolment is an obvious example but trends in assessment may be another):

> An analysis of the enrolment trends over the last 3 years indicates a downturn in the numbers taking Mathematics, both as an absolute value (2 412 fewer since 2008) and as a percentage of the total enrolment. *The reduction in numbers is linked to increases in numbers of those taking Mathematical Literacy.* (Western Cape Education Department Strategy, p.1, *my emphasis*).

Admittedly, this is document dealing with strategies for Mathematics and Science, two key subjects for admission to certain mathematics and science based tertiary programmes. Nonetheless, the document does provide useful insights, albeit obliquely, into ML in the province. Table 3 offers such an opportunity.
Table 2. Overall mathematics enrolment in the Western Cape Education Department between 2008 and 2011.

<table>
<thead>
<tr>
<th>Overall Mathematics enrolment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 10</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>2008</td>
</tr>
<tr>
<td>2009</td>
</tr>
<tr>
<td>2010</td>
</tr>
<tr>
<td>2011</td>
</tr>
</tbody>
</table>

Source: Western Cape Education Department, 2013

In line with the decrease in Mathematics enrolment, one must assume, as the report states, an increase in ML enrolment. So as enrolment in mathematics declines from 35,036 in grade 10 in 2008, to 27,122 in grade 11 in 2009, allowing for failures and drop outs we are looking at a figure of 7914, some of which must end up in the grade 11 ML classes. Again from grade 11 in 2009 to grade 12 in 2010 the drop in Mathematics is 10 200. This figure contains failures in grade 11, dropouts and those migrating to ML. While the concern for the drop in Mathematics is huge and valid, the increase in ML, given what kind of assessments learners doing ML are exposed to, and the implications thereof, is surely of equal concern. As the Mathematics enrolment decreases from 18 668 in 2008 to 13 375 in 2011, this has to be taken in conjunction with the pass rate figures (notwithstanding that those achieving over 30% but less than 40% are considered having passed). Increases in the number who pass ML leads to dubious joy: it illustrates that more are passing ML than mathematics, both in number and as a percentage and it illustrates that more learners will be drawn to ML if it means a pass mark on a NSC certificate (ML) and not a fail (Mathematics). Such simple analysis is unfortunately born out at the local school level: In a Western Cape school, the principal reported that, on account that all his pupils have failed mathematics in grade 12 in 2007, the school decided that they would not offer “pure” mathematics the following year. Similarly, the principal of another school stated that the school did not have any mathematics pupils in 2014 (as reported in the newspaper Cape Times, 29 July 2014, p.3). In declaring its aim to increase the number of learners taking Mathematics and to improve the results in Mathematics in the province, the impression is strongly created that the departmental mathematical goal is geared towards Mathematics not ML. Is this an indirect acknowledgement that ML, certainly in its current form, is not desirable from the provincial point of view? No parallel document at present exists for improving ML.

In July, 2014 the Minister of Basic Education told parliament that there were 327 schools “at which no Grade 12 pupils has registered to write matric maths” (Cape Times, 29 July, 2014, front page). The newspaper article continues with a direct quotation from the minister: “This does not imply that the school did not offer mathematics, but rather there were no learners who registered for mathematics” (p.3).

Furthermore, the paper continued, the Department (of Basic Education) were unable to “indicate precisely” (their direct quotation from the minister’s response) why pupils were choosing maths literacy rather than mathematics. Five years into the new NSC Mathematics/ML divide, this is quite an extraordinary admission.
Assessments in Mathematical Literacy

Benchmarking of ML was done for the first time in 2013. The verdict was that “the paper (2013 ML) was considered to assess critical thinking and was deemed comparable to General Mathematics offered by the Board of Studies New South Wales (NSW) in Australia” (Technical Report, Department of Basic Education (DBE), p.44-5).

The present reality of ML assessments is that emphasis have been placed on level 1 and 2 of the assessment taxonomy (knowing; and, apply routine procedure in familiar context) which together can account for 60% of the final mark, while level 3 (apply multi-step procedure in a variety of context) and level 4 (reasoning and reflecting) type questions are minimally represented. Where they are, they are often reduced in complexity (thus cancelling them as a complex problem solving type) due to the “multi-scaffolding” that is present, resulting in a multi-procedural series of steps, often with additional information about the steps to take provided. (Venkat, Graven, Lampen & Nalube, 2009, p.50)

Venkat et al. (2009), present a cogent argument that the shortcomings in the structure of the ML assessment taxonomy preclude the development of reasoning and problem solving competencies in the ML assessment. These, they say, lead to the kinds of questions found in ML assessments, questions which place a high emphasis on routine procedures. They follow four threads of mathematical development in the taxonomy to determine how complexity is conceptualised and suggest that there is a “tendency towards procedural orientations to progression” and “the notion that the degree of “immediacy” of information availability and/or “explication” of the required mathematical tools provides a sub-thread contributing to mathematical progression” (Venkat et al., 2009, p.50)

The General Mathematics NSW Australia pre-2014, on the other hand, contains sections not covered by ML (NCS, 2003), such as algebraic modelling and financial mathematics (covered in greater depth) (NSW, Australia, 2001). A brief overview of a sample of questions, taken from NWS Board of Studies Specimen Paper (copyrighted) indicated that their standard may be higher: although on the face of it some questions looked similar in standard, the absence of sub-scaffolding in the NSW questions placed the questions on a higher scale. Also there were questions which dealt with cognitive areas that were not tested in the ML question papers, such as algebraic modelling. Learners have to reason without any cues in the NSW papers, thus adding to the cognitive load.

The ML questions are typical of the style and intention of the question papers, as also pointed out in explicit detail five years ago by Venkat et al. (2009): questions in the main direct the candidate to what needs to be done in order to successfully answer them. Examples abound where formula are provided in the question and candidates are instructed to “use the formula” to answer the question. In some cases the formula has been manipulated (by, for example, changing the subject beforehand), requiring the candidate to substitute the facts from the information provided, without indicating that the formula is understood. The candidate is also prevented, on account of such direct instructions, to seek alternative solutions. The questions tend to be closed. The general approach in the two ML examination papers appears to be to set questions of a decent standard (in the main) but then break these down into sub-sets and guide (or direct) candidates towards solutions, including providing formulas at appropriate times, with details about what information to input. This excessive dumbing down of the ML assessment appears to be geared towards the group of learners with the least chance of understanding what ML (the proper version) is about, given their circumstances. And given that teaching tends to follow assessment, the options for improving the quality of the ML assessment appear few.
It is perhaps in that vein that the benchmarking process revealed some gaps in our assessments: learners did not get “adequate opportunity to demonstrate critical skills” in the assessments. (Department of Basic Education, Technical report, p.45)

Alarmingly, therefore, in spite of strong evidence that ML is not delivering to its original curriculum policy intentions, and despite the many career doors at tertiary institutions that are closed to the student who completes grade 12 with ML not Mathematics, consistent trends seem to indicate that ML enrollment in schools continues to increase. The corollary to that is that Mathematics enrollment is reducing, or where it shows an increase, in not increasing at the same rate. This is highly worrisome to many in education and broadly in society, especially those concerned with growing our numbers in the engineering, science, technology, certain business and the medical fields. In the following we describe ML learning environments, and we give example of a level question that can be used in ML high level (levels 3 & 4) authentic assessments.

ML authentic learning environments

The National Curriculum Statement (NCS) (2011, p.8) identifies authentic real-life contexts as one of the tenets of ML. The question that follows from this is: Are students exposed to authentic assessment questions/activities during the high stakes ML examination and/or tests during the FET phase? In this section we consider: what should an authentic learning environment for ML entail? Secondly, what learning theory supports authentic learning environments?

In this study we use the term ‘authentic learning environments’ to refer to an ambient that “provides a context that reflects the way knowledge and skills will be used in real life” (Gulikers, Bastiaens & Martens, 2005, p. 509). Establishing a connection between classroom and the contemporary world will both enhance student learning, and keep students abreast on whatever is taking place in their immediate environments (Madison, 2014). In this study we call these types of tasks: authentic tasks. Palm (2002, p.7) posits that:

“Authentic task” refers to one in which the situations described in the task compares favourably with a real-life situation outside the world of school mathematics. In addition, the task situation is truthfully described and the conditions under solving the task takes place in the real situation are simulated with some reasonable comparison in the school situation.

In ML learning environments, students solve mathematical activities that are embedded in authentic tasks. The main sources of the authentic sources are varieties of media articles, and contexts in non-school environments, for example, policy documents (Vos, 2011, Mhakure, 2014). It is important to note that if media articles and policy documents are used, it then it has to be assumed that these authentic learning environments change rapidly, thus necessitating the exploration of ways to develop characteristics of adaptive expertise (Madison, 2014). In this study we argue for the teaching of ML in authentic learning environments. If this is acceptable, then authentic assessments should typically include authentic tasks where students are expected to “demonstrate the same (kind of) competencies, combinations of knowledge, skills, attitudes, that they need to apply in criterion situations” in real-life. (Gulikers et al., 2004b, p.5).

Cognitive apprenticeship as theory that support ML learning environments

The cognitive apprenticeship theory – is foregrounded within the broader social constructivist paradigm. Cognitive apprenticeship has roots in and is strongly influenced by traditional apprenticeship model, which most of us are aware of, where learning takes place as novices
and experts interact socially emphasizing teaching skills in the context of their use (Collins, 2006; Dennen, 2006; Wang & Bonk, 2001). The difference though between the two apprenticeships is that “cognitive apprenticeship emphasizes the solving of real world problems under expert guidance that fosters cognitive and metacognitive skills and processes” (Wang & Bonk, 2001, p.132) whereas traditional apprenticeship focus on the completion of tasks in the psychomotor domain where the apprentice “owns the problem” of moving on to the acquisition of the next skill (Berryman, 1991). Central to the cognitive apprenticeship theory in formal education is that students learn to become practitioners – not simply learning about the ML practice but by engaging with real world authentic contexts (Dennen, 2006). Thus, the cognitive apprenticeship, as an alternative to other conventional approaches to formal education and training, aims to “produce graduates with equal thinking and performance capabilities” (Bockarie, 2002, p.48). We find this theory of learning quite useful in understanding of how the teaching of ML using authentic tasks could help students in acquiring lifelong mathematical skills.

An example of a ML authentic assessment questions

This particular example which shows how ML can be assessed (or taught) uses an excerpt from the Cape Times newspaper (4th March 2009). The mathematical content skills required in the NCS (2011) fall under the topics “Patterns, relationship and representations” and “Measurements”. The specific learning outcomes from the two topics are “compound growth and other non-linear relationships” and “measuring length, conversions, and calculating area” respectively. The mathematics of the excerpt is aligned to the key five tenets of ML, that is, ML involves: the use of elementary mathematical content, authentic real-life contexts, solving familiar and unfamiliar problems, decision making and communication, and the use of integrated content and/or skills in problem solving (NCS, 2011). In the NCS (2011, p.8) it states “learners must be exposed to real accounts containing complex and “messy” figures rather than contrived and constructed replicas containing only clean and rounded figures”. What is also intriguing about this excerpt and was highlighted by Gulikers et al. (2005, p.510) is that it deals with issues “from real life outside mathematics itself that has occurred or that might well happen”

The extract below appeared in the newspaper Cape Times on 4 March 2009. There are some errors and inaccuracies in the data given in the extract. The following questions will guide your investigation into these errors.

Development land poser in Cape Town as city size expected to double by 2030.

Reporter - Anel Powell.

The City of Cape Town, estimated to be almost 35 000 hectares (ha) in size in 2007, will double by 2030. This means it is expanding by 650 ha annually, increasing by what is the equivalent of two rugby fields in size every day.

Using the latest available aerial photography, taken in 2007, the city's urban growth monitoring project has analysed the city's urban development since 1945.

The city in 2007 was seven times larger than it was in 1945. Since 1977, the city has been growing at a fairly constant 2 percent until 1988, with the rate declining to 1.5 percent thereafter. City and population growth were similar until 1988, when the population growth outstripped the city's growth.

Questions

1. This question refers statements “The City of Cape Town, estimated to be almost 35 000 hectares (ha) in size in 2007, will double by 2030. This means it is expanding by 650 ha annually, increasing by what is the equivalent of two rugby fields in size every day”.

Do a calculation to correct the statement “by what is the equivalent of two rugby fields in size per day”.  

The extract below appeared in the newspaper Cape Times on 4 March 2009. There are some errors and inaccuracies in the data given in the extract. The following questions will guide your investigation into these errors.
(Hint: a rugby field measures 50m by 100m)

2. This question refers to the statement “The city in 2007 was seven times larger than it was in 1945. Since 1977, the city has been growing at a fairly constant 2 percent until 1988, with the rate declining to 1.5 percent thereafter.”

Use the data in the statement to find the size of the city in 1977.

**Figure 1. An example of a high level ML question**

This example typically shows the type of data that learners in ML could be exposed to as critical citizens. Earlier in the paper we raised the question whether FET students are exposed to these types of questions during the teaching and assessment of ML. The answer to this question appears to be negative. There is doubt that the NCS is clear on the type of learner who should graduate with certain skills in quantitative reasoning. Our analysis of the assessments questions show that learners are not sufficiently developed in order to cope with the higher cognitive level authentic tasks that we propose here. The question which should be posed to ML stakeholders, especially, those involved in teacher education, is: What can be done to the teaching and learning of ML so the learners can acquire the higher cognitive skills they require in order that they can function in careers and fields where those generic skills are vital. They also need them, in line with the general impetus for creating a mathematical subject for all: to function as critical citizens in a democratic society?

**Conclusion**

In this paper we surveyed trends in student enrolments in Mathematical Literacy and Mathematics since the inception of the NSC Mathematical Literacy. Our objective was to determine whether there were signs of migration by students, schools and provinces from Mathematics to ML. We also wanted to determine what the official Education Department (national and provincial) position was, via reports and public statements, with regard to these trends. We found evidence that there were declining enrolments for Mathematics across the board and that, as a parallel movement there were increases in enrolment for ML. The picture is not static, because one provincial department was shown to be working hard at changing the pattern and was actively supporting schools that had shown a degree of success in Mathematics, in order for them to increase their enrolment and future success. We questioned whether this emphasis on improving Mathematics, in absence of statements about ML did not in effect follow an international trend whereby ML and courses like those are essentially marginalised. Given the huge numbers of students who do ML we want to raise the question about the long term implications allowing this trend to continue, especially in light of the critique of ML, which was our next focus.

We critiqued the current assessment standards and show that the critique at the onset of ML still applies today. In light of the benchmarking process the DBE undertook for ML in 2013, we interrogated the standards of the New South Wales, one of the bodies DBE chose for its benchmarking process, against the current standards of ML and found the latter to be wanting in certain respects. We found that there was no algebraic modelling in ML and that the financial mathematics resembled the DBE Mathematics better than ML. We also found that the standard of the questions was higher than ML.

Lastly, we assessed the contexts created in ML against a notion of authentic environments and illustrate through an example the possibilities which exist for bringing the classroom context closer to a real life situation to give effect to the motivation that ML is designed to “make sense” of real world contexts. We found that some of the contexts that are used in the
teaching of ML do not mirror the way the ML content is experienced in everyday lives as evidenced by media excepts.

**Implications**

What are the long term implications of huge numbers of students migrating from mathematics (as a learning subject) to ML (as a learning subject) at the FET phase? What are the implications for the individuals (career choices, life choices), education departments (admission into certain disciplines) and the country as a whole (reduced numbers in specific fields in our economy and the implications for our place in the world)? In light of the critique about ML in its own right, as a subject of choice for those who genuinely have chosen not to pursue careers which require Mathematics (notwithstanding the concern about a lack of numbers doing mathematics) that concern takes on a different flavour: what should such students be doing and why? We have no doubt that the sense of the purpose for introducing the subject ML as part of the NCS is valid and just; our analysis shows that the implementation, as outlined against the assessments, falls far short of delivering the kind of subject that is promised in the policy statement for ML. By drastically lowering the assessment standards, by “multi-scaffolding” tasks which may go some way to being “authentic” and of a higher level, and thereby watering down their cognitive demand, by perpetuating a pattern which lends credence to the notion that ML does not benchmark favourably with more established versions elsewhere, and more pertinently, does not prepare students for roles in society outside of certain disciplines and careers, we do those who take ML as a subject a great injustice. We also do ourselves, in all respects, a great disservice.

**Recommendations**

There is no doubt that ML has become the refuge of all who fear Mathematics. As such the fate of ML is bound up with that of Mathematics. As a first win-win, the crisis surrounding Mathematics needs to be sorted out as a matter of urgency. Fortunately, stakeholders are engaged in this process in an on-going basis, but more can always be done. In parallel, because ML is a subject in its own right, ML needs to be raised in terms of quality and attraction for the right reasons. Benchmarking must be followed by change to the ML curriculum where this is found to be needed. The use of authentic environments is a suggestion that can be explored more fully and assessments must follow classroom based methods as closely as possible, and less the other way around. The use of multi-scaffolding should be downgraded in order that a greater differentiation can be introduced into the ML assessments. Problem solving as a concept, which does not necessarily require the use of numbers or data, can and should be explored, in line with the NSW, a benchmarking ally of standing, and other education departments elsewhere. The use of algebraic modelling should be introduced into the ML curriculum.

**References**


Mathematical Knowledge for Teaching in Africa – A Review of Empirical Research

Arne Jakobsen 1 & Reidar Mosvold 2

1 Department of Education and Sports Science, University of Stavanger, Norway.
2 Department of Education and Sports Science, University of Stavanger, Norway.

1 arne.jakobsen@uis.no; 2 reidar.mosvold@uis.no

A particular knowledge is needed for teaching mathematics, and mathematics teachers’ knowledge has an impact on students’ learning. Investigating the knowledge particularly needed for the teaching of mathematics thus constitutes a relevant focus for research. Much of the research in this area does, however, seems disconnected and fails to build upon previous results. We report from a systematic review work of papers reporting empirical studies about mathematical knowledge distinctive for teaching, but in this paper we limit the discussion to papers reporting from studies in Africa. Our aim is to shed light on what has already been investigated in Africa, how previous studies have approached the issue and what needs to be investigated further. Based on the review, we suggest that more studies should use existing measures of mathematical knowledge for teaching. More research on primary teachers’ knowledge is needed, as well as research on how mathematical knowledge for teaching can be developed among pre-service teachers.

Introduction

Ever since Shulman (1986, 1987) called for a more coherent theoretical framework for teacher knowledge, investigations of the knowledge that is particular for teachers have flourished. A consensus seems to exist among researchers about the existence of a body of mathematical knowledge important for teaching mathematics that is specialized for teaching and is distinct from the mathematical content being taught, the content of advanced mathematics courses or content knowledge needed by professionals other than teachers. Despite such a seeming consensus, different approaches have been used to investigate the mathematical knowledge needed to carry out the work of teaching mathematics. Along these lines, several conceptualisations of teachers’ knowledge have emerged such as knowledge quartet (Rowland, Huckstep, & Thwaites, 2005; Rowland, & Ruthven, 2011); knowledge for teaching (Davis & Simmt, 2006); conceptualizations focusing on pedagogical content knowledge (Baumert et al., 2010) and mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) – often referred to by the acronym MKT.

In order to avoid limiting our review to any specific conceptualisation, we refer to such distinctive knowledge inclusively; sometimes we also refer to beliefs about such knowledge. Numerous studies from all over the world have been published about the knowledge that is distinctive to teaching mathematics. In this paper, we review research literature that focuses on the knowledge distinctive for teaching mathematics in Africa. The aim is to present an overview of the work that has been done, provide a critical discussion of the focus, methods and findings of these research efforts, and also to provide implications and suggestions for further research. First we give a brief introduction to mathematical knowledge for teaching.

Mathematical knowledge for teaching

In his presidential address to the American Educational Research Association in 1986, Shulman (1986) argued that there is a kind of content knowledge that “goes beyond
knowledge of subject matter per se to the dimensions of subject matter knowledge for teaching” (p. 9) and he called this kind of knowledge pedagogical content knowledge. That teachers may need to know mathematics differently from people using mathematics in non-teaching profession has become focus of research by many. One such direction of research was initiated by Deborah Ball and colleagues at the University of Michigan in the U.S. who developed the theory of mathematical knowledge for teaching (MKT). Based on extensive classroom studies, they identified tasks that teachers need to do as they teach, and describe different types of knowledge needed in the work of teaching mathematics (see e.g. Ball et al., 2008). The notion of specialized content knowledge emerged from this work, and is a MKT domain that has received much attention. They describe specialized content knowledge as a type of mathematical knowledge that is unique for teaching, knowledge that is not typically needed in other professions using mathematics (Ball et al, 2008, p. 400). This kind of knowledge comes in addition to common content knowledge that is needed by everyone who needs to solve a particular mathematical task; common content knowledge is also the type of knowledge teachers are responsible for facilitating the development of among students (Hill, Sleep, Lewis, & Ball, 2007).

Along with their presentation of a practice-based theory of content knowledge for teaching, Ball and her colleagues also developed measures to assess teachers’ MKT (Hill, Ball, & Schilling, 2008), and they have shown that teachers’ MKT contribute to gain in student achievements (Hill, Rowan, & Ball, 2005). Many researchers have been intrigued by these results, and the MKT measures have been adapted and used in different countries, like Ireland (Delaney, Ball, Hill, Schilling, & Zopf, 2008), Norway (Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012), South-Korea (Kwon, Thames, & Pang, 2012), Indonesia (Ng, 2012) and Ghana (Cole, 2012). A goal in this research, it can be argued, is to adapt and validate MKT measures for use in these countries so they can be used to identify teachers’ mathematical knowledge for teaching in these countries. Another, and perhaps more important, goal is to develop common measures.

**The method of review**

In this systematic review, the search included peer-reviewed journal articles published in English in the period 2006–2012. Articles were searched for in the following databases:

- PsycInfo
- Eric
- Francis
- ZentralBlatt
- Web of Science

The search criteria we used for searching the databases were threefold. After extensive experimentation, we required that the text should include the term mathematics or some variant of it and the term knowledge or some variant. Then, we added the additional terms teaching, pedagogy or pedagogical, didactic, and instruction to inclusively capture the different ways in which the distinctive character of the mathematical knowledge might be conveyed. This broadened search did not limit the outcome to articles favouring any specific approach towards mathematical knowledge needed for teaching, but aimed at targeting all articles that related to the specific knowledge teachers use when teaching mathematics. In this paper we use the term “mathematical knowledge for teaching” in this broad sense. When referring to the framework of Ball and colleagues (2008), we use the acronym MKT.
After the initial database search in the above-mentioned databases, over 3000 articles were identified, but this number was decreased to 349 after two researchers from the research team examined the abstracts and titles. These 349 articles were then included for a more extensive reading by the review research team. Each article was read independently by two researchers and coded using guidelines developed by the research team. When the two researchers had read and coded an article, they reconciled and reached mutual agreement about a final set of codes for each article.

When coding articles, the first question was whether or not the article should be included in the review altogether. In order for an article to be included, it had to report on empirical research results as grounded in the empirical tradition of the social sciences (American Educational Research Association, 2006). Following this, we excluded review articles, theoretical articles, commentaries, book reviews etc., and concentrated on articles focusing on mathematical knowledge for teaching (or beliefs thereof) as a conceptual tool or measured variable. A total of 190 articles were eventually included in a more extensive review. All the included articles were coded according to what genre the study was, what variables were used in the study, whether or not causality was investigated and how, what problem motivated the study, sample size, whether or not a (standardised) measure was used, what levels the teachers were teaching, what stage they were in (i.e. prospective, beginning, experienced etc.), what region the study was conducted in, what mathematical topic(s) were investigated, and what main findings were made.

In this paper, we use data from our coding of these articles as a starting point, but we also go into a deeper discussion of the articles that report from studies from Africa.

**Focus and scope of the studies**

Among the 190 articles that were finally included in our review, the following frequencies can be observed when sorted after regions:

**Table 1.** Number of articles sorted after region.

<table>
<thead>
<tr>
<th>Region</th>
<th>Number of articles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>7</td>
</tr>
<tr>
<td>all</td>
<td>4</td>
</tr>
<tr>
<td>Asia</td>
<td>27</td>
</tr>
<tr>
<td>Europe</td>
<td>22</td>
</tr>
<tr>
<td>Latin America</td>
<td>3</td>
</tr>
<tr>
<td>Northern America and the Caribbean</td>
<td>112</td>
</tr>
<tr>
<td>Oceania</td>
<td>15</td>
</tr>
</tbody>
</table>

As seen from Table 1, we had seven articles from Africa in our review (Adler, 2010; Adler & Davis, 2006; Bansilal, 2012; Huillet, Adler, & Berger, 2011; Kazima, Pillay, & Adler, 2008; Pournara, 2009; Sapiere & Sorto, 2012). All seven articles presented studies from the south and southeast part of Africa. Five of these studies were from the Republic of South-Africa (RSA), one was from Botswana and RSA, and one was from Mozambique. When looking at the focus in these articles and the research questions that were investigated, a diversion was found. Two articles were motivated by questions concerning what improves mathematical knowledge for teaching, three were focused on the nature and composition of such
knowledge, one was motivated by a problem related to what mathematical knowledge contributes to practice, and the final article focused on teachers mathematical knowledge for teaching as such.

**Table 2.** School level focused on in articles.

<table>
<thead>
<tr>
<th>School level (grade)</th>
<th>Total number of articles</th>
<th>African articles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary, K-8 or subset other than middle school</td>
<td>81</td>
<td>1</td>
</tr>
<tr>
<td>Middle grades, something in range 5-9</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>Secondary, 7-13, or subset other than middle</td>
<td>41</td>
<td>6</td>
</tr>
<tr>
<td>Tertiary, or post-secondary focus</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Broader than any group above, no particular group identified</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

None of the seven articles presented studies that looked for causality using a quantitative design. Two of the articles used qualitative analyses, and the five others did not look for causality.

One of the studies was based on a large sample (n=183, Sapire & Sorto, 2012), one was based on a medium sample-size (n=11, Adler & Davis, 2006), whereas the five others were all small-scale studies. None of the studies used standardized instruments for measuring mathematical knowledge for teaching. As can be seen from Table 2, all but one of the seven studies from Africa focused on secondary teachers. The one exception, which focused on primary teachers, was the study of Sapire and Sorto (2012). Most of the studies had a focus on practising teachers – only the study by Pournara (2009) did not.

**Investigating mathematics for teaching**

The article most extensively cited was that of Adler and Davis (2006), and it can thus be argued that this was the most influential study. Building on Bernstein’s (1996) educational code theory and the notion of “unpacking” by Ball, Bass and Hill (2004), they investigate what they refer to as mathematics for teaching in (in-service) teacher education in South Africa. Their focus is on the mathematics middle and senior school teachers need for teaching in the South African context. The study is part of the QUANTUM project, a large research and development project on quality mathematical education for teachers in South Africa. In their article, Adler and Davis are concerned with how and in what ways, in-service programs in higher educations in South Africa provide opportunities for learning the mathematics needed in order to teach successfully in South Africa. In this study they worked across institutions in five of the nine provinces in South Africa.

A particular focus of their analysis was on assessment tasks, assuming that these tasks would “reveal the kinds of mathematical and pedagogical or teaching competencies that teachers in these courses were expected to display and so, too, the kind of mathematical knowledge that was privileged” (Adler & Davis, 2006, p. 281). On the one hand, in their coding of the assessments tasks they distinguished between the object of the task and whether or not it required principled or procedural elaboration (following Dowling, 1998). On the other hand, they discussed the mathematical knowledge as well as teaching practice that was privileged in the tasks. From this analysis, they conclude:
Overall, however, the analysis reveals the absence, rather than presence, of unpacked or elaborated mathematics for teaching in these across-site evaluation tasks, despite their courses being specifically designed for teachers. This finding confirms much of the discussion in the introduction to this article; this kind of mathematical work is not well understood and is hard to do in the context of formalized teacher education (Adler & Davis, 2006, p. 291).

The notion of mathematics for teaching is important in this study, and the authors call for a critical discussion of the genesis as well as elaboration of this notion. They also suggest that issues of equity should receive more attention in future studies.

A further exploration of the notion of mathematics for teaching is presented by Adler (2010). With the theories of Shulman (1986) as a background, Adler (2010, p. 1) argues: “strengthening our understanding of the mathematical work of teaching, what some refer to as mathematics for teaching, is a critical dimension of enhancing its teaching and learning”. She also relates this to the further development of Shulman’s ideas by Ball, Thames and Phelps (2008) and other work by Ball and her colleagues at the University of Michigan.

Adler (2010) uses two examples from the QUANTUM project to illustrate the substance of teachers’ mathematical work. In doing this, she highlights four tasks of teaching that she argues are inter-related. Two of these – designing, adapting or selecting tasks, and managing processes and objects, and valuing and evaluating diverse learner productions – are discussed more in detail. Based on these discussions, she argues that mathematics for teaching matters – both for teaching and learning – and that task design and mediation are particularly important.

Another article that focused on mathematics for teaching was that of Kazima, Pillay and Adler (2008). The study included two case studies – one focusing on the teaching of probability in Grade 8, and the other on teaching functions in Grade 10. Both case studies were made in a South African context, and both studies were part of the QUANTUM project. When these authors refer to mathematics for teaching, they define it as the “specialised mathematical knowledge that teachers (need to) know and know how to use in their teaching” (p. 284). This definition is similar to the definition of MKT that was proposed by Ball, Thames & Phelps (2008), and Kazima and colleagues (2008) frequently refer to earlier work by Ball and her colleagues. Like Adler and Davis (2006), Kazima and colleagues (2008) also have a strong focus on the issue of unpacking mathematical ideas.

In their analysis of data material from the case studies of the two teachers – called Nash and Vuyo – Kazima and colleagues had a particular focus on the introduction of mathematical concepts or ideas, “the mathematical work of teaching” (p. 287) and the different resources used by the teachers in their work of teaching mathematics. The analytical framework used draws upon the tasks of teaching described by Ball and colleagues (2004). From these tasks of teaching, the following six categories were developed:

- Defining – attempts to provide a definition
- Explanations – teachers explain an idea or procedure
- Representations – teachers represent ideas and in various ways
- Working with learners’ ideas – teachers engage with both expected and unexpected learners’ mathematical ideas
- Restructuring tasks – teachers change set tasks by scaling them either up or down
- Questioning – teacher asks questions to move the lesson on (Kazima et al., 2008, p. 288).
With these categories, Kazima et al. (2008) wanted “to capture what the teachers did irrespective of whether this was correct, appropriate, or productive” (p. 288). When analysing the lessons of the two teachers, they found some interesting differences. Whereas Vuyo had a strong focus on working with learners’ ideas and restructuring tasks, this was not visible at all in Nash’s lessons. The two teachers evidently had completely different emphases in their mathematical work of teaching, and Kazima and colleagues discuss possible reasons for this. The reasons for the teachers’ different emphases can be found, they argue, both in the mathematical content of the lessons and the teachers’ different approaches to present this content. They further argue that the tasks of teaching identified by Ball and colleagues (2004) “are not only mathematical, but they take on specific meanings across topics, and across different approaches to teaching” (Kazima et al., 2008, p. 296). These findings have implications for teacher education, but also, we suggest, for further investigations of mathematics for teaching as well as tasks of teaching mathematics in different African contexts.

What teachers know and how it influences practice

In the only study in this review with a large sample-size (n>70), Sapire and Sorto (2012) investigated the quality of mathematics teaching in Botswana and South Africa. Teaching quality was measured by coding of videotaped lessons. 183 classrooms in Botswana and South Africa were randomly selected, 100 of these were from the North West province of South Africa and 83 from South East Botswana. The sample consisted of 126 teachers in 116 different schools. The classrooms from Botswana had an average of 28 students per class, whereas the classrooms from South Africa had an average of 36 students per class. Classroom videos were coded according to four dimensions: “content area, mathematical proficiency, level of cognitive demand, and the teacher’s knowledge we observed during the teaching” (Sapire & Sorto, 2012, p. 434). Sapire and Sorto (2012) looked for similarities as well as differences between Botswana and South Africa within these four domains, but they used the last three domains in measuring teaching quality. In their investigations of how teachers apply what they know, Sapire and Sorto (2012) build upon the notion of the mathematical quality of instruction – a concept that is strongly connected with that of MKT (Hill et al., 2008).

In their analysis of the observed knowledge of teachers from Botswana and South Africa, Sapire and Sorto (2012) focused on three aspects that were derived from Shulman’s (1986) notion of PCK. The first aspect relates to knowledge of the mathematics relevant for the grade level being taught – what Ball et al. (2008) would refer to as common content knowledge. The second aspect is general pedagogical knowledge, which relates to teaching methods etc. Finally, the third aspect is referred to as “mathematical knowledge in teaching”, and they define this as “the degree to which the teacher can appropriately integrate the use of the instructional techniques with the mathematical concept being taught, and its effectiveness for student learning” (Sapire & Sorto, 2012, p. 441). They concluded that few lessons invited students to engage in high-level cognitive tasks. There was also little focus on engaging discourse and collaborative work, and hands-on activities were seldom used. “Most teachers presented lessons to their students with the intention of communicating that knowledge but not with the purpose of helping them to learn the material” (Sapire & Sorto, 2012, p. 443). The major difference between the lessons in these two countries was, however, the mathematical topics covered; Botswana teachers seemed to be more faithful in their implementation of the official curriculum in this respect.
Among the studies with a small sample-size, Bansilal (2012) presented an investigation of an experienced teacher’s use of circular reasoning in a Grade 9 classroom in South Africa. Previous studies indicate that mathematics teachers in South Africa have poor content knowledge, and students (consequently) display low performance in mathematics on test given by Trends in International Mathematics and Science study (TIMMS) of 1995, 1999 and 2003 (Howie, 2001, 2004), and the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SAQMEQ) tests. Compared to other African countries, South African Grade 8 and Grade 6 students perform poorly, and with this as a background, Bansilal set out to investigate how a teacher with poor mathematical knowledge can teach mathematics for understanding. By the use of narrative analysis, Bansilal tried to “identify ways in which the teacher’s poor mathematics knowledge affected the way in which she mediated the task” (Bansilal, 2012, p. 38). Bansilal found that the teacher did not formulate clear explanations, and she omitted key ideas when trying to help students. Based on her analysis, Bansilal suggests that, “research should go further than just categorising the gaps in knowledge. It should also identify the ways in which such poor knowledge manifests in the classroom” (Bansilal, 2012, p. 46). She continues to suggest that a collegial and supportive environment is important for such teachers’ further development. Teachers with poor content knowledge also need more detailed explanations, and such explanations are also important for the students.

**Teacher education and professional development**

The only study in our review that relates to pre-service teachers is that of Pournara (2009). His study was made in a course in financial mathematics for pre-service teachers in South Africa. The use of spreadsheets as a resource constitutes an important focus in the study, and the aim was that the pre-service teachers should be able to unpack or decompress the formulae embedded in a spreadsheet representing “a scenario of 12 monthly payments of R250 at 6% p.a. compounded monthly” (Pournara, 2009, p. 49). Like with Adler and Davis (2006), the idea of unpacking in Pournara’s study was also inspired by the work of Ball et al. (2004). In conclusion, Pournara (2009, p. 52) argues that:

(...) the students’ ownership and manipulation of the spreadsheet, their insight into the embedded relationships, and their ability to transcend the numeric representation are examples of productive and promising ways in which (future) teachers can learn (and/or relearn) mathematics in ways that are useful for the work of teaching.

The final article in our review – which is part of the QUANTUM project – reports from an action research project among secondary teachers in Mozambique (Huillet, Adler, & Berger, 2011). The overall aim of the project was to investigate the development of mathematical knowledge among teachers who participated in a research project. Four teachers participated in the study, and each of them was investigating aspects concerning the limits of functions. Their individual projects were part of the participating teachers’ Bachelor or Master Degree dissertation. Transcripts from interviews and seminars were analysed with a focus on five aspects of mathematics for teaching limits:

- The organisation of the first introduction of the concept
- The social justification for teaching limits
- The essential features of the limit concept
- The graphical register
- The \(\varepsilon-\delta\) definition (Huillet et al., 2011, p. 25).
Findings from their study indicate that teachers’ mathematical knowledge developed substantially in some aspects, but not so much in other aspects. The research process also proved problematic for one of the experienced teachers in the study, who was confronted with the mathematical content of his own practice in a challenging way. The authors suggest that teacher research has potential, but they argue that, “the content of the practice studied needs to be at the centre” (Huillet et al., 2011, p. 30). Such research, they suggest, can help teachers develop knowledge of certain aspects of mathematics for teaching. This type of studies can also lead to identification of content aspects that needs further development among practicing teachers.

Conclusions

Overall, it can be argued that mathematical knowledge for teaching needs to be further investigated in Africa. A few studies have already been done in this area – all of them in southern Africa – but more research is called for. The few studies that have been done are mainly qualitative studies, and none of them used existing instruments for measuring mathematical knowledge for teaching, and in that respect, the research appears disconnected from research done in other parts of the world. Based on this, we suggest that more studies should be organised where African teachers’ mathematical knowledge for teaching is measured with existing instruments. With such a suggestion, we do not want to argue that quantitative research and measurement is better than qualitative research; both qualitative and quantitative studies are important and can be used to answer different kinds of research questions. When attempting to measure knowledge needed for teaching mathematics in Africa, we do, however, suggest that it might be beneficial to have high-quality and common measures. This could be reached either by developing new measures or adapting existing ones for use in Africa. Some efforts have already been made in this respect (e.g., Cole, 2012), and future studies could build upon the experiences from adapting and using instruments in other countries.

One interesting observation from Table 2 is the level of teachers. Globally, the majority of papers on knowledge for teaching mathematics focus on primary teaching (and middle school), whereas four out of five articles in Africa focus on secondary teachers. The number of articles in Africa is small, but we think a stronger focus on the mathematical knowledge needed for teaching primary mathematics might be relevant in future studies. There is also a disparity of studies that focus on pre-service teachers; most of the existing studies focus on in-service teachers. Studies of the development of knowledge for teaching mathematics among pre-service teachers appear to be of particular importance since many African countries have challenges in educating enough (primary) teachers.

References


**Acknowledgements**

The authors would like to acknowledge members of the University of Michigan’s Mathematics Teaching and Learning to Teach research team who engaged in a larger review of the literature from which we have drawn (with financial support from the National Science Foundation (1008317)): Mark Hoover, Yeon Kim, Minsung Kwon and Yvonne Lai. We also thank Mark Hoover for reading and commenting on this paper.
Mathematics Education in South Africa: The Problems and the Perceived Causes

Marie Joubert
African Institute of Mathematical Sciences
marievjoubert@gmail.com

It is generally agreed that mathematics education in South Africa is problematic. This paper reports on research into the reasons for, or causes of, the problems. It uses a web 2.0 methodology to ‘draw on the wisdom of the crowds’, first attracting contributions and then using a questionnaire to gather views. Analysis of the 62 responses reveals that South Africans perceive that poor teaching, negative societal values towards mathematics and the language of learning and teaching are major causes of the problems in mathematics education. Other important factors are: the curriculum, the option of mathematical literacy for school students and assessment. The nature of mathematics (it is ‘hard’) was seen as less of a cause for the problems. Implications for policy are discussed.

Introduction

This paper reports on research which aims to draw on the ‘wisdom of the crowds’ to better understand the mathematics landscape in the UK and in South Africa. The strand of the research reported within this paper is concerned with the problems with mathematics education in South Africa and aims to understand more about the causes of the problems. It is underpinned by the belief that unless and until policy acknowledges the real causes of the problems it cannot effectively address the problems.

The research is ongoing, and here the emerging results of the initial phase are reported.

Background

It is generally agreed that mathematics education in South Africa has ‘problems’ although, interestingly, anecdotal evidence suggests that when asked what the problems are, many people instead describe the causes. The problems, which relate to poor educational outcomes, are not restricted to mathematics and as Spaull (2013b) suggests:

‘however one chooses to measure learner performance, and at whichever grade one chooses to test, the vast majority of South African pupils are significantly below where they should be in terms of the curriculum, and more generally, have not reached a host of normal numeracy and literacy milestones’ (p. 3).

The picture is complex, however, as South Africa’s education should perhaps more properly been seen as two (or more) systems rather than one (Spaull, 2013a). As the EFA Global Monitoring report points out, inequalities persist with only 14% of grade 8 students from poorer backgrounds reaching the minimum levels in mathematics in 2009, whereas 40% of students from richer backgrounds reach the minimum level (EFA Global monitoring, 2013).

[Note, Spaull refers to pupils above, and much literature such as the EFA refers to students (meaning school students); in South Africa school students are called ‘learners’.]

Although education generally is troubled in South Africa, it does seem that mathematics is particularly problematic (Howie, 2003; Human Sciences Research Council, 2011a; Mji & Makgato, 2006) with, for example, South Africa at ‘rock bottom’ in mathematics in the most recent international benchmarking tests known as TIMSS. Even the highest achievers in
South Africa performed less well than the average students in top performing countries (Human Sciences Research Council, 2011b).

In the UK mathematics education is also seen as problematic, and of 50 reports, published between the beginning of 2011 and December 2013, 24 make claims that there is a problem with mathematics education in England. Overall the picture painted by these 24 reports is far-reaching and complex. Harris provides an overview:

‘England is viewed as having a mathematics ‘problem’ at all age groups .... ‘the mathematics ‘problem’ in England is a multi-faceted one with many important stakeholders, all of whom can have different views of the problem’ (2012, p.5, 7).

For example, there is a widespread view that in Higher Education (HE) and the workplace, there are too few young people with mathematical skills, knowledge and confidence at a sufficiently high level (ACME, 2011c, 2012a; British Academy, 2012).

The reports suggest reasons for, or causes of, the perceived problems in mathematics education. These relate to three key areas: 1) Young people’s (and adults’) mathematical activity, and in particular a) the perception that maths is difficult and b) negative attitudes in society generally towards mathematics 2) The curriculum, qualifications and assessment, and in particular a) that the curriculum does not meet the needs of HE and employers and b) that assessment (which is seen to ‘drive’ classroom teaching and learning) does not assess what it ideally should and 3) Teachers and teaching, and in particular poor teaching.

It is likely that the situation in South Africa is not dissimilar although there may be further causes owing to South Africa’s particular context.

The South African research literature suggests a range of causes for the problems. For example, Spaull (Spaull, 2013a) identifies factors such as teacher education, parents’ education, speaking English at home as being strongly associated with mathematics performance. Howie (2005) explored the relationships between background variables and mathematics achievement at the school level and the classroom level. She found that the variables at the combined school and classroom level most strongly associated with mathematics performance were the community where the school was located, size of classes, attitudes, beliefs and commitment of the teachers including dedication towards lesson preparation and the workload of the teacher. In a separate, earlier study, (2003) she also explored the relationship between language proficiency and mathematical achievement and found that students who spoke English or Afrikaans at home, and those coming from classrooms where these languages were mainly used, overall gained higher scores in mathematics. In this same study, she also found that there was an association between achievement in mathematics and socio-economic status of the student, their ‘self-concept’ in terms of finding mathematics difficult and their perception of the importance of mathematics (their own perception as well as their mother’s and their friends’).

Howie’s research allows her to make claims about associations between variables and mathematical achievement, but she cannot make claims about the causes of the problems in mathematics education.

Research by Mji and Makgato (2006), however, investigated the causes of poor achievement in mathematics and science by seeking the views of teachers and learners in a small number of schools. They found that the participants’ views fell into two main areas: direct influence and indirect influences. The former included teaching strategies, content knowledge and
understanding (of teachers), motivation and interest (of both teachers and learners) and non-completion of the syllabus. The indirect influences included parental role and language.

The research reported in this paper takes this work forward, by asking a wider South African group for their views and with a different methodology. Whereas research such as Spaul’s and Howie’s, and the reports produced in the UK, can be seen to take a top-down approach, the approach used by both Mji and Makgato and within this research is perhaps more ‘bottom-up’ or grass-roots. Building a comprehensive understanding may require both.

Methods

As suggested above, this research draws on the wisdom of the crowds to develop an understanding of the mathematics education landscape. The aim is to reach a wide and diverse audience of interested people, through social media channels such as twitter, discussion forums, email lists etc (opportunity sampling), and then to gather their views. In-line with the main study based in the UK, and drawing on the author’s experience of using social media and web 2.0 within research (Joubert & Wishart, 2012; Joubert, 2011; Timmis, Gibbs, Manuel, & Barnes, 2008), the principle underpinning the data collection is that it must be quick and easy, whilst acknowledging that this sort of data collection runs the risk of superficiality. A very short online questionnaire was devised, citing seven possible causes. Five are taken from the main study and two were later added to take account of the South African context. Participants were also provided with the opportunity to add more detail.

The first two possible causes, taken from the UK reports but almost definitely relevant in the South African context, are 1) assessment focuses on the wrong things and 2) the curriculum is not fit for purpose.

Generally within society it seems to be acceptable to be bad at mathematics. This is true not only in the UK (see above) but also probably in South Africa. Within the questionnaire this related cause was cited as ‘Societal attitudes towards mathematics are negative’.

As far as teachers are concerned, it is widely agreed that teachers and poor teaching are to blame for low levels of achievement in mathematics and the way the related cause was phrased was: ‘Teaching is not good enough’.

As far as the learners are concerned, however, the literature and the political rhetoric tend not to put the responsibility for learning on the learner. However, there seems to be an important relationship between the nature of mathematics and student commitment to learning. The questionnaire included the cause ‘Mathematics is hard’ to address this.

The first of the two causes added later was included because of obvious concern amongst the mathematics education community about it: ‘Too many learners opt for maths literacy’ (see for example Southern African Catholic Bishops’ Conference, 2012). Maths literacy was introduced in 2008. The final cause, also added later is ‘The language of learning is not the home language’ (thus taking into account the findings by Spaul, Howie and Mji and Makgato, see above).

Results

Between 30th June 2013 and 30th July 2014, 185 people, from a range of different countries but mostly the UK and South Africa, responded to the questionnaire. Of these 126 were school and university teachers. The focus for this paper is on the 62 South African responses.
For the South Africans, the majority are teachers (school and university). Occupations are shown in Figure 3, below. For the category ‘other’, the majority are education professionals (e.g. teacher trainer, educational consultant).

![Figure 3. Occupation of South African respondents](image)

The results presented below provide the quantitative results with further comments taken from the longer answers.

In terms of the original five potential causes of the problems, the results are shown in Figure 4 below.

![Figure 4. Causes of the maths problems](image)

### Teaching

It seems that poor teaching is seen as a major cause for the mathematics problems in South Africa. Half (50%) the respondents said they thought that poor teaching was a key reason and another 26 (42%) said they thought it had some influence. The qualitative responses strongly support this result. Twenty-five suggested that in some way teaching is not good enough. In some cases this was simply stated (e.g. ‘teaching which is inadequate’, ‘the main reason why there is a problem in SA is because of very, very poor teaching’). In other cases some reasons were given, normally suggesting that teachers lack knowledge of mathematics (usually) and/or teaching skills (in a few cases). For example: ‘Some Maths teachers do not fully
understand the content they are supposed to be teaching, this affects their confidence in class and the amount of work that is done in their classes’ and ‘I think that many South African mathematics teachers have below-basic levels of content knowledge, with high proportions of teachers unable to answer questions aimed at their pupils.’ A number of respondents suggested reasons for teachers’ lack of subject knowledge, suggesting for example, that they are ‘inadequately trained’ or did not take mathematics as a major subject.

There were also six comments suggesting that there is a shortage of mathematics teachers, stating for example that there is a ‘dire shortage’ and that ‘the shortage of experienced and qualified teachers is a great issue in the senior grades’.

**Societal attitudes**

In terms of negative societal attitudes, the quantitative responses look much like those for poor teaching, with 29 (45%) respondents choosing ‘key reason’ and 28 (44%) choosing ‘has some influence’. Only eleven comments from the longer answers addressed this issue, in comparison to the 25 addressing poor teaching. One group of comments relates to both learners and teachers accepting low standards because maths is seen as difficult as demonstrated in the quote below:

“In some (underprivileged) societies, maths is considered to be difficult, and the learners set low standards for themselves….”

Another (related) set of comments suggested that teachers and others influence learners to think maths is a subject that is not for them but only for ‘intelligent’ people. For example, one teacher commented (quoted directly) ‘They Said Mathematics is difficult, even big brothers and sisters believe that Mathematics is for intelligent People.’ In another comment, a teacher suggested that ‘learners always says mathematics is too difficult but it is misconception.’.

These comments are about learners’ (and societal) perceptions about maths, but interestingly there were no comments about the nature of mathematics itself. The suggested cause ‘maths is hard’ on the questionnaire had been intended to probe views about the nature of mathematics. On the other hand, as shown in the graph in Figure 4, seven respondents (all teachers) thought that ‘maths is hard’ is a key reason for poor achievement and 27 thought it had some influence (of which 17 are teachers).

**Curriculum**

25 of the 62 respondents (over 40%) thought the curriculum was not a concern. Twelve of these are school teachers and four are university teachers. However, within the longer answers there were 23 comments about the curriculum, which suggests that for many it is actually a concern. Six of these comments relate to overall concept of the curriculum, suggesting that the curriculum does not meet the needs of the employer or higher education (e.g. ‘One size fits all - you cannot provide training for employment as well as background for university in the same classroom’) and that its design has not kept up with the times.

A further set of three comments discusses the options in the final years of school, where learners can opt for mathematics or mathematics literacy, which is apparently seen as a ‘weak choice’. Two of the comments suggested that the removal of the old ‘Standard Grade’ maths at this level is a mistake.

Seven comments relate to the design of the curriculum suggesting that it is ‘too vast’, is disjointed, repetitive, saying “The SA curriculum tends to do the same thing over and over
again from Gr 4 - 6, just with bigger numbers.’ One specific comment relates to the introduction of Euclidean Geometry, not taught in South Africa for the past ten years.

A further seven comments tell of the ‘failures’ of the curriculum. First, the concern is that learners at all levels lack basic arithmetic skills (four comments). For example, one teacher said:

‘Dependence on calculators and insufficient recall of bonds and tables, as well as limited understanding of basics such as fraction work prevents them from feeling confident about their work.’

Second, there seems to be a concern that there is not enough emphasis on problem solving skills and critical thinking (four comments). For example, one teacher stated that ‘not enough is being done to teach problem solving from lower grades and upwards’ and another said that learners’ skills in problem solving are ‘almost non-existent’ (sic) and that both teachers and learners ‘think you learn maths through examples’ and ‘students don't get the opportunity to “re-invent” their own maths’.

**Assessment**

Twelve respondents (20%) selected ‘not a reason’ to the option ‘assessment focuses on the wrong things’. Of these, ten are teachers. On the other hand, eight (13%) selected ‘key reason’, and of these, seven are teachers. Four of the longer answers referred to assessment (but interestingly not really to concerned with the focus of the assessment); one suggested that assessment takes place too frequently, one points to the gap between Grade 9 and Grade 10 in terms of assessment and the other two suggest that there is a tendency to teach to the test (my words, not theirs): ‘children are taught how to pass exams not how to do Maths.’

**Maths is hard**

Nearly half (47%) of respondents gave ‘not a reason’ in response to ‘maths is hard’. The only comments related to this cause have already been reported in the societal attitudes results, where it seems the essence of the message is ‘because maths is hard you can expect to fail’.

**The South African context**

For the additional causes, added to take into account South African context, the results are given in Figure 5 below.

![Figure 5. Additional causes within the South African context](image)
Whereas 15 respondents (27%) (eight of whom are teachers) said they thought that maths literacy is a key reason for the problems in maths education, 17 (30%) (all but one are teachers) suggested that it was not a reason. Four comments refer to maths literacy; these suggest that learners choose to take literacy, or their schools ‘force’ them to, rather than mathematics, as they are likely to achieve a higher grade in maths literacy.

In terms of the language of learning and teaching, only six (16%) of the 38 respondents who answered this question thought it was not a cause. Seventeen (45%) thought it was a key reason and (39%) thought it had some influence. The comments mentioning LoLT (six) suggest that in many cases, although mathematics is taught in English or Afrikaans, many teachers struggle with the language of instruction with ‘new concepts being taught by teachers for whom English or Afrikaans is not mother-tongue.’

Other causes

The longer answers suggested three further broad areas of concern: learners, home life and parents, and schools and policy.

In terms of the learners, a number of comments (six) suggest that they are somehow at fault, saying that the learners are lazy, unmotivated, disinterested and do not have a culture of working. A further comment suggests that learners experience difficult times, fall behind in mathematics and never catch up. The final comment here states that ‘Children lack self-confidence, communication skills, reading skills, etc. They are often not exposed to career guidance and what is needed to qualify for STEM-related careers.’

Six comments suggest that home life and parents cause problems in mathematics learning. Of these, four suggest that parents do not support their children in their school work, sometimes explaining how difficult it is for parents:

‘Majority of our learners have no support at home and therefore have no culture of learning. Parents are too busy trying to make a living and do not have time to check up on their children's progress or lack thereof. The learners only do the Mathematics during the Mathematics period and do not consolidate/practice/do homework, at home.’

One of these four comments also explains that many children are not exposed to mathematical thinking and vocabulary from an early age, adding that this is ‘obviously compounded by the fact that indigenous languages do not have such terms.’

Two further comments explain that children are hungry and lack the basics in life.

The third major area of concern, schools and policy, included a wide range of comments (17 in all). Many of these comments relate to education more generally, rather than mathematics specifically, but are still perhaps relevant to the research. At the level of schools, three respondents mentioned the school environment, one suggesting that they are ‘often unsafe and not conducive to learning’ and the others saying that the classes are too big. Two comments relate to educational philosophy, saying first that schools do not provide opportunities for learners to develop holistically and second saying that ‘we do not provide enough ways for our young people to succeed, just ways to fail’.

At the level of policy and the department of education, one respondent cited ‘an education department in shambles’. Two further comments suggested (implicitly) that the department does not communicate well enough with stakeholders. Another two comments refer to interventions and investment at Grade 12, saying it is ‘too late’.
Two comments mentioned teachers, saying that teacher unions ‘have a stronghold in many schools and cover for their members’ and teachers are not held accountable.’

In terms of mathematics, one respondent stated that there is ‘a political drive to maintain an "acceptable" level of pass rate by effectively lowering the standard (below that required of employers) instead of improving the teaching quality.’

Finally, one respondent commented: ‘Ours remains a problem of seriously unequal access to quality learning opportunities at all levels of the system: Grade R to university.’

**Discussion**

The data suggests that, for South Africans, the three most important causes for the problems in mathematics education are teacher quality, societal attitudes towards mathematics and LoLT. It seems that teacher quality is already well recognised as a major cause of the mathematics problems, and within South Africa there are a number of initiatives and organisations that aim to improve teacher quality by providing professional development (Mostert reports on one such research project at SAARMSTE 2015). Clearly South Africa has a long way to go, but it seems that there is a will to address the concern.

How negative attitudes towards mathematics might be tackled is perhaps more problematic. In the UK, there has been a ‘push’ towards breaking down these negative attitudes, for example by establishing National Numeracy which is rolling out various initiatives (For example, see National Numeracy for everyone for life, 2012). It is probably too soon to make an evaluation of the success of these initiatives, but it may be that South Africa can draw on some of the lessons learned.

The LoLT is clearly problematic, and not only within mathematics, and it is not clear whether and how policy might tackle this problem. It could be argued that the first step would be to educate the teachers, and this might occur as a by-product of the professional development discussed above.

For other areas of concern, such as assessment, curriculum and maths literacy, the data is less clear. There have been many changes in the curriculum and the introduction of maths literacy was an initiative partly aimed at addressing concerns about the curriculum; it is probably too early to know how successful it has been. Finally, based on the data, ‘maths is hard’ is arguably the least important cause of the problems. It may be that respondents disagree that it is hard and certainly, policy cannot change the nature of mathematics. However, could it be that acknowledging that it is hard might be a first step to addressing the causes of the problems in mathematics education?

The paper began by stating that unless and until policy acknowledges the real causes of the problems it cannot effectively address the problems. This paper has attempted to add another brick to our wall of understanding but has no simple answers. It seems that all the causes suggested in the questionnaire are, to a greater or lesser extent, important. The idea of identifying ‘the’ cause or the main causes, is perhaps naïve as the picture is clearly highly complex, and it might be interesting to investigate further the apparent contradictions in the data; for example why are opinions so divided on whether maths literacy causes problems in mathematics education?

Finally, as indicated in the introduction to the paper, this research is still at an early stage. Whilst the sampling methodology has some advantages (such as gathering data relatively quickly and easily), there are also some disadvantages (such as participation requires internet access and no control over the demographic of participants (e.g for teachers or learners, their
kind of school). Future research might need to consider sampling further. Further methodological questions concern the design and content of the questionnaire, which may need to be reviewed in the light of the responses. For example, it seems that ‘assessment focuses on the wrong things’ was not understood. However, many of the participants have indicated a willingness to be contribute further and it may be that re-design of the research could draw on their expertise.

References

ACME. (2012). Raising the bar: developing able young mathematicians (pp. 1–4). London.
The Relevance of Mathematics Teacher Identity in the Context of a Mathematics Teacher Development Programme (MTEP).

Nyameka Kangela ¹ & Marc Schafer ²
¹ FRF Mathematics Education Chair –Mathematics Education Department, Rhodes University, South Africa
² FRF Mathematics Education Chair –Mathematics Education Department, Rhodes University, South Africa
¹ g11k7223@campus.ru.ac.za, ² m.schafer@ru.ac.za

The focus of this paper is to explore the role of identity in a Mathematics Teacher Professional Development Programme (PDP). The Mathematics Teacher Enrichment Programme (MTEP) is a mathematics teacher PDP which is located at Rhodes University and aimed at improving teaching and learning of mathematics in the Grahamstown Education District. The MTEP emphasizes that conceptual teaching is the cornerstone of effective mathematics teaching. The content of the MTEP session therefore foregrounds conceptual understanding of mathematical ideas and concepts. This paper focuses on the professional identity formation of selected teachers through an exploration of conceptual teaching as the key role promoted in MTEP. Findings from data gathered so far indicate that the participating teachers have an understanding of the importance of promoting conceptual understanding in their teaching. The data also show that these teachers see themselves differently with respect to conceptual teaching from how the saw themselves before they joined MTEP. How the participating teachers’ identities impact on their classroom functioning will be revealed by data gathered from video recorded lessons and post observation interviews.

Introduction

Many of the South African teacher professional development programme designs of the past adopted a traditional approach which was generally aimed at certification of unqualified teachers, specific upgrading, and preparation of teachers within the confines of a new curriculum (Villegas-Reimers, 2003). Research, however shows that this traditional approach to professional development does not always result in the improvement of teaching (Darling-Hammond & Mc-Laughlin, 1995; Ball & Cohen, 1999; Shumar & Sarmiento, 2008) because as Reed & Schaefer (2005) argue, this traditional approach to professional development often ignored the critical importance of the context within which the participating teachers work. A traditional approach to professional development has also been criticized for failing to provide serious and sustained teachers’ learning about curriculum, students and teaching (Ball et al., 2001). Ball et al (2001) argue that a traditional approach to professional development often lacks curriculum for teachers’ learning, a curriculum that considers the practices in which teachers are being asked (and expected) to enact, and the mathematical knowledge that such practices entail.

Clarke and Hollingsworth (2002) argue that if we are to facilitate effective professional development of teachers, we must understand the process by which teachers grow professionally and the conditions that support and promote that growth. The two authors define professional growth as an inevitable and continuing process of learning. An example of a professional development programme that has been perceived as successful was the Lesson Study Project in Japan (Ono & Ferreira, 2010). A ‘lesson study’ is a professional development process that Japanese teachers engage in by systematically examining their own practice. The goal of a ‘lesson study’ is to improve the effectiveness of the experiences that
the teachers provide to their students. The core activity in a lesson study is for teachers to collaboratively reflect on a small number of ‘study lessons’. These lessons are called ‘study lessons’ because they are used to examine the teachers’ practices (Jita, Maree, & Ndhalane, 2007). Lesson study is typically characterized as classroom-situated, context-based, learner-focused, improvement-oriented and teacher-owned (Ono & Ferreira, 2010). These features have all informed the establishment of the First Rand Foundation (FRF) Mathematics Education Chair Project at Rhodes University.

The First Rand Foundation (FRF) Mathematics Education Chair located at Rhodes University is one of several initiatives aimed at improving the quality of mathematics teaching, particularly in less privileged communities, by explicitly taking cognizance of individual teacher needs. The specific objectives of the Chair are a) to research sustainable and practical solutions to the mathematics challenges in South Africa, b) improve mathematics learner performance (pass rate and quality of passes) in these public schools, c) to provide leadership in mathematics education and d) to increase the dialogue around solutions for the South African mathematics education crisis. The MTEP contact sessions are framed by a concept-driven model of teacher development, an approach which allows a broad philosophy of styles and approaches, a platform for constantly expanded and enriched mathematical ideas (Schäfer, 2011). Although curriculum is not the departure point of MTEP, the contact sessions are however linked to both curriculum and classroom practice, and specifically take into account sections of the curriculum that are problematic to learners.

The MTEP sessions continuously foreground conceptual teaching as advocated by Kilpatrick et al. (2001). The MTEP’s non-traditional approach to professional development emphasises the need to understand contexts of teachers (Little, 1993; Joubert & Sutherland, 2008). Further, teachers who participate in MTEP have willingly opened their practice for learning which means they participate in trying to understand problems facing mathematics teaching and learning. Effective professional development programmes change passive teachers to teachers as active learners in shaping their professional growth (Clarke & Hollingsworth, 2002). The associated in-school teacher support programme which aims to link MTEP sessions with classroom practice is important in situating professional development of teachers in realistic contexts (Clarke & Hollingsworth, 2002).

One of the five key research questions of the FRF Mathematics Education Chair relates to how a positive professional identity can be grown. This is a key question in the search for solutions to the South African mathematics crisis as it directly speaks to restoring a positive disposition in mathematics teachers (Schäfer, 2011). Kilpatrick et al. (2001) talks of positive disposition in terms of productive disposition. Productive disposition refers to the tendency to see sense in Mathematics, to perceive it as both useful and worthwhile, and to see oneself as an effective learner and doer of Mathematics (Kilpatrick et al., 2001). An integral part of MTEP is the in-school support programme. The intentions of the in-school support programme is to offer support in applying in the classroom what was learnt during MTEP sessions so as to ensure a meaningful link of new knowledge to classroom practice. The in-school support programme provides key insights and inputs for the planning of MTEP sessions.

Kilpatrick et al. (2001) argues that teachers should not just acquire knowledge. Teacher preparation and professional development programs must challenge teachers’ development, and help teachers to apply and analyze that knowledge in the context of their own classrooms (Kilpatrick et al., 2001). Opportunities for teachers to discuss mathematical ideas and their representation as well as discuss student learning are assumed to be important for teachers to learn how to teach mathematics effectively (Reed & Schaefer, 2005). MTEP therefore
presents opportunities for teachers to enhance their own professional development and transform their own identities as teachers in order to enrich their own teaching practice.

**Objectives of this paper**

This paper draws on an ongoing study that is aimed at exploring processes of change of mathematics teachers’ identities through their participation in MTEP. We are interested to explore:

1. the relevance of identity in a mathematics teacher enrichment programme.
2. what are the indicators of conceptual teaching, as this is the key role promoted in MTEP.

The identity formation process in the broader study is conceptualized in accordance with Wenger’s (1998) and Sfard and Prusak’s (2005) notion of identity. According to Wenger (1998), in order to make sense of the process of identity formation and learning, it is important to understand these three distinct modes of belonging, *engagement, imagination and alignment*.

- **Engagement** - active involvement in mutual processes of negotiation of meaning.
- **Imagination** – creating images of the world and seeing connections through time and space by extrapolating from our own experience.
- **Alignment** - coordinating our energy and activities in order to fit within broader structures and contribute to broader enterprises.

Sfard and Prusak (2005) define identity as collections of stories about persons, or more specifically, as those narratives about individuals that are reifying, endorsable and significant. The reifying quality comes with the use of verbs such as be, have or can rather than do, and with the adverbs always, never usually, and so forth, that stress repetitiveness of actions (Sfard & Prusak, 2005, pg 16). According to the two authors, a story is endorsable if the identity builder, when asked, would say that it faithfully reflects the state of affairs in the world. A narrative is regarded as significant if any change in it is likely to affect the storyteller’s feelings about the person. The most significant stories are often those that imply one’s membership in, or exclusions from various communities.

They further identify two sub-categories of stories, namely *current identities*, which consist of stories about the actual state of affairs, and *designated identities*, which consist of narratives of what is expected to be in future. Actual identities usually are told in present tense and formulated as factual assertions and designated identities are stories believed to have the potential to become a part of one’s actual identity. Learning may be thought of as closing the gap between current and designated identities (Sfard & Prusak, 2005). Identities are likely to play a critical role in determining whether the process of learning will end with what counts as success or with what is regarded as failure (Sfard & Prusak, 2005). The work of Wenger (1998) complimented by the work of Sfard and Prusak (2005) will be used in this study as theoretical and conceptual lenses to track how participating teachers’ identities grow (if at all) as they participate in the MTEP.

Identity formation and extends from the past and stretches into the future. Identities are malleable and dynamic, an ongoing construction of who we are as a result of our participation with others in the experiences of life (Wenger, 1998). When exploring teachers’ mathematics professional identities we wish to specifically focus on how their practice is informed by conceptual teaching as this is a central role promoted in the MTEP and in-school support
programme. Wenger (1998) argues that roles can be designed, but one cannot design identities that will be constructed through these roles. The MTEP 2010 teacher evaluation feedback revealed that teachers wanted more explicit support on how to teach mathematics effectively and conceptually. It would thus be fair to say that the designated identity of the teachers as envisaged by MTEP is one of a mathematics teacher who is proficient in teaching mathematical procedures that are underpinned by a solid conceptual understanding.

The MTEP and the in-school support programme provides a participation space to enable teachers to work on their current identity towards their designated identity (Sfard & Prusak, 2005). This study aims to research this journey to a designated identity. The conceptual teaching role provides the context within which to elicit participating teachers’ stories and how MTEP is shaping their identities. Through conceptual teaching MTEP teachers are provided with a language which they can identify with, and talk about their identities and their practice. The broader study is aimed at exploring mathematics teacher’s journey from their actual to their designated identities.

Figure 1 illustrates the theoretical framing of this study.

Figure 1. Theoretical Framing of the study

The three research questions of the broader study align with Wenger (1998)’s three modes of belonging and they are:

1. How does teacher participation in MTEP encourage or discourage teachers to accumulate shared histories of learning with respect to conceptual teaching?
2. How do teachers see themselves with respect to conceptual teaching through their participation in MTEP?
3. To what extent do teachers’ styles and discourses align with the broader vision of the MTEP activities?

Indicators of Conceptual Teaching

Although the broader study is aimed at exploring mathematics teacher professional identity using conceptual teaching as a means to elicit teachers’ stories, it is important to discuss some indicators of conceptual teaching as defined in MTEP. They are:

- teaching that promotes conceptual understanding.
- teaching that promotes productive maths talk.
- promoting effective use of manipulatives in teaching of mathematics.
- teaching that promotes visualization.
- positive self-efficacy.

**Teaching that promotes conceptual understanding**

Firstly, conceptual teaching is teaching that aims to develop and grow conceptual understanding. Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. It refers to the knowledge of the underlying structure of mathematics – the relationships and interconnections of ideas that explain and give meaning to mathematical procedures (Kilpatrick et al., 2001). Rittle-Johnson & Alibali (1999) define conceptual understanding as explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain. Researchers (Hiebert & Wearne, 1986; Rittle-Johnson & Alibali, 1999) argue that for students to have a good intuitive feel for mathematics, conceptual and procedural knowledge should be connected and this also help to promote understanding. According to Kilpatrick et al., (2001) a significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes.

The ability to engage with learners in productive mathematical conversations about multiple ways of problem solving thus indicates proficient conceptual understanding of mathematics (Taylor & Vinjevold, 1999; Kilpatrick et al., 2001; Wilson et al., 2005). Kilpatrick et al., (2001) argues that teachers with limited conceptual understanding can hardly engage with students in productive and challenging mathematical conversations. Results of research conducted by Kazemi & Stipek (2001) show that when conceptual understanding is promoted in a classroom, students are accountable for participating in an intellectual climate characterized by argument and justification. They also argue that classrooms practices that are characterized by intentional encouragement for conceptual understanding allow mathematics to drive students’ engagement in activities. Establishing classroom norms that support children’s development of conceptual understanding of mathematics requires teacher knowledge about both mathematics teaching and children’s mathematical thinking. Sound knowledge of mathematical concepts help the teacher to develop mathematical confidence and a freedom to move away from textbooks and a rigid curriculum (Wilson et al., 2005). The degree of teachers’ conceptual understanding relates to the richness and extent of the connections they make, and this has a positive contribution to confidence in teaching (Kilpatrick et al., 2001)

MTEP sessions are designed and presented in a manner that promotes learning and teaching for conceptual understanding. Facilitators invited internally (within the institution) and externally present their sessions such that they do not only sharpen teachers’ content and pedagogical knowledge but foreground conceptual understanding. To mention, but one example, of MTEP sessions that promote conceptual understanding, is a session on statistics and probability. Teachers were grouped into pairs and were asked to describe without use of verbal words, using gestures concepts such as mean, mode and median. This provided an excellent opportunity for the teachers to think deeply about these concepts and how they would present them to the learners for meaningful application of formula. Hiebert & Lefrevre, (1986) argue that conceptual understanding acts as a screening agent to reject inappropriate procedures. Below is data gathered from a pre-observation interview, which indicates how teachers’ practice has been influenced by conceptual teaching.

**Teacher 2 ‘96 Well before I became a member of MTEP, I think in most cases I would say**
I was basically teaching a procedure way of doing things eh eh, because really I wouldn’t go to asking why such a thing is happening, and then after joining MTEP it is then that I started to realise that there is more than to a procedure, because these learners would understand it better if they understand the concept.

102 so after I joined MTEP workshops then those problems were addressed and I began to grow in understanding the effects of conceptual teaching’

107 As for now, I am still growing in that part, I would say partly I am more procedure and my other part now is beginning to grow into a conceptual way of doing things.

Teacher 4 ‘90 Before I joined MTEP, I was not concentrating in the concepts, like the learners before they do a topic, they must know what that topic is meant, like to learn conceptual, learners must do projects, like hands on activities. Before I came to MTEP, I saw those activities as just wasting of time, you know but now I know, though they are wasting of time, they last longer, in fact they last for life, because learners don’t forget what they learn conceptual.

108 Conceptual teaching learners to gain a deeper understanding of what they are doing, at times when you discover something on your own, because sometimes I give them that chance to discover a formula on their own, so when you discover on your own, it is very difficult to forget, so that is why I would encourage other teachers to use conceptual teaching.

Teaching that promotes productive maths talk

To engage learners in productive mathematical talk means that teachers take student’s ideas seriously in their attempts to support students understanding (Wood et al., 1991). Productive classroom discourse requires that the teachers engage all students in discourses by monitoring their participation in discussions and deciding when and how to encourage each student to participate. By actively listening to students’ ideas and suggestions, teachers demonstrate the value they place on students’ contributions to the thinking of the class (White, 2003).

Educators who promote productive mathematics talk according to White (2000) do not stop at asking challenging questions and listening to student’s answers, but should go beyond that and interpret student’s responses as indicators of their levels of understanding and adjust their strategy accordingly.

According to Stein et al., (2008) there are five practices which help teachers support students’ productive mathematics engagement. They are anticipating likely student responses to cognitively demanding mathematical tasks, monitoring students’ responses to the tasks during the explore phase, selecting particular students to present their mathematical responses during discuss and summarize phase, purposefully sequencing the student responses that will be displayed, and helping the class making connections between different students’ responses and between students’ responses and the key ideas. Productive maths talk is about allowing students to access maths concepts, ask learners to justify their reasoning, follow-up on a student answer, question with encouragement to think more deeply (Morrone, et al., 2004)

Research (Stein, et al., 2008) shows that productive mathematics talk is a key to effective mathematics teaching. Productive maths talk supports student’s learning of mathematics, make student’s thinking public. Productive maths talk according to White (2003) is encouraged when teachers value students’ ideas, explore students’ answers, incorporate students’ background knowledge and encourage student-to-student communication. Teaching that promotes productive maths talk from research suggest that teachers must create an opportunity for learners to participate (Cobb, Wood & Yackel, 1993) and explain their thinking (Walshaw & Anthony, 2008). However some researchers (Doyle & Carter, 1984;
McClain & Cobb, 2001) warn that keeping a discussion going on in a mathematics classroom does not necessarily advance student thinking. There is a difference between students’ cooperation and students engaging in a productive maths talk. Fraivilling et al. (1999) argue that in order to promote productive mathematical talk teachers should intervene in advance. We therefore argue that teacher selection of questions and thought provoking questioning style is key to promoting productive maths talk in a maths classroom, because it is what determines whether the talk is productive or not.

Productive maths talk is promoted in MTEP sessions because these sessions are interactive. Teachers are encouraged to share their ideas on how they would approach a particular mathematics concepts in their teaching. Some teachers’ lessons were video recorded and the recordings are used during MTEP sessions to provide a space for teachers to engage with their own teaching and also engage in a discussion that sharpens their maths and pedagogy. Teachers indicated a need for support in designing and assessing mathematics projects for their learners. They presented their learner’s projects in MTEP and this helped to engage in a mathematical talk and experienced the value of that in their own learning. Below is data gathered from pre-observation interview in response to the question, ‘In what ways as a result of your participation in MTEP has your teaching changed?’

**Teacher 4** ‘52 I have confidence, I have more confidence now in teaching maths as I told you that I have gained more teaching strategies, also many ways, I have got many ways to solve problems, at times I just ask learners to come up with their solutions, so I am confident now because I can even welcome the solutions from learners, I engage learners in discussions in class.

Although this does not provide sufficient evidence of productive maths talk taking place in the classroom, however it illustrates that the teacher is aware of the importance of discussion and engaging learners in talk in a maths class.

**Promoting effective use of manipulatives in teaching of mathematics**

Manipulative materials are objects designed to represent explicitly and concretely mathematical ideas that are abstract. Kennedy & Tipps (1994) argue that manipulatives make even the most difficult mathematical concepts easier to understand and they enable students to connect abstract concepts to real objects. Uttal et al., (1997) warns that they must be chosen and used carefully to facilitate learners’ perception of the relation between the manipulative and what the teacher hopes learners will learn. An important characteristic of successful use of manipulative is that instruction and manipulative use are linked from the outset (Uttal et al, 1997). Although Ball (1992) argues that the use of manipulatives enhance perception and thinking, she also warns that they do not always carry meaning or insight, ‘understanding does not travel through the fingertips’, as she puts it.

Manipulatives cannot substitute for instruction, teachers should link instruction with learners’ developing conceptions of what manipulatives represent. When students use manipulatives, they need to be helped to see its relevant aspects and to link those aspects to appropriate symbolism and mathematical concepts and operations (Kilpatrick et al., 2001). Teachers play a vital role in helping learners use of manipulatives to support understanding of mathematical concepts (Clements, 1999). Manipulatives should always be seen as a means and not an end in themselves. They require careful use over sufficient time to allow students to build meaning and make connections (Kilpatrick et al., 2001). A concrete manipulative may be interesting to young children, but this is not sufficient to advance their knowledge of mathematics and concepts (Uttal et al., 1997). Teachers therefore play a crucial role in
successful use of manipulatives in learning and teaching of mathematics.

Manipulatives are used in almost all the MTEP sessions and are used purposefully. MTEP teachers experience the value of use of manipulatives in their own learning. During the MTEP sessions they are asked to discuss the features of these manipulatives and share how they would use them in their teaching. Teachers are also encouraged to design the manipulative both during MTEP and in their classrooms. To mention but a few examples of manipulatives used in MTEP sessions are wooden cubes which could be used to teach number patterns, sequences and series, area and volume of shapes and geo-boards which teachers designed and could be used to teach geometry, perimeter and area of shapes. The teachers were also provided with an opportunity to explore use of technology in teaching of mathematics. Geogebra is a mathematics software programme for learning and teaching mathematics suitable for mathematics at high school and university level. It deals with algebra, geometry, trigonometry and aspects of linear programming. According to the CAPS document, teachers are expected for example to revise the effects of parameters a, q and p on graphs of parabolas, hyperbolas and exponential functions, and the geo-gebra lessons have been helpful to support teachers to meet that expectation. Some teachers from the project have through these sessions and in-school support visits some presented how I teach lessons in AMESA. These lessons did not only highlight the importance of conceptual understanding, but encouraged the importance of the use of manipulatives in teaching of mathematics.

Teaching that promotes visualization

There is evidence from research (Rivera et al., 2007; Makina, 2010) that promoting visualization in teaching of mathematics is rich and it enables learners to gain deeper understanding of maths concepts. According to (Rivera et al., 2007) visualization in algebra enables learners to justify a formula and its parts, offers an alternative way to understand structures and relationships which necessitates good use of variables. Mathematical visualization is the process of forming images (mentally, or with a pencil and paper, or with aid of technology) and using such images effectively for mathematical discovery and understanding (Zimmermann & Cunningham, 1991). Presmeg (1997b) defines visualization to include processes of constructing and transforming both visual mental imagery and all the inscriptions of a spatial nature that may be implicated in doing mathematics. According to Makina(2010), “Visualisation incorporates those mental processes that make use of, or are characterised by, visual imagery, visual memory, visual processing, visual relationships, visual attention and visual imagination.” The intuition which mathematical visualization seeks is not a vague kind of intuition, a superficial substitute for understanding, but the kind of intuition that penetrates to the heart of an idea. It gives depth and meaning to understanding, it serves as a reliable guide to problem solving, and inspires creative discoveries (Zimmermann & Cunningham, 1991)

Diagrams and other visual representations are essential components of mathematics curriculum, for they convey insight and knowledge Hanna (2001) however teachers can promote visualization even in their absence. Duval (2006) warns by highlighting a strong discrepancy when we focus on visualization, between the common way to see figures, generally an iconic way, and the mathematical way they are they are expected to be looked at. He argues that there are many ways of “seeing”. Success in promoting visualization rest on teachers having a clear understanding of the role of visualization asking carefully selected questions which will help learners connect the visuals with the mathematical idea.

One of the activities which was practiced in MTEP to promote visualization, was to ask teachers to provide proofs of theorems without use of traditional procedural way. These
activities do not imply that procedural knowledge is not important, but teachers are encouraged to use their visual intuition so they see its power in their own learning. In this process teachers became familiar with the kind of questions to ask when promoting visualization in their teaching. Teachers were also provided with on-line references such as, the Visual Technology for the Autonomous Learning of Mathematics (VITAL) website which provides teachers not only with suggestions on use of manipulatives in maths teaching, but with promoting visualization in teaching of maths.

**Self-efficacy**

Research (Matoti, 2011; Flores & Day, 2005; Beijaard et al., 2000) shows that self-efficacy is an essential ingredient which influences effective teaching. Self-efficacy refers to beliefs that individuals hold about their abilities and it is said to have a measure of control over an individuals’ thoughts, feelings and actions (Matoti, 2011). Self-efficacy refers to beliefs in one’s capabilities to organize, and execute the courses of action required to produce given attainments (Bandura, 1977). The task of creating learning environments conducive to the development of cognitive competencies rests heavily on the talents and self-efficacy of teachers (Bandura, 1977).

Productive disposition is the habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one’s own efficacy (Kilpatrick et al., 2001). The more mathematical concepts that teachers understand, the more sensible mathematics becomes. Also, their attitudes and beliefs about themselves as learners and doers of mathematics become positive(Kilpatrick et al., 2001). Self efficacy beliefs would indicate teachers’ evaluation of their abilities to bring about positive student change (Gibson & Dembo, 1994). Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics (Kilpatrick et al., 2001).

The teacher of mathematics plays a critical role in encouraging students to maintain a positive attitude towards mathematics. How a teacher views mathematics and its learning affects the teachers’ teaching practice, which ultimately affects not only what students learn but how they view themselves as mathematics learners (Kilpatrick et al., 2001). Key characteristics that are common between self efficacy and productive disposition are, belief in diligence, belief in ability to bring about positive change, and seeing (mathematics) or whatever task they have to perform as sensible and worthwhile hence we find a relationship between the two.

**Conclusion**

Although gaining access to teacher practice (with the good intentions) has not been easy (Aldous, 2004; Wilson, Cooney, & Stinson, 2005), there is a need for meaningful and supportive professional development opportunities for mathematics teachers in South Africa. Graven (2003) argues that teacher development is far more complex than the simple retraining of teachers. In our context of change, teacher development needs to be supported by developing new identities of teachers (Graven, 2005). It is thus important, if we wish to improve teaching and learning in South Africa, that we understand how mathematics and teaching combine in teachers’ development and identities (Adler, Ball, Krainer, Lin, & Novotna, 2005). Identity has been used in mathematics education research as a unifying concept because it draws together and potentially connects a range of interrelated elements that are integral to our understanding of mathematics teaching and learning (Grootenboer, Smith, & Lowrie, 2006). A study conducted by Graven (2005) reveals that strengthened
mathematical identity increases teacher investment in the mathematics education profession. For mathematics teachers, establishing a positive professional identity involves positioning themselves within discourses of education in general and mathematics teaching in particular in ways that allow them to be seen by others and by themselves as ‘good’ teachers of mathematics (Morgan, 2005). Professional development initiatives that neglect the significance of identity are thus unlikely to be successful in improving teaching and learning of mathematics. For mathematics teachers, establishing a positive professional identity involves positioning themselves within discourses of education in general and mathematics teaching in particular in ways that allow them to be seen by others and by themselves as ‘good’ teachers of mathematics (Morgan, 2005).

References


ML Teachers’ Non-recognition of Realistic Constraints in Solving a Problem Set within a Real Life Context

Cathrine Kazunga1 & Sarah Bansilal2
1,2School of Education, University of KwaZulu-Natal
1kathytembo@gmail.com, 2Bansilals@ukzn.ac.za

There have been calls across the globe for mathematics to be made more relevant to real life experiences of people. The introduction of the subject Mathematical Literacy (ML) in South African schools was intended to achieve this purpose. ML assessment tasks play an important role in influencing the extent to which the aims of ML can be met. The purpose of this study was to examine the extent to which a sample of 83 in-service ML teachers were able to consider the realistic constraints associated with finding the cheapest price of a quantity of cereal. Data were generated from the teachers’ written responses to an assessment item that the teachers completed as part of the assessment in the university programme. The findings reveal that many teachers, in trying to locate a suitable mathematics algorithm opted to use equivalent ratios inappropriately. In applying the algorithm these teachers ignored the situational constraints. The results illustrate some of the complexities of the process of mathematisation.

Introduction

Globally, there has been a recent focus on developing numeracy skills in ordinary citizens so that they can use mathematics to make sense of data and information that appears in everyday life experiences. A useful definition of numeracy (seen more broadly than just number sense) is:

The ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life” (PIAAC Numeracy expert group, 2009).

A more rigorous definition of the broader concept of Mathematical literacy is defined by the Organisation for Economic Co-operation and Development/ Programme for International Student Assessment (OECD/PISA) as:

Mathematical literacy is an individual’s capacity to recognise, do and use mathematics, including to reason mathematically in a variety of contexts, and to identify the role that mathematics plays in the world by describing, modelling, explaining and predicting phenomena. Mathematical literacy is a continuum—they more mathematically literate individuals are better able to use mathematics and mathematical tools to make well-founded judgments and decisions required by constructive, engaged and reflective citizens (OECD, 2010, p.5).

The definition highlights processes that mathematically literate individuals should be able to engage in—reasoning, describing, modelling, explaining, and predicting—to make well-founded judgments leading to informed decisions. South African education authorities have gone a step further than most other countries by introducing a school subject based on these principles. The subject Mathematical Literacy or ML as the subject is commonly referred to, is described in South African curriculum documents:
The competencies developed through Mathematical Literacy allow individuals to make sense of, participate in and contribute to the twenty-first century world — a world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology. (DoBE 2011, p.8)

This excerpt reveals that the intention of the subject is to develop learners’ competencies in understanding and utilising numerical or mathematically based information in real life situations. As emphasised in all three definitions the main purpose is to help people make informed decisions in situations which use mathematical information. As a school subject, the teaching and learning form taken by ML has been coverage of some basic mathematics topics and exposure to activities set within commonly encountered contexts. It is hoped that by engaging with simulated real life situations, interpretation, reasoning and decision making skills may be enhanced.

In this article we consider the responses of 83 in-service teachers to a question which asked for a judgement about which of the combinations of two different packages of cereal was the cheapest. In this setting there is no mathematical procedure or algorithm that they could carry out to reach the correct answer. We study their decisions and explore their interpretations and reasoning used to make the decision. Accordingly the research questions that underpins the study are: 1. What are the strategies used by the teachers to decide upon the cheaper option? 2. To what extent do the teachers consider the realistic constraints when responding to the task?

We acknowledge that to understand the issues that people consider when making judgements in real life settings is a complex task and that there is substantial difference between a real life situation as actually experienced and one that is recontextualised into a textual representation used in a classroom teaching or assessment activity. We note too, that when we try to design authentic tasks drawn from the real life experiences of participants, the transformation process that moves the real life experience to a class based task results in some loss of authenticity. However studies of teachers’ or learners’ engagement with assessment tasks set within a real life context contribute to an understanding of the complexities of designing and responding to such tasks. We hope that this study extends the existing research about assessment in ML.

Literature Review

In this review we briefly consider studies that have been conducted about the learning of ML concepts. Since the introduction of the subject Mathematical Literacy in South African schools in 2006, there have been studies that have focused on ML learners’ conceptual understanding. Venkatakrishnan (2010) reported how ML learners’ mathematics proficiency improved, while engaging in ML activities. In her study, Venkatakrishnan found evidence in the lessons, of useful mathematics that was produced in the course of the ML lessons. She wrote that some strands that may be less commonly represented in mathematics classrooms, such as “strategic competence, adaptive reasoning and the development of a productive disposition, feature strongly in ML lessons” (Venkatakrishnan, 2010, p.66). The purpose of Debba’s (2012) study with Grade 12 ML learners was to identify possible factors that influenced learners’ success in the provincial preparatory examination paper. He found that there have been calls across the globe for mathematics to be made more relevant to real life
experiences of people. The introduction of the subject Mathematical Literacy (ML) in South African schools was intended to achieve this purpose. ML assessment tasks play an important role in influencing the extent to which the aims of ML can be met. The purpose of this study was to examine the extent to which a sample of 83 in-service ML teachers were able to consider the realistic constraints associated with finding the cheapest price of a quantity of cereal. Data were generated from the teachers’ written responses to an assessment item that the teachers completed as part of the assessment in the university programme. The findings reveal that many teachers, in trying to locate a suitable mathematics algorithm opted to use equivalent ratios inappropriately. In applying the algorithm these teachers ignored the situational constraints. The results illustrate some of the complexities of the process of mathematisation.

Introduction

Globally, there has been a recent focus on developing numeracy skills in ordinary citizens so that they can use mathematics to make sense of data and information that appears in everyday life experiences. A useful definition of numeracy (seen more broadly than just number sense) is:

The ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life” (PIAAC Numeracy expert group, 2009).

A more rigorous definition of the broader concept of Mathematical literacy is defined by the Organisation for Economic Co-operation and Development/ Programme for International Student Assessment (OECD/PISA) as:

*Mathematical literacy* is an individual’s capacity to recognise, do and use mathematics, including to reason mathematically in a variety of contexts, and to identify the role that mathematics plays in the world by describing, modelling, explaining and predicting phenomena. Mathematical literacy is a continuum—thus more mathematically literate individuals are better able to use mathematics and mathematical tools to make the well-founded judgments and decisions required by constructive, engaged and reflective citizens (OECD, 2010, p.5).

The definition highlights processes that mathematically literate individuals should be able to engage in—reasoning, describing, modelling, explaining, and predicting—to make well-founded judgments leading to informed decisions. South African education authorities have gone a step further than most other countries by introducing a school subject based on these principles. The subject Mathematical Literacy or ML as the subject is commonly referred to, is described in South African curriculum documents:

The competencies developed through Mathematical Literacy allow individuals to make sense of, participate in and contribute to the twenty-first century world — a world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology. (DoBE 2011, p.8)

This excerpt reveals that the intention of the subject is to develop learners’ competencies in understanding and utilising numerical or mathematically based information in real life situations. As emphasised in all three definitions the main purpose is to help people make informed decisions in situations which use mathematical information. As a school subject, the
teaching and learning form taken by ML has been coverage of some basic mathematics topics and exposure to activities set within commonly encountered contexts. It is hoped that by engaging with simulated real life situations, interpretation, reasoning and decision making skills may be enhanced.

In this article we consider the responses of 83 in-service teachers to a question which asked for a judgement about which of the combinations of two different packages of cereal was the cheapest. In this setting there is no mathematical procedure or algorithm that they could carry out to reach the correct answer. We study their decisions and explore their interpretations and reasoning used to make the decision. Accordingly the research questions that underpins the the design of the tasks were problematic, with errors appearing in some tasks. In some cases the contextual information was too far from the instructions, causing many learners to assume that the crucial information was not there. Some contexts containing complex information confused learners with no experience of those contexts. The strategies employed by learners included: number grabbing; guessing without checking; scanning for crucial information; and making the task easier by making additional assumptions.

Bansilal, Mkhwanazi and Mahlabela (2012) carried out a study on 108 ML teachers. The purpose was to explore the teachers’ interpretations and use of the transfer duty rule. The study found that 81% of the teachers were able to calculate the transfer duty payable when given the cost of the house. In contrast when asked to find the cost of a house for which a given transfer duty was provided, only 55% of the teachers were successful. Thirteen teachers did not recognise the mathematical demand, while 29 teachers recognised the inverse nature of the question but lacked the requisite algebraic skill to successively manipulate the formula. None of these studies have focused on teachers’ or ML learners’ understanding of ratio and its applicability to situations which is a focus of this study.

Ratio is a multiplicative comparison between two entities in a situation (Behr, Harel, Post, & Lesh 1992; Lamon 2007; Shield & Dole 2013). The ratio and proportion domain occupies a central role in the wide range of mathematical topics studied at both primary and secondary level. Many research studies provide considerable evidence which indicate that students perform poorly in proportion related tasks (Hart 1984; Behr et al. 1992; Olivier, 1992; Kaput & West,1994; Dole 2008; Mahlabela, 2012). In a recent South African study Mahlabela (2012) carried out a study with a class of 30 Grade 9 learners, by exploring their misconceptions and errors. He used the Concepts in Secondary Mathematics and Science (CSMS) study that was carried out in England in the 1980’s (Hart, 1988). Mahlabela’s study found that half of the learners could not solve problems in which the rate was not given but was easy to find. They were only able to solve problems which did not require a rate or where the rate was given. This implied that these learners did not achieve the Grade 9 Learning Outcomes for ratio and proportion. Only six percent of the learners could comfortably solve problems on ratio and proportion that are at Grade 9 level or beyond. Learner errors in Mahlabela’s study indicated that learners sometimes did not view ratio and proportion as concepts that belong to the multiplicative conceptual field. The multiplicative conceptual field entails all “situations that can be analysed as simple and multiple proportion problems and for which one usually needs to multiply or divide” (Long, 2011, p.34). Some learners conceptualised ratio as an additive relationship, pointing to a misconception about the meaning of ratios. Mahlabela (2012) also noted that learners did not portray the correct conceptualisation of ratio. A large proportion of the learners could barely set up a proportion correctly. He recommended that teachers need to ensure that learners can correctly set up ratios, and thus correctly set up a proportion.
Conceptual Framework

Solving problems in ML requires a great deal of movement within various representations (verbal, algebraic, symbolic, graphical). According to OECD/PISA (2003, p. 26) context-based problems can be solved by following the general strategy used by mathematicians, which the PISA mathematics framework - refers to as mathematising. Mathematising has five aspects which are represented in Figure 1.

![Figure 1. Schematic Diagram showing the process Cycle of Mathematisation –adapted from OECD (2003, p.38)](image)

Table 1 below adapted from Vilakazi (2011) shows the links between aspects of mathematisation and the associated mathematical or contextual demands.

**Table 1. Aspects of mathematisation as a process (adapted from Vilakazi, 2011)**

<table>
<thead>
<tr>
<th>Number</th>
<th>Aspect of mathematisation</th>
<th>Mathematical or contextual demands</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Problem situated in a real life context</td>
<td>Knowledge of contextual language, rules or representations.</td>
</tr>
<tr>
<td>2</td>
<td>Organizing it according to mathematical concepts and identifying the relevant mathematics.</td>
<td>Knowledge of relevant mathematical concepts, and the conditions under which a concept can be applied</td>
</tr>
<tr>
<td>3</td>
<td>Gradually ‘trimming away’ reality through processes such as making assumptions about what are the important features of the problem, generalizing and formalizing, which promote the mathematical features of the situation and</td>
<td>Making assumptions to render problem solvable. Identifying relevant process/formula or rule that is suitable. Reconciling the</td>
</tr>
</tbody>
</table>
transforming the real-world problem into a mathematical problem that faithfully represents the situation.

<table>
<thead>
<tr>
<th>4</th>
<th>Solving the mathematical problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Carrying out the computation, correct substitution, simplification, algebraic manipulation, obtaining a solution, converting units.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5</th>
<th>Making sense of the mathematical solution in terms of the real situation, including identifying the limitations of the solution.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interpreting answer within the context, to see if the solution makes sense in the context. Engaging in contextual reasoning.</td>
</tr>
</tbody>
</table>

Although the model is presented as separate aspects, in reality these are continuous processes with each merging into the other. In this study it will be shown that many teachers experienced problems with aspects 2 and 3, because they were unable to recognise that the mathematical procedure they chose was not suitable for the situation. This non-recognition stopped further progress in further aspects of mathematisation which required them to ascertain whether the solution made sense in the context.

**Methodology**

This exploratory study was carried out with participants who were enrolled as part of an in-service programme for practising ML teachers. This study forms part of a larger study that aims to examine ML teachers’ conceptions of topics (Bansilal, 2011; Bansilal et al., 2012). In this article we consider the responses of 83 teachers to an assessment item comprising one task with three questions which asked the teachers to make a judgement about finding the cheapest way of combining packs of bran to make up a stipulated quantity. The item was taken from an assessment comprising 6 items in total.

The teachers’ written responses were analysed and themes relating to the ways in which they utilised the contextual information were then identified. The analysis of the teachers’ written responses can be viewed as content analysis which is used in the analysis of educational documents and throws “additional light on the source of communication, its author, and on its intended recipients, those to whom the message is directed” (Cohen, Manion & Morrison, 2007, p.165). In addition, Neuman (2011, p.323) states that with content analysis the people being studied are not aware of that fact and “the process of placing words, messages, or symbols in a text to communicate to a reader or receiver occurs without influence from the researcher who analyses its content”. Hence the teachers’ responses were not tailored to suit the researchers’ (and authors’) expectations.

The task appears below in Figure 2.
At the Pak store on 28 April 2007, a 500g box of Bran Flakes was R15,99, while a 750g box of Bran Flakes was priced at R26,59. I need 2 kg of Bran Flakes.

1. Write down at least two different ways in which I could get a total of 2 kg of Bran Flakes, by using the 750g and 500g boxes.

2. What combination of the 750g and/or 500g boxes will result in the cheapest cost for 2 kg of Bran? Show all working.

3. If I need to buy 2,25 kg of Bran, what combination of the 750g and/or 500g will result in the cheapest option? What is the cost, using your option?

Figure 2. The assessment task

Results

In reporting on the strategies used by the teachers, the reference to the teachers contain a number from 1 to 83, so T5 for example, refers to the fifth teacher. The scripts were not arranged in any particular order and a higher or lower number does not indicate any difference in ability or performance.

Responses to Question 1

The intention behind Question 1 was to direct the teachers’ attention to the different ways in which the 500g and 750g packs could be combined to give a total of 2 kg, so they could then solve the problem in Question 2. There are two possible combinations; four 500g packs, or two 750g packs and one 500g pack.

Only 40 out of the 83 teachers wrote down the two different ways of getting 2 kg of Bran Flakes. Ten students provided only one combination. There were 33 teachers who did not provide at least one way of getting a total of 2 kg of Bran Flakes, using the 750g and 500g boxes. Of these 33 teachers it was clear that 23 misunderstood the instruction because they tried to work out the cost of buying 2 kg of Bran Flakes. In fact 8 of these teachers were able to provide the correct response to Question 2, showing that they were able to find a combination of packages adding up to 2 kg, hence their problem must have been with misunderstanding the instruction of Question 1. However many of the others tried to use techniques of proportional reasoning to work out the cost.

A response by T69 appearing in Figure 3, shows how a student used a complex strategy based on equivalent ratios to work out firstly, that 750g is equivalent to 0.75 kg and secondly, 500g is equivalent to 0.5 kg!

<table>
<thead>
<tr>
<th>1kg = 1 000g</th>
<th>1kg = 1 000g</th>
</tr>
</thead>
<tbody>
<tr>
<td>2kg = 2 000g</td>
<td>2 kg = 2 000</td>
</tr>
</tbody>
</table>

\[ x = 750g \]
\[ 2000x = 750 \times 2 \]
\[ x = \frac{1500}{2000} \]
\[ = 0.75kg \]

\[ x = 500g \]
\[ 2000x = 500 \times 2 \]
\[ x = \frac{1000}{2000} \]
\[ = 0.5kg \]

Figure 3. Response of T69 who used ratios to convert grams to kilograms
**Responses to Question 2**

For Question 2, 53 teachers provided the correct answer of four 500g packs that cost R63.96. Two 750g and one 500g pack would cost R69.17, making it the more expensive combination. Of these 53 teachers, 46 presented the cost of each option and then selected the cheaper one, while seven teachers just wrote the cheapest combination and the associated cost.

One participant incorrectly judged the combination of two 750g boxes and a box of 500g which was R69.17 to be the cheapest. Eleven participants wrote down the cost of the two combinations and did not go on to identify the cheaper combination. There were also two teachers who used the incorrect cost for packs. Three teachers did not attempt the question and five teachers’ responses were taken as incomplete because it did not have any completed calculations.

The responses of the remaining teachers are of interest. Many forced in the use of ratios to try and get to an answer. Some drew upon details from a previous item to form a proportional relationship. The previous item in the assessment was about extending a recipe for 24 biscuits to enable one to bake 84 biscuits. The item included an ingredient conversion table showing the equivalent masses and volumes of 23 ingredients including bran. The corresponding conversion for Bran that 20g is equivalent to 25ml was used by some participants such as T12 which is illustrated in Figure 4.

![Figure 4. The response of T12](image)

In Figure 4, the participant (T12) has set up a proportion between mass in grams and volume in ml. This proportion enabled her to work out the equivalent volume of 750 g of bran, as 937.51 ml. The participant then went on to divide this value by 1000 and then multiplied it by 2.5, for some reason. This indicates that the teacher was unclear what meaning could be assigned to the result (937.51) arising from the equivalent ratio calculations, and she did not know how this could be related to the question.

Another response (by T17) is presented in Figure 5 below.

![Figure 5. The response of T17](image)
Note that the teacher’s method of working out the price would not always work in this context. He obtained two different prices for the cost of 2kg of Bran Flakes by using the cost of the 500g and the 750g pack respectively as the base cost. The underlying assumption is that the cost is a continuous measurement and given the cost of one quantity Q the cost of another quantity P could be calculated by working out the factor \( k = \frac{\text{mass of } P}{\text{mass of } Q} \) and then multiplying the cost of Q by this factor. That is, cost of P = \( k \times \text{cost of Q} \). When Q is the 500g pack and P is 2kg the method works well because \( k = 4 \) which is a whole number so the cost of P is equal to the cost of four boxes of Q. However the method used by T17 fails in this setting when Q is 750 g and P is 2000g, because \( k = \frac{2000}{750} = \frac{8}{3} \) and it is not possible to get \( \frac{2}{3} \) of a box of Q.

Other teachers in trying to force the use of ratios, made incorrect assumptions similar to that of T17 that one could buy smaller amounts of Bran flakes using the cost of the 500g or 750g packs on a pro-rata basis. The response of T25 is represented in Figure 6.

| 2kg = 750g + 250g and 250g is \( \frac{1}{2} \) of 500g |
|-----------------|---------------------|
| 250g = 15.99 ÷ 2 = R7.995 |
| 250g = R8 Cost |
| 750g = R26.99 |
| 2kg = 750g + 250g |
| = R26.99 + R8 = R34.59 |

**Figure 6.** Response of T25

Here T25 has worked out the cost of 1 kg using the 750g pack and \( \frac{1}{2} \) of the 500g pack, and has presented this as the cost for 2kg. The incorrect assumption here is that one could get \( \frac{1}{2} \) a box, which is not possible in a supermarket setting.

**Responses to Question 3**

Thirty one teachers provided the correct combination of the 750g and/or 500g boxes giving the cheapest cost for 2.25kg of Bran. This number was less than the 53 teachers who were able to correctly work out Question 2. The expected answer was the combination of three packs of 500g and one pack of 750g which cost R74.56 as compared to three boxes of the 750g which would cost R79.77. It is interesting that of the 53 teachers who answered Question 2 correctly, only 21 answered Question 3 correctly. Conversely there were 10 teachers who did not provide the correct answer to Question 2, but were able to provide the correct answer this time.

Twelve of the 31 participants who provided the correct answer just wrote down the cheapest combination and its cost. The other 19 participants provided the full details of the two options, their costs and then compared the costs and selected the cheapest one. Three participants calculated the cost of the two options but did not select the cheaper one. One chose the combination of three 750g boxes costing R79.77 which was not the cheapest.

There were six participants who did not provide a response and 16 who wrote down incomplete calculations.

A common assumption of the teachers was that a proportional reasoning approach could be used (similar to the reasoning employed by T17 in Figure 4). This approach assumes that any
sub-quantity of the 500g or 750 g pack was available, which is not possible in the real and constrained world of the supermarket.

In a similar manner to T25, three responses used fractional portions of the 500g and 750 g boxes to calculate the cost of 2.25kg. Three participants calculated the cost of a quantity exceeding 2.25kg such as 2.5kg or 2.75kg and 11 participants used the incorrect cost of the 500g or 750g e.g. using R15.66 instead of R15.99 for the 500g pack.

There were 7 teachers who first worked out the cost of 1g using the 500g price and 1 g using the 750g price to find cost of 2250g respectively. This strategy is referred to as the unitary method used in ratio calculations (Hart, 1988; Mahlabela, 2012). An example of this is the response by T16 which appears in Figure 7. There were twenty two teachers who employed reasoning similar to that used by T17 for Question 2 (Figure 5). An example of this is the response of T19 that appears in Figure 7.

<table>
<thead>
<tr>
<th>T19</th>
<th>T16</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.25kg = 2250g</td>
<td>500g cost R15.99</td>
</tr>
<tr>
<td>2250÷750g×3</td>
<td>1g cost = ( \frac{15.99}{500g} ) = R0,03198 = R0.032 of 500g</td>
</tr>
<tr>
<td>3×750 = R79.77</td>
<td>750g cost R26.59 1g cost R0.035 of 750g</td>
</tr>
<tr>
<td>Again 2.25kg=2250g</td>
<td>1kg=1000g if using 500g: 2.25kg will cost 2,250 × R0.032=R72</td>
</tr>
<tr>
<td>2250÷500 = 4.5</td>
<td>If using 750g 2.25kg will cost 2250 × R0.035=R78.75</td>
</tr>
<tr>
<td>4.5×15.99 = R71.96</td>
<td>Cheaper to buy using 500g boxes i.e. ( \frac{2.25\times1000}{500} = 4 \frac{1}{2} ) boxes</td>
</tr>
<tr>
<td>The cheapest cost buy 3×750g of R79.77</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 7.** Two teachers’ responses who used proportional reasoning

Similar to T17 in Figure 2, the approach of T19 is correct for working out the cost of 2250g as 3 times the cost of the 750 g, since \( \frac{2250}{750} = 3 \) and it is possible to get three boxes of the 750g quantity. However the method does not work for finding the cost of 2250g using the 500g pack as a unit. Teacher 16 (T16) used a similar approach to that of Teacher 19 (T19) but used the unitary method of first reducing the cost for the 500g to the cost of 1g and then using that to find the cost of 2250g. He also reduced the cost for the 750 g pack to the proportional cost for 1g and then multiplied that by the mass of 2250 g to get the corresponding cost. These teachers’ methods of working out the cost of a different quantity P given the cost of a quantity Q would be valid in context where rates are given per kg or part thereof, such as in the case of buying 2250 g of tomatoes where one can choose any quantity and the cost is given per kg. In this context, the quantities were restricted to combinations of the given packs so the unitary method and the factor method were not applicable.

**Discussion**

One of the issues that emerged in the responses to Question 1 was that its intention in assisting the teachers to work out the next question was not achieved. Some teachers did not understand the instructions and provided inappropriate answers. For example one teacher, T69, seemed to search for some sort of algorithm to work out the problem, eventually setting up a proportion and using that to calculate 500g in terms of kilograms. The teacher did not seem to recognise the obviousness of the answer he obtained because he then repeated the strategy to express 750 g in terms of kilograms. This was done even though the question did not ask for any calculations. In fact only 53% of the teachers provided the expected response to a question that was included as an intended scaffold to the more difficult Question 2. This
shows that the intentions of the task designers were not realised in practice. In this case, for 47% of the teachers, the inclusion of Question 1 was more of a disadvantage than a scaffold to help them find a solution to Question 2.

In general the most common reason for not obtaining the correct response for Questions 2 and 3 were struggles with those aspects of mathematisation which requires the identification of the mathematics concept and checking the conditions under which the mathematics procedure can be used. This failure to align the solution to the context meant that they were unable to check if the solution made sense in the context. In terms of the aspects of the mathematisation cycle described in Figure 1, they faltered in aspects 2 and 3 thereby limiting their mathematisation efforts in aspects 4 and 5 of the model presented in Figure 2.

What emerged was that many teachers in trying to locate a suitable mathematics algorithm opted to use equivalent ratios. They then tried to represent relationships using equivalent ratios. Some teachers were influenced by the fact that the previous item was based on ratio, and even forced in information from a previous unrelated question in an effort use the algorithm associated with solving one of for quantities in a proportional relationship. The teacher T12 for example set up equivalent ratios by relating the mass of the cereal to the volume. She was unable to make sense of the answer that she obtained, and then carried out a series of arbitrary operations on the number. Mahlabela (2012) recommends that teaching should consider that a ratio as a multiplicative comparison of two quantities or measures. A key developmental milestone is the ability of a student to begin to think of a ratio as a distinct entity, different from the two measures that made it up. It is clear that the teachers in this study have not considered that this situation was one which did not lend itself to the use of ratios. They did not consider whether the conditions under which the algorithm worked, was applicable in this case. The use of ratios was not appropriate because there were only two possibilities of boxes available, that of the 500g and the 750g. Some teachers such as T25 assumed that fractional quantities of the packs were available that could be used to make up 2kg or 2.25 kg. In some cases it was assumed that it was possible to get ½ a pack. In other cases, by using proportional relationships such as T16, it was assumed that it was possible to get any amount of the cereal from the 500g and 750 g pack. If the required quantity was not a multiple of one of the available sizes, the solution arising from the equivalent ratio method would not be relevant in the situation.

A second reason why the ratio strategy was not appropriate when considering combinations of two different packs is that a ratio denotes a multiplicative relationship between two sets. Olivier (1992) defines a ratio as “a property belonging to two sets; a special kind of relationship between the sets, namely a relationship that is described by formulae of the type y = kx ; where x and y are variables and k a constant” (p. 309). The formula indicates that the numbers x and y have a multiplicative relationship and y can be obtained by multiplying x with a certain number k. Hence using this relationship would lead to equivalent ratios of the form \( x_1: f(x_1) \) and \( x_2: f(x_2) \). Alternatively the same relationship could be represented as the set of equivalent ratios \( x_1: x_2 \) and \( f(x_1): f(x_2) \), where \( f(x) \) is the cost of \( x \) g of cereal based on either one of the 500g pack or the 750 g pack. To consider the cost as determined by the second pack, one would need a second function say \( g(x) \), to denote the cost according to the second pack. This would lead to a second set of equivalent ratios arising from this setting \( x_1: g(x_1) \) and \( x_2: g(x_2) \). It is not possible to reduce these to one set of equivalent ratios, which the teachers tried to do. Hence it was short sighted of the teachers to try and use the equivalent ratio strategy.
These results show that many teachers ignored the fact that cereal in supermarkets is sold in boxes which cannot be broken down in smaller quantities and neither can it be made up into bigger quantities. Studies about the use of contextualised assessment have shown that learners from different countries often fail to bring realistic considerations to bear in their solution of short word problems where the task designers expect them to take these into account. (Verschaffel, Greer & de Corte, 2000). This study has shown a similar tendency by the teachers many of whom ignored the contextual constraints of buying boxes of cereal.

**Conclusion**

In this paper we studied 83 teachers’ responses to a question set within a real life context of buying boxes of Bran Flakes, in order to understand the strategies they used and the extent to which they were able to take realistic constraints into consideration. The findings revealed that many teachers tried to force the use of a mathematics algorithm associated with equivalent ratios to solve the problems. In fact in their quest to try and use a mathematical procedure, they did not consider whether the answer they arrived at was possible. Many teachers ignored the realistic constraints associated with buying boxes of cereal from a supermarket and based their solutions on assumptions that were not valid.

As was demonstrated, mathematics questions set within real life settings may not always be as simple as assumed. Teachers, like learners find it difficult to navigate around the realistic constraints while trying to find mathematically appropriate strategies. The results suggest that ML teachers also need help in navigating such problems.

The study has also cast the role of task design into the spotlight. Firstly it was shown that one question whose intention was to help teachers answer the next question, did not function in the expected manner for many of the teachers. In fact it seemed as if many teachers were disadvantaged by the question because they produced an incorrect answer even though they knew the correct response as demonstrated in their solutions to the next question. Secondly, the study showed that some teachers ignored the realistic constraints of the setting. The subject ML sets out to help learners make informed decisions in situations which require numerically based arguments; curriculum and assessments include activities set around simulated real life contexts. Trying to understand the issues that people consider when making judgements in real life settings is a complex task. In a real life situation, judgements may be made based on extraneous factors that may not be known to the instructors. In an assessment setting the driving force may be the need to do well in the assessment. Thus students’ responses to the tasks may be influenced by their desire to obtain high marks, and may not be particularly concerned with demonstrating the ability to use mathematics and contextual tools appropriately (Bansilal et al., 2012). Hence there is a need for further research in this area to help us find out the extent to which learners/ teachers dissociate the classroom based activities set within real life settings from their actual real lives. More research is needed about the possible cues in assessment activities which prompt learners (or teachers as was the case in this study) to recognise or ignore the realistic constraints of the setting.

**References**


D. Grouws (Ed.) Handbook of research on mathematics teaching and learning (pp296-333). New York: MacMillan


Grade 11 Mathematics Learners Approaches to Working with Vertical and Horizontal Shifts of Parabolas

Happy Kunene 1 & Sarah Bansilal2
1,2School of Education, University of KwaZulu-Natal
1hnkunene@gmail.com, 2Bansilals@ukzn.ac.za

The purpose of this was to investigate learners’ attempts at sketching graphs and finding equations of graphs that were obtained from translations of given graphs. The sample for the study was a group of 49 Grade 11 learners from KwaZulu-Natal. Data was generated by the written responses of the learners to a questionnaire, which comprised items based on translations of various graphs, two of which were parabolas. The findings show that the learners found vertical shifts of graphs easier to work with, than horizontal shifts. Most learners were unable to express the equation of a function whose graph was obtained from a horizontal shift of another graph. It was also found that many learners struggled with basic skills such as substitution into algebraic expressions, following scales on axes and calculation of coordinates of points.

Introduction

The idea for this study emerged from reflections by the first author about the ways in which she was taught graphical representations of functions. She recalled being taught certain rules for drawing graphs as a learner. The teacher would usually just tell the learners the rules which they then practised. In terms of sketching the parabola they first had to find the shape of the graph by looking at the sign for a in the equation y = ax^2 + bx + c or y = a(x - p)^2 + . The second step that was stipulated was to find the x and y-intercepts by making y and x zero in the given equation and then solving for x and y respectively. The third step involved working out the coordinates of the turning point. The formula x = -b/2a was used to find the x-coordinate. This value was then substituted into the original equation to find the value of the y-coordinate of the turning point. Once these were established it was then possible to sketch the graph of the parabola. There was no space for any deviation from this method and each graph was drawn in the same manner.

Changes in the mathematics curriculum for South African schools brought in a more flexible and connected way of looking at graphs. The curriculum includes the study of translations of various graphs, where learners are expected to understand the effects of various parameters on the graphs. For example given a function of the type \( f(x) = x^2, f(x) = \frac{k}{x} \) or \( f(x) = \sin x \) or \( f(x) = \cos x \), learners are expected to know how the graph of \( y = f(x) + b \) and \( y = f(x - k) \) and \( y = f(x + k) + b \) can be obtained by shifting the graph of \( f(x) \). (DoBE, 2008; 2011)

However the first author found that her learners still preferred to generate the new graphs by using the table method instead of carrying out vertical or horizontal translations on the original graph. It was then decided to set up a study to investigate the challenges experienced by learners when asked to sketch graphs or to determine the equations of graphs obtained from a shift on another graph. It is hoped that the insights learnt in the study will contribute to knowledge about the teaching and learning of the drawing and interpretation of parabolic graphs and the effect of horizontal and vertical shifts on the graphs.
Literature Review

A study conducted with ten Grade 11 learners in South Africa by Mudaly and Rampersad (2010) explored their conceptual understanding of graphical representations of functional relationships. The findings indicated that the learners used procedural knowledge to explain simple concepts and their visual understanding was weak and dependent on diagrams that teachers use in the classroom. The graph is a visual representation of a function which can also be represented symbolically as an equation and attention to visual aspects is necessary. The authors recommended that teachers should focus on developing visualization skills in the teaching and learning of graphical representations of functional relationships which they believed was essential for conceptual understanding.

A study by Even (1998) focused on the flexibility of students in moving from one representation to another. Even’s study was conducted with 152 college mathematics students. It was found that the participants struggled to link different representations of functions; e.g. they were not well acquainted with the roles of the parameters in different representations of functions including common functions such as the quadratic function. Knowledge about different representations is not independent, but is interconnected with knowledge about different approaches to functions, knowledge about the context of the presentation and knowledge of underlying notions. The author recommends that there is a need to explore interconnections with other aspects of knowledge as well. Borba and Confrey (1996) conducted a study focussing on a student’s graphical constructions showing the transformation of functions in a multiple representational environment. The study suggests that it is helpful for learners to visualise the graphs followed by the use of tables followed by the symbolic equation. The recommendation from the analysis of the results is that visual reasoning and seeing graphical transformations are the most powerful forms of cognition. Borba and Confrey (1996) also state that students must be given time and resources to make constructions, investigations and conjectures. Furthermore for this teaching environment to be successful, a teacher must carefully construct the task and sequences to be followed and must be able to listen to the student.

Deeper knowledge of different representations of functions was also emphasized by Lloyd and Wilson (1998). The aim of the study was to investigate the content conceptions of an experienced high school mathematics teacher and to link those conceptions to their role in the teacher’s first implementation of a reform-oriented curricular material during a 6-week unit of functions. The results of the study suggest that teacher’s comprehensive and well-organized conceptions contributed to instructional strategies comprising conceptual connections, powerful representations, and meaningful discussions. The recommendation was that in order to support teachers in making long term instructional changes, it is crucial to investigate the process through which the curriculum reform is interpreted and personalised by teachers.

According to Leinhardt, Zaslavsky and Stein (1990) a teacher’s knowledge of the subject content is of vital importance. Their review found that many studies focused on interpretation tasks, with only a few focusing on construction tasks. They further state that learners learn by intuitions and misconceptions. The use of the scale is very important and learners need to understand how to work with scaling first before drawing or interpreting graphs. The authors also noted that is easier for learners to sketch the graph from the equation than to find the corresponding equation from a given graph.

Eisenberg and Dreyfus (1994) noted that visual reasoning is crucial. Their study investigated the effects of a teaching unit whose aim was to get students to think of function
transformations in a visual way. The sample for this study was drawn from a boys senior high school in Israel. The study found that that transformations in a vertical direction are easier for students than those in a horizontal direction. They also found that most students did not transform the functions at all, but simply treated each transformed function as the individual function. Their conclusion emphasises the need for a well instilled and developed function sense in students which should be the central aim of the high school and beginning of the collegiate curriculum. Visual reasoning should be the essential component of a high school graduate’s mathematical skill. The authors assert that visual reasoning can be taught by using computer programmes for sketching graphs when the graphs of functions are to be sketched by students. Students should sketch the graphs on their own, and the teacher must be there to respond to questions from the students by giving hints of possible solutions.

Zazkis, Liljedahl and Gadowsky (2003) on the other hand investigated the horizontal translation of functions focussing on the horizontal translation of a parabola. The participants of that study were grade 11 and 12 learners, and secondary school teachers. Participants were required to draw the graphs of \( y = x^2 \) and \( y = (x - 3)^2 \) and explain what they found out about the graph of \( y = (x - 3)^2 \). The results confirmed that the horizontal shift of the parabola is initially inconsistent with expectations and counterintuitive to most participants. The authors concluded that teachers should start by translating points first using other letters as symbols rather than \( x \), in order to help learners see the translations of the points and thereafter to explore how the new function equation can be derived. Zazkis et al., (2003) recommended that the teaching of the section about the translations of graphs should be re-routed starting with the teaching of transformation geometry first so that learners are not confused by the horizontal translation.

**Framework**

Shifts of graph are sometimes interpreted as a physical movement of the graphical image itself which may sometimes be taken as a concrete or physical entity. On the other hand the manipulation of the equation representing a graph requires operations on the symbolic expressions. Hence learners may find it hard to reconcile the shifting of the graph with the manipulation of the corresponding equation and these two actions may be taken as unrelated. The two processes (shifting of the graph and symbolic manipulation) may be taken as examples of the concrete-embodied versus symbolic representational divide as elaborated upon by de Lima and Tall (2008). The authors (see also Tall, 2004) assert that there different modes of working mathematically. They describe three worlds of mathematical thinking:

- **A conceptual embodied** world of human perception and action (including pictures, mental images and the internal connections we make in our mind)
- **A proceptual symbolic world** of mathematical symbols [or process-object encapsulation of symbolism] that operate flexibly either as concepts to think about or as processes to make calculations and perform symbolic manipulations…
- **The axiomatic formal world** of formal mathematics expressed as axiomatic systems, formal definitions and mathematical proof that is met in pure mathematics at university. (de Lima and Tall, 2008, p.5)

In terms of concrete embodiments of concepts, the authors distinguish between a conceptual and procedural embodiments. A conceptual embodiment is one that is supported by the symbolic representation while a procedural embodiment is exhibited for example when symbols are moved without meaning. In their study, de Lima & Tall (2008) suggest that a robust concrete embodiment may help learner move to a conceptual embodiment, which will
then facilitate further movement in understanding that is aligned to the symbolic world. However, many learners develop procedural embodiments which may work for one situation, and in trying to extend it to other situations, they develop misconceptions. de Lima and Tall (2008) in their study on learning algebra in Brazil found that many learners constructed their own ways of working by using embodied actions performed on the symbols. For example they performed actions such as “mentally picking them up and moving them around with the added ‘magic’ of rules such as change sides, change signs” (de Lima & Tall, 2008, p.3). These concrete ways of working with symbols were described as procedural embodiments which could not be supported in the symbolic mode. In the situation of vertical and horizontal shifts of graphs, the instructions seem to support a concrete-embodied representation which make the symbolic representation harder to accept. In this study the shifting of graph may be guided by a concrete embodied image of the graph as a physical entity that can be shifted up, down, left or right. When learners operate on the graphs as if they were shifting an object, the transition from the graphical to the symbolic cannot be accomplished because the two modes are operating in a contradictory manner.

Methodology

This qualitative study was conducted with a class of 49 Grade 11 Mathematics learners from a rural high school in KwaZulu-Natal. The learners were given a questionnaire with six tasks based on vertical and horizontal shifts of a number of graphs. For the purposes of this article we discuss the results for the first two tasks related to shifts of parabolas.

Results

In this section we first provide the results to Task 1, followed by the results for Task 2.

Results for Task 1

The instructions for Task One appear below in Figure 1. This is followed by the presentation of the results for each question from Task 1.

<table>
<thead>
<tr>
<th>Task 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consider the graph of ( f(x) = x^2 ) is shifted in different ways. What will be the equation of the new graph if ( f(x) ) is shifted by</td>
</tr>
<tr>
<td>1.1 two units upwards : Write the new equation as ( y = ) …………..</td>
</tr>
<tr>
<td>1.2 three units down: ( y = ) …………..</td>
</tr>
<tr>
<td>1.3 one unit to the left : ( y = ) …………..</td>
</tr>
<tr>
<td>1.4 two units to the right: ( y = ) …………..</td>
</tr>
</tbody>
</table>

Figure 1. Task 1

Responses to Question 1.1 and 1.2

Sixty three percent of learners answered the question correctly. There were 10% whose response were of the form \( y = 2x^2 \) and \( y = -3x^2 \) to question 1.1 and 1.2 respectively. The approaches followed by 22% of learners were not easily understood and included examples
such as \( y = 2x^2 - 2; \ y = 2x^2 - 3x \) and \( y = \frac{1}{x^2} - 2 \). However all these responses included the numbers 2 and 3, suggesting that learners tried to find ways in which they could account for the values 2 and 3, but they did not know exactly how the horizontal and vertical shifts could be taken into consideration in the equations.

**Responses to Question 1.3 and 1.4**

Eighteen percent (18%) wrote correct equations for both questions. There were 39% of learners who produced answers as \( y = x^2 + 1 \) and \( y = x^2 - 2 \) respectively to the two questions. Twelve % of them wrote the answers for the above mentioned question as \( y = x^2 - 1 \) and \( y = x^2 + 2 \) respectively and 8% wrote \( y = 1 \) and \( y = -2 \) respectively. These responses suggest that learners know that the number of units by which the graph is shifted should be reflected in the new equation, but they do not understand exactly how the formula is affected. Twenty two percent of answers were not easily classified such as \( y = \frac{x^2}{2} + 2 \) for the shift to the right. This shows that students mixed up shifts of parabolic graphs with general equations for hyperbolas.

**Results for Task 2**

Task Two is presented below and this is followed by a question by question analysis of responses to each sub question.

<table>
<thead>
<tr>
<th>Task Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given ( f(x) = 2x^2 )</td>
</tr>
<tr>
<td>2.1 Sketch the graph of ( f ) in the space below.</td>
</tr>
<tr>
<td><img src="image_url" alt="Graph" /></td>
</tr>
<tr>
<td>2.2 If ( g(x) = 2x^2 - 3 ), and ( t(x) = 2(x-3)^2 ), sketch the graphs of ( g ) and ( t ) on the same system of axes above. <em>Use a different colour if possible.</em></td>
</tr>
<tr>
<td>2.3 Now consider the graph ( g(x) = 2x^2 - 3 ) again. If the graph of ( g ) is shifted 2 units to the right, sketch the new graph on the same set of axes. <em>Use a different colour if possible.</em></td>
</tr>
<tr>
<td>2.4 Write down the equation of the new graph of Q2.3. in the form ( h(x) = \ldots ).</td>
</tr>
</tbody>
</table>

**Figure 2. Task 2**
**Responses to Question 2.1**

82% of the learners were able to draw a correct graph while eight learners did not draw the graph. The response of the remaining one learner accompanied by comments is presented in Figure 3 below.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph" /></td>
<td>In studying the learner’s response, there are many conceptual difficulties that can be identified. Firstly the plotting of the points are inaccurate. Secondly for some x values the learner disregarded the value of 2 in the expression $2x^2$, that is she seemed to have worked our $x^2$ in some instances, e.g for $x=2$, it seems she got $y=4$ (but the plotting of the points are not aligned). Thirdly for negative values of x the learner seemed to have computed the value of $-x^2$. The point for $x=1$ seems to be the only x value which was substituted correctly. Fourthly the learner displays problems with the scales – the points 2, 4, 9, 16 are all equally spaced on the y-axis. This shows that this learner has many problems with substitution in simple algebraic expressions, plotting of the coordinates of points and working with scaling of the axes. All these prerequisite skills have not been attained making it impossible for her to understand the relationship between the shifts and the new equations of the shifted graphs.</td>
</tr>
</tbody>
</table>

**Figure 3.** Response of learner to Question 2.1

**Responses to Question 2.2a: graph of $g(x) = 2x^2 - 3$**

Fifty five percent drew a correct graph. The remaining learners drew graphs which reflected various misconceptions that they had. We present examples of graphs that learners drew with some comments next to each graph.
<table>
<thead>
<tr>
<th>Graph</th>
<th>Explanation</th>
</tr>
</thead>
</table>
| ![Graph](image) | **Graph that is concave down**
Eight percent of the learners drew a parabola which was concave down as shown alongside. It appears as if this learner neither translated any graph nor plotted any points. He just drew a graph which is concave down. The learner seems to associate the negative sign appearing in the equation $y = 2x^2 - 3$ with a downward concavity, perhaps because of a confusion with the rule that when the value of $a$ in the graph of $y = ax^2 + bx + c$, is negative then the graph is concave down. |
| ![Graph](image) | **Graphs with incorrect scales**
Eight percent drew correct graphs however the scale was not correct as shown in the figure alongside. The learner has used the multiples of 2 for the values on the positive $y$ axis (2, 4, 6, 8...) and the multiples of 1 for the values on the negative $y$ axis. He used different scales on the same axis. |
| ![Graph](image) | **Just a stretching of the turning points**
Ten percent of them just seemed to have stretched the turning point as shown alongside, while other points from the original graph were not indicated. Only the turning point was indicated and the parabola was drawn passing through that turning point without any evidence of other points being plotted first or obtained by shifting. |
Unsymmetrical graphs

Four percent drew the parabola with the correct turning points but it was not symmetrical as shown. As can be seen in the graph alongside, the learner translated (-2; 8) to (-2; 5), (0; 0) to (0; -3) and (1; 2) to (1; -1) which are all shifts of the points 3 units down. However the point (-1; 2) was translated to (-1; 1) which is incorrect, because it is not a shift of 3 units downwards. That is why the graph is not symmetrical.

Figure 4. Examples of graphs drawn by learners for Q2.2a

Responses to Question 2.2b: graph of $t(x) = 2(x-3)^2$

Twenty percent of the learners produced the correct graph. Fifty five percent of the learners did not draw the graph. Again to summarise the various attempts made by learners we will present examples of actual graphs, with comments alongside in Figure 5

<table>
<thead>
<tr>
<th>Graph</th>
<th>Explanation</th>
</tr>
</thead>
</table>
| ![Graph showing the left half only](image) | **Graph showing the left half only**
Sixteen percent of them drew the left half of the graph.
In the figure alongside, the learner has used the domain -2 to 2 and that domain limited him because these points were not sufficient to get the correct shape of the graph. He correctly substituted for the values $x = -2, -1$ and 0 and got plotted the points correctly. The problem that is manifested in this scenario is one commonly experienced when learners use point–by–point plotting methods, but do not consider values of the domain that span the whole graph. The graph that is then produced is incomplete. |
Responses to Question 2.3
Twenty two percent of the learners drew a correct graph while 55% did not draw the graph. Other responses included a shift of the graph of \( g(x) \) by two units to the left; a shift of the graph of \( g(x) \) 2 units down; a shift of the graph of \( f(x) \) 2 units down; a shift of the graph of \( f(x) \) 2 units to the right; a shift of the graph of \( f(x) \) 2 units up and a shift of the graph of \( g(x) \) 2 units up.

Responses to Question 2.4
Twenty two percent did not write the answer while 14% percent wrote \( h(x) = 2x^2 - 5 \). Other responses included adding 2 to the expression for \( g(x) \) to get \( g(x) = 2x^2 - 1 \); subtracting 2 from the expression for \( f(x) \) and getting \( h(x) = 2x^2 - 2 \); adding two to the expression for \( f(x) \); multiplying the coefficient of \( x^2 \) by 2 in \( g(x) \) and getting \( h(x) = 4x^2 - 3 \).
Discussion

The learners’ responses reveal that many have fundamental problems with drawing graphs. Many learners did not use uniform scales in the axes, and some did not even represent any points on the axes. There were also learners who had problems with working out the values of expressions such as \(2x^2\) and \(2x^2-3\) for certain values of \(x\). These learners will need help in working with basic algebraic expressions otherwise they will always be disadvantaged. It seems for many of these learners key skills related to drawing graphs were not developed when they were introduced to the various graphs.

It was also found that learners preferred the table method of drawing instead of working with shifts in the graphs. This is observed when the learners were supposed to draw the graph of \(y = 2(x - 3)^2\), they substituted the \(x\) values -2; -1;0;1;2 into the equation and got the \(y\) values. Drawing graphs using point by point plotting is an initial strategy and as learners develop their understanding, they are meant to use other skills which help them see global properties. The table method restricts their view to the values that appear in the table only.

Sometimes learners were unable to make connections between a given equation and the original one that it had emerged from, for example participants did not consider \(t(x) = 2(x - 3)^2\) as the translation of \(f(x) = 2x^2\). They treated \(t(x)\) as an individual function not related to \(f(x)\) because they used the table method to sketch \(t(x)\) as we saw from the results that others drew part of the graph. This was also identified by Eisenberg & Dreyfus (1994) where they got that most students in their study did not transform other functions at all. They seemed to look at each transformed function as independent and unrelated functions. Lowenthal and Vandeputte’s (1989) suggestion is that students should have graphs of certain stock functions in their repertoire and should feel at ease to manipulate them. However the learners in this study do not seem to have this repertoire at their disposal and perhaps they missed out on some essential visualisation experiences. As mentioned by Mudaly and Rampersad (2010), one cannot even attempt to foster conceptual understanding in graphs if visualisation skills are poor.

Overall we found that for Task 1, 63% of the class got Question 1.1 and Question 1.2 correct. These questions asked for the equation of the new graph when the graph of the given equation was shifted vertically. In contrast only 18% were able to get the new equation when the graph was shifted horizontally as in Question 1.3 and 1.4. Similarly sketching a graph which was shifted vertically was experienced as easier by this class because 55% of the class got this right for Question 2.2a). In the case of horizontal shifts, only 22% where able to sketch the graph after a horizontal shift as for Question 2.2(b). However for Question 2, it was evident that the biggest struggle was to produce the equation of a parabola which had undergone a horizontal shift, since nobody was able to produce the correct equation. These results are supported by the studies by Eisenberg and Dreyfus (1994) as well as Zazkis et al. (2003) where they noted that finding the equations of graphs that have undergone horizontal shifts are harder to conceptualise than those which have been shifted vertically.

There were also instances where learners were confused by the signs in the equation of the horizontal shift of the parabola. Some learners substituted \((x - a)^2\) in the place of \(x\), when asked to express the equation for a shift to the left and \((x + a)^2\) for a shift to the right (note \(a>0\) in this explanation). Many learners however seemed to be stuck on the concrete movement of the graph. When the graph shifted further right in the direction of the positive \(x\)-axis, the response was to add a positive number to the expression as in Questions 1.3, 1.4 2.4. For vertical shifts, it seemed as if they were taking the expression on the RHS and performing operations on the expression as if it was a concrete entity. An upward movement
meant take the expression and add a positive number to it; a downward movement meant add a negative number. The responses to Question 1.3 and 1.4 and Question 2.4 show that a large number associated a shift to the right with adding a positive number to the expression and a shift to the left meant an addition of a negative number on the RHS and a downward movement by the addition of a negative number. Their responses seem to suggest that they are moving the expression as a concrete entity similar to the way in which de Lima and Tall (2008) observed the learners try to physically shift the symbols as if they were physical entities. The problem is that symbols and expressions cannot be manipulated as if they were objects.

This perspective may be inadvertently supported by rules about slides (translations) in transformation geometry. When a point \((x, y)\) is shifted \(a\) units up and \(b\) units to the right the coordinates of the shifted point are \((x+a; y+b)\). However this is just the calculation of the new position of the point, and if one considers the set of points on a graph say a parabola, which has shifted up by \(a\) units and to the right by \(b\) units, then each point \((x,y)\) on the original graph \(F\) has a matching point on the translated graph \(F'\) that is \((x+a; y+b)\). However the change in the equation describing the graph \(F'\) is a completely different matter. For the parabola, there is an equation which constrains the value taken on by the dependant or \(y\)-variable according to the value taken on by the independent or \(x\)-variable. When the graph \(F\) is translated to \(F'\), the original equation is no longer descriptive of the relationship and there is a new equation which now describes the relationship between the two variables. The changing equation is about the change in the relationship between the variables.

Trying to understand why the graph of \(f(x)\) shifts to the right by \(a\) units when drawing the graph of \(f(x-a)\) while \(f(x)\) shifts to the left when drawing the graph of \(f(x+a)\) (where \(a\) is positive) seems to be counter-intuitive to many learners especially in the light of the fact that when the graph of \(y=f(x)\) is shifted vertically by \(k\) units up or down (where \(k\) is positive) it is represented by \(f(x) + k\) (upward shift by \(k\) units) and \(f(x) - k\) (downward shift of \(k\) units). This situation has been described as an example of an epistemological obstacle by Zazkis et al., (2003) and they have suggested that teachers must re-route their teaching of the translations of functions by teaching transformations of individual points first, and thereafter generating a new equation based on those translated points.

**Conclusion**

The purpose of this study was to explore Grade11 learners’ difficulties when working with horizontal and vertical shifts of parabolic graphs. It was found that learners experienced horizontal shifts as more difficult than vertical shifts, when sketching graphs as well as when producing equations for the new graphs. This trend is supported by other research conducted by Eisenberg and Dreyfus (1994) and Zazkis et al (2003) and demonstrates that it is not unique to the learners in this study. It however calls for an interrogation of the ways in which teachers introduce the section. Perhaps an introduction that is more conceptually grounded instead of the rule based method may be more successful with learners. Another finding of the study was the low levels of skills that some learners exhibited with respect to drawing graphs. This may be the result of impoverished experiences of drawing graphs at the lower grades. When learners have gaps in knowledge and skills that should have been developed earlier on, conceptual development of more complicated concepts such as shifts in graphs is severely retarded as shown by the responses of some learners. This is an ongoing problem which is a contributor to the poor results in mathematics and only detailed attention to this problem by all stakeholders will result in improvements.
References


Introduction

This paper engages with how one of Jim’s (pseudonym) lessons is analysed using models and tools that are borrowed from the work of Heyd-Metzuyanim and Sfard (2012) and Sfard and Prusak (2005) in order to understand how he makes sense of his initial mathematics teaching. We argue that for a BT, the process of teaching and making sense of his teaching is an essential starting component for shaping his professional identity. This process includes understanding one’s own experience in and of teaching, and one’s meaningful participation in the mathematics classroom (Wenger, 1998). Sfard and McClain (2002) suggest that participation is important when attempting to understand the process of shaping an identity.

They talk of mathematizing, which refers to the participatory role of teachers to disseminate mathematical ideas to the learners in a mathematics classroom. Heyd-Metzuyanim and Sfard (2012) suggest that mathematics learning can be seen as interplay between two activities; that of mathematizing (communicating about mathematical objects) and that of subjectifying (communicating about participants’ mathematical discourse). These constructs form the basis of the models that we use in this study to analyse Jim’s classroom activities.

What do we mean by identity? Wenger (1998) considers identity as a way of talking about how learning changes who we are. Identity refers to the personal histories of becoming in the context of our communities. Gee (2001) sees identity as a “certain kind of person” in a given context. With regard to this paper, this person is Jim in the context of his classroom. Gee (2001) also views identity as a person’s own narrativization. Sfard and Prusak (2005) elaborate that foregrounding a ‘person’s own narrativization’ and telling who one is, is an important element of identity. Narrativization could connect identity to attitudes and beliefs a teacher has while he engages in teaching.

Shaping an identity for BTs is therefore inseparable from issues of practice, community, and the meaning they form from these issues. Through Jim’s story, we analyze and explore his classroom practices in order to understand his certain way of teaching. The overall study draws from the following theoretical perspectives: (a) Wenger’s (1998) community of practice theory and (b) Lave and Wenger’s (1991) legitimate peripheral participation theory.

The backdrop of this study is the observation and concern that the performance of students in mathematics at secondary level of education in Lesotho is deteriorating (MoET, 2011). There are many reasons for this, and our assumption is that the teacher’s role is key to student performance – hence our focus on BT’s classroom practice. From our own experiences and observations, the induction years of a BT are not easy. There are many anxieties, challenges and uncertainties that face him. These could be attributed to many factors such as having to deal with a novel school environment (Brown and McNamara, 2011) and school culture. Often there is insufficient time for BTs to adjust to this novel situation. In addition, they may lack confidence in selecting the appropriate mode of teaching when embarking on new topics prescribed by the curriculum and the mathematics textbooks. BTs are under pressure to align
their practice to official policy and the curriculum statements. There is thus a danger that their role could be reduced and limited to being mere curriculum-implementers rather than being meaningful and innovative facilitators in the classrooms. This alignment is generally perceived as ‘good teaching’ in the eyes of school administrators, yet such perceptions of ‘good teaching’ could possibly compromise BTs’ emerging professional ‘identity’.

While exploring and trying to familiarize himself with norms and expectations of the school, a BT also often struggles to find his own space in the mathematics classroom. His greatest challenge sometimes lies in dealing with students who have yet accepted him as a mathematics teacher. Who am I, and what do I want to become are some key questions frequently asked by a BT in order to realize his sense of self and to shape his professional identity (Samuel and Stephen, 2000).

Against this background, mathematics learning is seen in this study as the interaction between two concomitant activities as mentioned earlier; that of mathematizing and subjectifying (Heyd-Metzuyanim and Sfard, 2012). Making sense of mathematics and the way learning mathematics is disseminated in the classroom are therefore important for Jim as he shapes his professional identity.

The main question that this paper addresses is: How does Jim realize his dream of becoming an experienced mathematics teacher, and how does Jim shape his professional identity. This paper is part of an ongoing project and the findings are thus tentative and provisional.

**Literature review**

The overall study is framed by Wenger’s (1998) social theory of learning, community of practice theory, Lave and Wenger’s (1991) legitimate peripheral participation theory as well as aspects of the work by Beijaard, Verloop and Vermunt (2000); Gee (2001); Sfard and Prusak (2005).

The works of Wenger (1998), and Lave and Wenger (1991) have gained significant traction in mathematics education research, particularly with regard to practice and participation. Their theories inter alia focus on how newcomers grow into old-timers. BTs’ ‘growth’ can be described through their stories of who they are and what they are doing (Gee, 2001). In our study, BTs’ stories are narrated using these theories as background to understanding how they shape their professional identity. Lave and Wenger (1991) assert that learning and a sense of identity are closely associated. Therefore, it is important to find out what beliefs and practices BTs come with and how they integrate or change these beliefs and practices as they grow their own professional identity. In this regard, we argue that the meaning BTs form of their practice will ultimately shape their professional identity.

Wenger (1998) identifies four components of learning namely: Community, Identity, Meaning and Practice as shown in Figure 1.
According to this model learning is the central focus of Wenger’s social learning theory. In her own words:

“Participation here refers not just to local events of engagement in certain activities with certain people, but to a more encompassing process of being active participants in the practices of social communities and constructing identities in relation to these communities” (Wenger, 1998).

A BT might thus realize his designated identity through active participation in the classroom in order to grow his professional identity. While growing his identity, a BT may need to talk about and engage with his practice to cope with the new situations that face him. This narrativization of his experience may provide him with tools and a better understanding of his own practice to plan for the future (Sfard and Prusak, 2005).

Lave and Wenger (1991) consider learning as a situated activity. They refer to it as legitimate peripheral participation, which provides a way to speak about the relations between new comers and old timers, about activities, identities and communities of practice. We also recognize that learning for BTs is closely related to the students’ learning of mathematics. Thus, learning becomes a vehicle for the evolution of practices and for growing professional identities as Wenger (1998) argues. Practicing and participating in mathematics classrooms thus shape and re-shape BTs’ professional identity.

Accordingly, we argue that learning for BTs is closely related to their identity. We propose in our model that identity is thus moved to the center of Wenger’s model as illustrated in Figure 2 below:

---

**Figure 1.** Wenger’s theory of social learning
Our observation indicates that learning shapes one’s identity. Identity thus becomes the centre of our study whereby learning becomes one of the pillars for shaping this identity. A BT’s making sense of his ‘actions’ therefore becomes the learning process in relation to his activities in the mathematics classroom. We argue that this is an important aspect in shaping his professional identity. Sfard and Prusak (2005) see identities as collections of stories about persons or narratives about individuals. They suggest that these stories are reifying, endorsable, and significant.

The focus in this paper is therefore what BTs do in the classrooms and what they express in their reflections. When we say ‘what BTs do’, we thus also mean how they make sense of their own actions in the mathematics classrooms. Making sense within the situated context is a way of shaping the professional identity and facilitates reflection for improved practice.

**Methodology**

An ethnographic approach is used to track Jim’s story. Ethnography is inquiring, describing, understanding and interpreting a culture or situation in depth, and understanding a way of life from the participants’ perspectives (Cohen, Manion and Morrison, 2000; LeCompte and Preissle, 1993; Punch, 1998).

Jim is young and enthusiastic. He obtained a general BSc degree in Mathematics and Physics but did not have any formal training in teaching. He has a dream to have a thriving future and be a successful mathematics teacher. He accepted an offer as a mathematics teacher at a secondary school which has high academic expectations from their students to perform well.

The meanings of the teaching situations which Jim actively constructs (Gee, 2001; Sfard and Prusak, 2005) are analyzed through observing classroom practices and using a semi-structured interview. We specifically focus on Jim’s narrativization (how he shapes his professional identity). He was purposefully selected, as he was in his first year of teaching.

**Ethical issues**

Ethical clearances were sought from relevant authorities and from Jim. Care was taken to ensure that he was not compelled to answer any question that he did not want to. He was
given a chance to scrutinize the recorded and transcribed activities and interviews.

Analytic tools

The situated meanings that are developed from Jim’s utterances in the mathematics classroom can be linked to subjectification—eg. ‘This is how I feel’- (Heyd-Metzuyanim and Sfard 2012). This analysis includes certain subjectifying utterances such as it can’t etc, in order to understand how Jim feels about his or students’ mathematizing activities. We will also be looking at utterances such as it can’t be converted; they are not equal etc., which could be linked to mathematizing. In short, this paper explores certain subjectifying utterances that are concomitant to mathematizing utterances which may link to mathematics learning and teaching, as well as developing a professional identity.

At this juncture, we present a mini-model of the analytic tool that is used in this study. This model is adopted and modified to suit our study in order to extract the subjectifying utterances – the activity of talking about properties of persons - rather than what the persons do (Heyd-Metzuyanim and Sfard 2012). The study explores these utterances that introduce mathematical conversations and that shape interactions (eg. We have to compare …). We then explore these subjectifying utterances further as those narratives about individuals that are reifying, endorsable and significant. The reifying quality comes with the use of verbs such as have, is, can etc (Sfard and Prusak, 2005). This would mean reifying a person’s actions into mental properties’ thus attributing the person with certain permanent qualities (Heyd-Metzuyanim and Sfard 2012). The reifying utterances are then linked to other utterances (eg. interviews) to identify if these are endorsable or significant. A story about a person counts as endorsable if the identity-builder would say that, it faithfully reflects the state of affairs in the world, Sfard and Prusak, 2005). A narrative is regarded as significant if any change in it is likely affecting the storyteller’s feelings about the identified person (Sfard & Prusak, 2005).

Table 1 below shows how utterances are classified (indicating the criteria on the selection of these utterances) into mathematizing, subjectifying, reifying, endorsable and significant as adopted from the above mentioned models. These categories are defined and interpreted further (2nd & 3rd column) with a few examples (4th column) for clarification. For the purpose of this paper we will focus mainly on mathematizing and subjectifying utterances.
Table 1. Analytic tool to understand how learning shapes identity

<table>
<thead>
<tr>
<th>Classification &amp; Criteria</th>
<th>Definition &amp; Description</th>
<th>Our understanding &amp; indicators</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematizing</strong> (Mathematical terms, numbers etc)</td>
<td>Communicating about mathematical objects (Heyd-Metzuyanim &amp; Sfard, 2012)</td>
<td>PS: Focus is on the BT’s mathematical utterances (as well as of students).</td>
<td>eg. Discussing/describing mathematical operations etc.</td>
</tr>
<tr>
<td><strong>Subjectifying</strong> (linking mathematical terms, ideas etc)</td>
<td>Communicating about participants of mathematical discourse and the activity of talking about properties (Heyd-Metzuyanim &amp; Sfard, 2012).</td>
<td>The BT will have specific performance - 1st level eg. You said - routine performance - 2nd level eg. Let us do this - and generalizing performance - 3rd level eg. Are we together?</td>
<td>eg. Would you like to do it on the board? This is how I see ... etc.</td>
</tr>
<tr>
<td><strong>Reification</strong> (Selected subjectifying utterances indicating a predictable trend)</td>
<td>Reification refers to the process of giving form to one’s experience, a process and its product (Wenger, 1998). Reification is the discursive activity of rendering the status of an object to something that was not necessarily treated this way so far (Sfard and Prusak, 2005).</td>
<td>Reification shapes one’s experience and gives meaning to their action. Possible indicators are, I don’t understand, how can you add? etc.</td>
<td>Reifying quality comes with the use of verbs such as be, have, can etc eg. Why don’t you understand this simple thing?</td>
</tr>
<tr>
<td><strong>Endorsable</strong> (reflective reified utterances)</td>
<td>Identity builder faithfully reflects on the affairs (Sfard and Prusak, 2005)</td>
<td>Meanings are interpreted I could have … by the narrator</td>
<td></td>
</tr>
<tr>
<td><strong>Significant</strong> (narratives become the person’s identity)</td>
<td>Narratives bring changes in identifier (Sfard and Prusak, 2005)</td>
<td>Narrator changes the Change in attitude and approach classroom practice</td>
<td></td>
</tr>
</tbody>
</table>

The samples below originated from the transcriptions of the observation of one of Jim’s lessons. The selected utterances are classified into subjectifying and mathematizing activities.

**Jim’s classroom practice**

Jim was teaching ‘Fractions’ to Form A students. He introduced the basics of the topic in a previous lesson. After demonstrating a few examples he gave three tasks, one after the other for students to work out on the chalkboard.

*Question: Copy and put signs >, < or = between each pair of fractions to make true statements.*
Jim’s approach was consistent with a typical instrumental model of teaching and learning whereby the teacher follows and demonstrates a series of rules, laws and algorithms for students to grasp the mathematical concept (Skemp, 1976; Tanner and Jones, 2000). The question required the comparison of two fractions. Students correctly worked out the first two questions. The prescribed textbook (Prism 2000 Plus, Book 1) that Jim used did provide a few models that explored relational understanding (Skemp, 1976). For instance, the textbook emphasized that when “the units of measurement are different”, one cannot conclude that one fraction is greater or smaller than the other one. In such cases (of having two fractions with different units of measurement), “(w)e have to use the same unit for all numbers before a meaningful comparison can be made” (Prism 2000 Plus, Book 1: P.72).

According to Jim (Jim Int 1), students understood the concept and knew that the denominator must be the same in order to compare two or more fractions (Jim Ob 1.064) possibly because the information is in the textbook (our assumption). While answering the questions a&b, students followed Jim’s approach that he demonstrated in the class. It appeared as if they were using the same ‘language’ while engaging with the concept of ‘Fraction’. In our view, the mathematical discourse that emerged in this mathematics classroom was as Jim expected (Jim Int 1). However, there was a twist in the story as the students attempted to answer question ‘c’.

Jim’s introductory attempt was successful

He introduced the subtopic, ‘simplifying fractions’ by describing the procedure using the example \(\frac{3}{6}\) (Jim Ob 1.022: *If you want to simplify the function, what do you do? You look at the common factor. What is the common factor between 3 and 6?*). The subjectifying utterances (Jim Ob 1.022: *If you want to ...; what do you do? You look at ... etc*) were used in order to introduce, describe and explain the concepts or to correct the error that might have occurred in these processes. As a result, mathematizing activities progressed as Jim expected. However, not all students could follow what was going on regarding the mathematical activities that were explored in detail indicating that mathemetizing and mathematics learning did not take place entirely successfully.

Once students satisfactorily answered the questions a&b, Jim concluded that the objectives of the lesson had been achieved (Jim Ob 1.001–060; Jim Int 1). However, as the lesson proceeded, he observed that ‘what students learned’ was equally important to ‘what was not learned’ because some of them were challenging his conclusion on question c, and argued that \(\frac{3}{4}\) *m and \(\frac{5}{8}\) km were equal.*

The Table 2 illustrates how student 10 answered question c. He systematically concluded the answer, evidently ignoring the stated information that it is necessary to “use the same unit for all numbers before a meaningful comparison can be made”, (Prism 2000 Plus, Book 1).
Table 2. The answer to question c, according to student 10 (Jim Ob 1).

\[
\frac{3}{4} m \overset{\text{by 1 in the form of } 4/4}{\rightarrow} \frac{3}{4} m = \frac{6}{8} \overset{\text{by 1 in the form of } 2/2}{\rightarrow} \frac{6}{8}
\]

In order to obtain 16 as the common multiple, \(\frac{3}{4}\) is multiplied by 1 in the form of \(\frac{4}{4}\) and \(\frac{6}{8}\) is multiplied by 1 in the form of \(\frac{2}{2}\). Considering the numerators are the same (12), the student concluded that the fractions are equal. He did not consider the values of meter and kilometer in relation to these fractions. Hence he concluded that, \(\frac{3}{4} \text{ m} = \frac{6}{8} \text{ km}\).

After working out this particular question, some students argued that both fractions were equal. However, they ignored the fact that there were two different units with these fractions. Jim invested some extra effort to convince them that there was a need to convert kilometers into meters (or vice-versa) in order to compare the fractions (Table 3).

The Table 2&3 narrates the twist in Jim’s story. The utterances that were quoted below are interpreted by the first author and were further categorized into Subjectifying and Mathematizing utterances (3rd & 4th columns). Though these utterances were separated into two categories, they are entirely subjectifying utterances that inform mathematizing actions and vice versa.

Table 3. Mathematizing and subjectifying in Jim’s classroom take the central role Key: *But are you aware that ...?*

<table>
<thead>
<tr>
<th>Transcription</th>
<th>Our interpretation</th>
<th>Subjectifying</th>
<th>Mathematizing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim Ob 1.064. Student 10: Sir, we have to compare these denominators. The denominator is to be the same.</td>
<td>Student identified that the task was on comparing fractions and therefore, denominators have to be the same.</td>
<td>We have to compare …</td>
<td>The denominator is to be the same.</td>
</tr>
<tr>
<td>Jim Ob 1.065. Jim: But are you aware that this one is in meter and that one in kilometers? …</td>
<td>These utterances indicate that Jim is guiding the student towards a desired direction.</td>
<td>But are you aware that …</td>
<td>This one is in meter and that one in kilometers?</td>
</tr>
<tr>
<td>Jim Ob 1.071. Student 13: We can’t … (We can’t compare meter and kilometer). The student seemed to understand the way fractions with different units are compared.</td>
<td>We can’t … We have to … … can’t compare meter and kilometer before converting them to be the same units … Convert kilometers into meters or meters into kilometers.</td>
<td>We can’t … We have to … … can’t compare meter and kilometer before converting them to be the same units … Convert kilometers into meters or meters into kilometers.</td>
<td></td>
</tr>
</tbody>
</table>

Jim’s utterances shown above are directly linked to mathematical figures or concepts and
Jim demonstrated an intensive engagement in mathematizing. Jim tried to identify the way that mathematizing was taking place (Jim Ob 1.064: We have to compare …) in the classroom. Mathematics identity was thus beginning to shape for Jim and his students as they all talked about the mathematical concepts and their processes. It was also evident that the process of learning mathematics progressed even though there were gaps in this process and Jim could ask; is that one difficult (Jim Ob 1.061). He needs to bridge these gaps effectively for smooth, efficient and successful learning. Jim’s story on mathematizing did not end here, but showed that he started learning to shape his own mathematics identity.

Jim experiences difficulties to explain why \( \frac{3}{4}m \) is not equal to \( \frac{4}{5}km \)

Table 4 shows Jim’s utterances when he tried to convince the students how to tackle question c. The subjectifying utterances quoted below were categorized into three levels for exploring how these utterances demonstrated Jim’s mathematizing actions. The first level is the starting point whereby the study gathers specific performances or a particular action of the BT. The second level of utterances discusses his routine performances. The final level of subjectifying utterances discusses his certain way of acting.

Table 4. Subjectifying & Mathematizing

<table>
<thead>
<tr>
<th>1st level: About specific performance</th>
<th>2nd level: About routine performance</th>
<th>3rd level: About the actor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. What he did here …</td>
<td>5. So that means you are comparing</td>
<td>3. I think we have to make them same.</td>
</tr>
<tr>
<td>2. He converted …</td>
<td>6. so actually you are comparing</td>
<td>8. That means we can compare…</td>
</tr>
<tr>
<td>4. He found that …</td>
<td>7. That means our answer is …</td>
<td>9. You know that …</td>
</tr>
</tbody>
</table>

Through these utterances Jim’s budding identity becomes apparent. Once he listened to how his students’ solved a given task, he would elaborate on certain issues related to the task. These processes included clarification (What he did) of certain mathematical procedures that were demonstrated by the student, interpreting the action (He converted, He found that) as well as concluding (That means our answer is) the answer. Jim’s intention was to ensure that his students followed the appropriate procedures and tackled the task as expected. He then categorically linked (That means we can compare) the concept to the next stage with an understanding and expectation (You know that) that the students knew the concept. When he interpreted the concepts or students’ activities, he frequently used the utterances such as; I think, that means, you know etc that indicated how he used these words to emphasize the way he interpreted their learning activities.

Jim’s reifying actions are identified

The next stage was to explore if these utterances demonstrated any significant pattern though reifying actions in the classroom. A few utterances are selected for exploring these as shown below:
Table 5. Jim required students to focus on learning

<table>
<thead>
<tr>
<th>Indirect reification (Forming an opinion about actions of someone or something)</th>
<th>Direct reification (describing a person): What does Jim observe about him or about others?</th>
<th>Direct reification Jim’s conclusion in a certain way</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim Ob 1.061. Jim: Thank you very much (Appreciation).</td>
<td>Jim Ob 1.065. Jim: But are you aware that this one is in meter and that one in kilometers? (Guiding)</td>
<td>Jim Ob 1.067. Jim: You do not want to say any more (Affirmation).</td>
</tr>
<tr>
<td>Jim Ob 1.067. Jim: But you moved your hand (Observation).</td>
<td>Jim Ob 1.067. Jim: I thought you wanted to say something (Conclusion).</td>
<td>Jim Ob 1.067. Jim: We have to compare these denominators …</td>
</tr>
</tbody>
</table>

The main concern that surfaced from these utterances indicated that Jim was disturbed with one particular student who raised his hand as if he wanted to say something. Jim assumed that the student was seeking clarification, but he responded; *nothing sir*. A series of utterances from Jim (Jim Ob 1.067) seemed to indicate that he refused to accept that there was *nothing* (as the student claimed) and demonstrated his gradual escalating emotions and frustrations. *You moved your hand (observation), I thought you wanted to say something (conclusion)* and finally, *you do not want to say any more (affirmation)* showed a pattern of how he assessed the student’s action. Jim perhaps misunderstood the situated meaning of this particular incident. In our view, through these utterances, he might have gathered some breathing space that possibly helped him to tackle the assignment (comparing the fractions) in a different manner, which he did in his own way!

**Jim’s reifying actions were significant**

Though this incident seemed to be an isolated one that not necessarily demonstrated how Jim interprets when such distracting events take place, it indicated how Jim would perhaps respond in similar situations. Usually, he was a calm person who talked in a neutral tone. He appreciated students’ efforts (Jim Ob 1.061) when tackling any mathematical task. If they addressed a task wrongly, he was there to alert them and to guide them (Jim Ob 1.065). These reifying utterances are therefore significant.

Table 6. Reifying and identifying (Jim Ob 1.098) Key: *That means our answer is …*

<table>
<thead>
<tr>
<th>Indirect reification</th>
<th>Direct reification</th>
<th>Direct reification</th>
</tr>
</thead>
<tbody>
<tr>
<td>That means our answer is … He did… he converted… he found… (opinion)</td>
<td>So after that conversion he found that … (explanation and conclusion).</td>
<td>I think we have to make them same…You know that … <em>That means we can compare and you know that 750 meters is actually 750/1.</em></td>
</tr>
</tbody>
</table>

Jim frequently pronounced certain utterances in order to ensure that mathematics learning was taking place. He allowed his students to work on the chalkboard and assisted them in their workings. In a way, he was interpreting and elaborating student’s performance on this particular task (eg. converted, found etc). These utterances were meant to be supportive that clarified, interpreted, linked or concluded certain mathematical procedures and concepts that needed to be established. The nature of these utterances (we, our, he, that means, you know that etc) also sent a message that he was part of the community and was very
compassionate with the students who were actively participating in the lesson.

However, there were a few reifying actions that needed to be explored further. Student 13 observed that student 10 answered the task incorrectly (*We can’t ... meters and kilometers. We have to convert kilometers into meters or meters into kilometers*). Jim then requested this student to help the class. This activated peer learning. Utterances such as *can you help us ...?* (Jim Ob 1.074) shows that Jim tried to create a culture in the classroom that encourages and motivates students to explore the activities (Teacher identity). In this regard, Jim could be identified as a person who motivated students to learn by practice (Personal identity). He affirmed and guided students when he observed that they were on the right track (*Is that one difficult?; But are you aware that*). When they were unable to conclude correctly, he intervened and tried to explain how to find the answer. Later, he reflected, *it is interesting the way the students thinking* (Jim Int 1.113). *I thought that the problem was not comparing the fraction, but it was with the conversion of one unit into another* (Jim Int 1.132). Students kept on insisting that the two fractions were equal. Therefore, he had to repeat the procedure to show why \( \frac{3}{4} \) m is not equal to \( \frac{5}{8} \) km. His classroom practice in this regard was significant as he had learned to be careful when multiple goals were explored within a single task (Jim Int 1.156: *We just make sure that they know how to do the conversion... so I think that issue has to be addressed on its own*).

The most significant action was the introduction of 750 meter as 750/1 (Jim Ob 1.098). The lesson took a turn by bringing the idea of whole number that convinced the students (at later stage) to realize that 750 m is greater than \( \frac{3}{4} \) m. The meaning that students formed was different from the meaning that Jim was trying to establish. He tried to reach out by explaining how this student concluded (*So after that conversion he found that, what he did here, he converted*). Later, he reflected that; *if I say 750 whole, then they would have said yes* (Jim Int 1.173), *it is a matter of language, I think* (Jim Int 1.175). From these utterances, we foresee Jim’s reifying actions were endorsed and therefore become significant for him.

We need to consider the following utterances that made the class to explore the task in a different manner. In this regard, we would like to explore certain utterances that Jim and his students made (e.g. *You know, actually you know, we normally know*). Table 7 demonstrates some of those utterances.

**Table 7.** Key: *We normally know that \( \frac{3}{4} \) is 750 meters.*

<table>
<thead>
<tr>
<th>1st level Reification</th>
<th>2nd level Reification</th>
<th>3rd level Reification</th>
</tr>
</thead>
<tbody>
<tr>
<td>About specific performance</td>
<td>About routine performance (Direct reification - describing a person):</td>
<td>About the actor (Direct reification):</td>
</tr>
<tr>
<td>Indirect reification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jim Ob 1.114. Student 16: I am saying that ( \frac{3}{4} ) meters is equal to 750 meters.</td>
<td>Jim Ob 1.110. Jim: <em>How</em> are they equal?</td>
<td>Jim Ob 1.111. Student 16: We normally know that ( \frac{3}{4} ) is 750 meters.</td>
</tr>
<tr>
<td>Jim Ob 1.115. Jim: … Because they are not equal (Conclusion).</td>
<td>Jim Ob 1.115. Jim: <em>How?</em> Can you show us? … (Jim positioned himself with respect to students by these utterances).</td>
<td>Jim Ob 1.116. Student 16: They are not equal, <em>how</em> sir? (Student insisted that the concept is correct).</td>
</tr>
</tbody>
</table>

Starting with the utterances, *how are they equal* to which the student responded, *we normally
know that, refers to the common understanding that $\frac{3}{4}$ is equivalent to 750. This was an indication of a flawed pre-requisite knowledge the students might have. However, he insisted that $\frac{3}{4}$ meter is equal to 750 meters. Jim then said; how, can you show us, because ... Now it was the student’s turn to ask this question, how sir?

These subjectifying utterances are evolving around mathematizing. Students are trying to figure out why $\frac{3}{4}$ m and 750 m are not equal. These reifying actions show that Jim was concerned about the challenge he met with this particular assignment and therefore, is significant for two reasons. Firstly, students talked about the value of the fraction ($\frac{3}{4}$ m) that according to them is equal to 750 m. We found that Jim’s utterance (Because they are not equal) was perhaps judgmental, however he tried to convince them (at a later stage) why they are not equal (Jim Ob 1.117). The statement (Jim Ob 1.111) could also be misleading because if students normally know that $\frac{3}{4}$ and 750 are equal, that means their concepts on whole number and fractions are flawed. At that stage, Jim failed to convince them that $\frac{3}{4}$ meter is NOT equal to 750 meters. This reifying utterance was significant as Jim endorsed afterwards (Jim Int 1.132: ... the problem was not comparing the fraction but it was the conversion of one unit into another). He also observed that the decision of exploring two concepts in the same question was premature (Jim Int 1.156). As a result, they spent more time trying to convince each other that 750 is greater than $\frac{3}{4}$ meter (Jim Int 1.189).

As the student 16 insisted that two fractions were equal regardless of the different units, Jim required an explanation of him on this. Jim and his students were trying to persuade each other of their side of the story. The argument was protracted and consumed a lot of time. Students found it difficult to consider the different units as part of the solution, and Jim struggled to convince them. In this regard, Jim admitted that the question that he copied from the textbook was prematurely designed in the textbook. He overlooked or failed to anticipate the difficulty students would have, regarding the two different concepts in the same question.

Jim’s utterances are therefore endorseable and significant because:

- Utterances 001 - 042 demonstrated that Jim focused on simplifying the fractions followed by the comparison of two fractions (Jim Ob 1.043-060). However, he suggested, now let us jump to ‘c’ (Jim Ob 1.061). As he observed later, it was too early to link units while comparing the fractions. From utterances 1.063 to the end of the lesson, Jim and students concentrated only on this particular question ‘c’ that consumed the major part of the lesson.

- Jim repeated the concept more than once, yet he could not achieve the objectives of the lesson as he had planned.

- The utterances brought some changes in the classroom atmosphere that made Jim to understand that he should not have rushed the students to explore two different concepts in one assignment (Jim Ob 1.117).

- Jim provided an elaboration on ‘why they are not equal’, yet students kept on insisting that $\frac{3}{4}$ and 750 are equal.

- Even though we did not discuss this issue during the interview, the presence of the first author with a camera might have had an impact on Jim in the classroom.
When students answered the first two questions correctly, Jim assumed that they understood the concept correctly. In the third example, students failed to follow his logic when he linked units to the fractions.

**Linking Jim’s story to the broader study**

Jim identified that question ‘c’ was **mathematically significant** because of its real life application (of measuring distance) and because he had observed that students failed to follow the appropriate rule (Heyd-Metzuyanim and Sfard, 2012; Sfard and Prusak, 2005; Tanner and Jones, 2000). Through this deliberate approach of elaborating (Table 1&3), Jim was trying to **negotiate a meaning** for this lesson (Wenger, 1998; Table 3) through participation and reification (Sfard and Prusak, 2005; Wenger, 1998) that was accepted by student 10 (Table3, 1.065). Jim later acknowledged that there was a problem and that he should have rectified the misconception students had (Jim Int 1: actually, it was a bit difficult). In other words, he did not prepare the ground thoroughly enough for students to explore the key to the solution. From this analytic narrative above, we explored how Jim tried to make sense of his classroom practice. Through his reflections he was able to make meaning of his own practice and learn from his mistakes. The process of reflection and making meaning of oneself is thus an important component of shaping a professional identity.

Jim realized that having only one certain kind of attitude and approach to teaching mathematics might not be sufficient to reach out successfully to all his students (Jim Int 1). He recognized that he needed to work on his teaching and adapt his approach.

Through Jim’s reifying, endorsable and significant utterances, it was clear that his membership in his class was still in the process of being established. We also observed that Jim was not yet convinced of his inclusive membership (I tried to show) in the classroom community. He was also not sure of his exclusion (do it your way) from the classroom community. It is thus important to recognize that identities of BTs are not only manifested in personal narratives and stories but also in the classroom activities that they engage with. It is difficult to understand the complex relationship between personal identity and professional identity in the identity formation process. Furthermore, it is the interplay between subjectifying and mathematizing that illustrates and demonstrates a BT’s emerging mathematical identity and community of practice identity. As Wenger (1998) indicates, learning becomes the vehicle for the evolution of practices and for growing professional identities. Practicing and participating in mathematics classrooms thus shape and re-shape Jim to become a successful mathematics teacher one day, as he dreams to be.

**Conclusion**

To conclude this story, I quote Jim’s reflective thoughts;

“I **like being a facilitator**, not somebody just takes a chalk and gives (ideas) to the students. At times, you tend to return to the students. Sometimes you just go there (to the classroom) and get something from you (the students). So, if you are (a) facilitator, you want to learn. You are not making them to learn. Help them to learn” (FG Int 1).

Jim undoubtedly is learning to become a teacher who wishes to provide opportunities for students to learn mathematics by encouraging them to actively do mathematics and practice. He is a supportive teacher and affirms students when they demonstrate familiarity with the concepts. From the above analysis it could be concluded that Jim did however not end his lesson entirely happy. Jim encountered a few challenges that compromised mathematizing. He tried to convince the students (through various reifying and significant subjectifying utterances) what went wrong and what did not go wrong, yet the students failed to accept his
arguments. Therefore he could not achieve the objectives of the lesson fully according to him. The question now is whether Jim would choose a different approach if he was given a chance to re-do this classroom activity.

References


Acknowledgements

This project was funded by First Rand Foundation, Mathematics Education Chair in Education Department, Rhodes University, Grahamstown, RSA.
Seeking Synergy: The Need for Research at the Literacy/Numeracy Interface

Sally-Ann Robertson

South African Numeracy Chair Project, Education Department, Rhodes University, South Africa.
s.a.robertson@ru.ac.za

This paper emerges from a broader longitudinal case study researching teachers’ use of classroom talk in supporting children’s numeracy development. This paper argues for the importance, particularly within the South African context, of drawing together theoretical perspectives and insights from language/literacy research and numeracy research. This argument is based on preliminary analyses of: the results of the South African Annual National Assessments [ANAs] (part of the Foundations for Learning campaign); aspects of the post-Apartheid South African curriculum revisions and their attempted implementation; and initial empirical data from the case study teachers pointing to the interrelationship between language and mathematics learning. I propose the merging of three theoretical constructs (from Vygotsky, Halliday and Bernstein) as particularly useful for researching the interface of language and mathematical learning in the first year of the intermediate phase (Grade 4), when the majority of South Africa’s learners switch from learning mathematics in their mother tongue to learning in English.

Introduction

A central purpose of education involves helping children move along the ‘mode continuum’ (Gibbons, 2003, after Halliday) from common-sense ways of thinking and talking about things towards more formalised ways of doing so. Gibbons observes that “children have to learn to use language for a range of purposes and in a range of cultural and situational contexts” (2003, p. 250).

This paper is part of a broader study focusing on the ways Grade 4 mathematics teachers use classroom talk to support learners’ linguistic and conceptual development, especially with regard to the way they help learners move from what Vygotsky (2012) termed ‘everyday’ and ‘spontaneous’ concepts towards the more ‘academic’, ‘abstract’, ‘scientific’ mathematical concepts that preponderate as children move up through the grades.

The study brings together insights from the teaching and learning of numeracy and literacy to explore the use of talk as a mediational tool in two English second language Grade 4 mathematics classrooms in the Eastern Cape Province of South Africa. Although it is the home language [HL] for less than 10% of the South African population (Statistics SA, 2012, p. 22), English is the prime language of learning and teaching [LoLT]. Language is a major source of frustration and a significant contributory factor to the inequalities plaguing South Africa’s primary education landscape (Fleisch, 2008), not least in terms of its impact on epistemological access to mathematics (Setati Phakeng, 2014).

This paper contributes mainly at the conceptual and theoretical level, but some empirical data are used to illuminate aspects of the discussion. I argue from my background in second language teaching and learning that there is much synergistic potential in using insights from language and literacy research to work at the literacy/numeracy interface. I argue that based on our ANA results for mathematics and language over the past three years, and particularly
noting how the ‘crisis’ (after Fleisch, 2008) is most keenly felt in our less advantaged schools, that the data point to a need to consider this interface more closely. I conclude by arguing for the drawing together of strands taken from three areas of theoretical construct: Vygotsky’s socio-cultural theory of learning (2012); aspects of Halliday’s systemic functional linguistics view of language as a social semiotic resource (1985); and Bernstein’s work on ‘pedagogic discourse’, and his ideas around the different speech codes children bring to school (1990).

**Research design for the broader study**

Much research has focused on the contribution of classroom talk to learners’ sense-making of ideas and concepts, and on appropriate strategies for analysing classroom talk (amongst others, Mercer, 2004; Barnes, 2008). There has also been extensive research on the place of language in teaching and learning mathematics (amongst others, Adler, 1998; Moschkovich, 2007; Setati, Chitera & Essian, 2009). I have not found much research explicitly focussed on language at the literacy/numeracy interface. Hoadley has worked in this area (2007; 2012), drawing mainly on Bernstein’s ideas, and – to a lesser extent – on Vygotsky. The contents pages of the *South African Journal of Childhood Education* reveal relatively little local research work at this interface.

The present study identifies opportunities for cross-pollination between what research has helped us learn about ways for developing learners’ literacy (particularly in contexts where this is occurring through a second language), and research into optimal ways of supporting learners’ numeracy development. Pursuit of this goal involves examination of the ways two case study teachers use talk (primarily in English) to mediate their Grade 4 learners’ numeracy development.

A case study within a mainly qualitative and interpretive paradigm (Patton, 1990) was seen as the most appropriate approach for this research, the case being classroom talk across the two sites. An interpretive view holds that participants in any social context act according to the meanings they ascribe to that context: “social action can only be understood by interpreting the meanings and motives on which it is based” (Haralambos, Holborn & Heald, 2000, p. 971). At the same time, the present study acknowledges the problematic nature of a view of ‘reality’ as being primarily socially constructed (Maxwell, 2012), and recognises the need to take account of very real external factors impinging upon the research context. (These include management structures; language policy issues; and aspects of South Africa’s broader socio-political history and socio-economic patterning.)

The research sites represent what Graven identified as ‘opportunity samples’ (personal communication, March 11, 2014). The Grade 4 Mathematics teachers at both schools are participants in the Rhodes University South African Numeracy Chair’s [SANC] NICLE project [Numeracy Inquiry Community of Leader Educators]: a programme targeting the needs and interests of primary school mathematics teachers. My own membership of NICLE affords me the opportunity of engaging with the two teachers in a co-learning capacity.

**Theoretical assumptions about language and learning**

Vygotsky proposed a close “reciprocal” / “interfunctional” relation between thought and language (Kozulin, in Vygotsky, 2013, p. xlvi), as well as an essential relationship between talking and thinking. He was also amongst the first to highlight the importance of cultural experience, hypothesizing on the central role played by cultural symbols (*inter alia* language, books, pictures and other man-made objects) in a child’s cognitive development. He argued that these symbols not only affect the *content* of a child’s learning, but more importantly, the
actual process of learning: “When the child learns a language, for example, he does not simply discover labels to describe and remember significant objects or features of his social and physical environment but ways of construing and constructing the world” (Wood 1998, p. 17). This ‘construing’ and ‘constructing’ process, Vygotsky argued, is most powerfully achieved in the zone of proximal development [ZPD] where “a child’s empirically rich but disorganised spontaneous concepts “meet” the systematicity and logic of adult reasoning” (Kozulin, in Vygotsky, 2013, p. l). For Vygotsky, children’s spontaneous concepts provide “the necessary, but not sufficient [my emphasis], conditions for progress toward more powerful forms of thinking” (Renshaw & Brown, 2007, p. 533).

Based on his work on the different speech patterns (language codes) across the social classes in Britain, Bernstein (1964) argued that language was one of the central obstacles facing working-class learners: difficulty in fully understanding and using the elaborated code of speech found in most classroom contexts prevents them from fully accessing the types of abstract thinking that prevail in classrooms. “As a child progresses through a school,” Bernstein argued, “it becomes critical for him to possess, or at least be oriented toward, an elaborated code if he is to succeed” (1964, p. 67). Countless researchers have used Bernstein in their classroom analyses, including local research by, amongst others, Hoadley and Muller (2009). Throughout his life, Bernstein expended considerable mental energy on unpacking “the inner logic of pedagogic discourse and its practices” (2000, p. 4), and the ways in which these are communicated, and continue to contribute to differences in educational attainment across social strata. Hasan put Bernstein’s ideas to the test in Australia, analysing how working-class and middle-class mothers’ ways of interacting verbally with their children contributed to their children’s ‘mental dispositions’ (2002). Her findings corroborated those of Bernstein, though she did draw attention to an important point, made by Bernstein himself, which many overlook. This was that, while the mental dispositions children develop in the home undoubtedly impact on the ways in which they subsequently relate to school knowledge, it is not inevitable that such forms of consciousness are fixed. New speech encounters in new environments may well lead to the appropriation of new speech systems (linguistic codes), more closely aligned to the “specialised discourses of the school” (2002, p. 17). South African research (e.g. Hoadley, 2006, 2007, and 2012; Hoadley & Muller, 2009) makes similar claims.

Writing about Halliday’s contributions to our understanding of how language works as a semiotic system, Foley (1991) explains that Halliday saw a child’s progress towards recognising and then realising the full meaning-making potential of language as being achieved through learning from more competent others, and that this “tutelage” constitutes “a vicarious form of consciousness” (p. 24). The parallels here with Vygotsky’s ZPD ideas are clear. The significance of Halliday’s systemic functional approach to analysing language is that it alerts us to “differences of orientation to meaning and acting” (Foley, 1991, p. 27) which, if linked to Bernstein’s ideas around the effects of early socialization on children’s speech patterns, helps in our analyses of classroom talk and of the ways it enables or constrains learners’ mathematical sense-making.

**Some analysis of the interface between language and mathematics in the South African context**

Notwithstanding South Africa’s post-Apartheid Government’s commitment to social transformation via increased educational equity, analyses such as those done by Fleisch (2008) and Spaull (2013) show we are a long way from realising this. The most recent ANA results (Department of Basic Education [DBE], 2013) reveal a widening achievement gap
across the different school quintiles (school catchment areas, based on socio-economic status [SES]), with learners’ numeracy and literacy achievements well below what might be expected relative to the considerable investment – in both financial and human terms – made towards improving learners’ educational circumstances and outcomes.

The DBE instituted its ANA strategy in 2011. This involves standardised nation-wide testing of all learners in Grades 1-6 and 9. Tables 1 and 2 show some ANA outcomes for Mathematics and Language respectively across 2011-2013.

**Table 1.** Average % marks in Mathematics by grade (2011-2013)

<table>
<thead>
<tr>
<th>Phase/Grade</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2011-2013 Grade Average</th>
<th>Phase Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Foundation Phase [FP]</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>63,6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>57</td>
<td>53,6</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1</td>
<td>3</td>
<td>40,6</td>
<td></td>
</tr>
<tr>
<td><strong>Intermediate Phase [IP]</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0</td>
<td>3</td>
<td>30,3</td>
<td>32,1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7</td>
<td>9</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td><strong>Senior Phase [SP]</strong></td>
<td>/a</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Data derived from DBE, 2012; 2013)
Table 2. Average % marks in Language by grade (2011-2013)

<table>
<thead>
<tr>
<th>Phase/Grade</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2012-2013</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>AL</td>
<td>L</td>
<td>AL</td>
</tr>
<tr>
<td>P</td>
<td>59</td>
<td>8</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>2</td>
<td>1</td>
<td>1,5</td>
</tr>
<tr>
<td>P</td>
<td>34</td>
<td>3</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>P</td>
<td>n/a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

(Data derived from DBE, 2012; 2013)

The data here show the marked fall off in children’s numeracy and literacy performance as they proceed up the grades. The fall off between foundation and intermediate phase average marks across the three years is notably steeper for mathematics (53.6% down to 32.1%) than for language (55.5% down to 46.6%); a range differential of 21.5% and 8.9% respectively. This differential may in part be a consequence of children having to contend with a double load: more challenging and abstract mathematical tasks, coupled with less scaffolding of the vocabulary and syntactic structure of the language in these tasks. This latter point is currently being investigated by Sibanda (2013). Such a fall off is not uniquely South African, however. American research, for instance, revealed that most children progress through the earlier stages of reading development in similar ways, but once the controlled and scaffolded reading typically found in the early grades gives way to texts “more varied, complex, and challenging linguistically and cognitively” (Chall & Jacobs, 2003, unpaged), unevenness in children’s achievements appears. Increased text complexity is often accompanied by expansion in the number of subject areas making up a curriculum. Encounters with new knowledge areas and a less highly controlled vocabulary and syntactic load places considerable strain on learners. Chall and Jacobs labelled this fall off the ‘fourth-grade slump’, noting that such slump is more prevalent amongst children of lower SES (2003).

Innumerable studies have attested to a quite intractable correlation between low SES and low educational attainment (amongst others, Pretorius & Naude, 2002; Hart & Risley, 2003). In his analysis of the crisis in South Africa’s primary school education, Fleisch (2008) drew attention to a marked ‘bimodal distribution’ of achievement in literacy and numeracy across the different socio-economic and racial sectors of our society. More recently, Graven (2014) noted that South Africa “provides an ‘extreme’ case of performance gaps between high and low SES learners even while political will and resource allocation for redressing inequality are identified as a national priority” (unpaged). The following table shows some poverty-
related achievement differentials in the 2013 ANA results.

**Table 3.** Average % ANA marks in Language (Home Language [HL] & First Additional Language [FAL]) and Mathematics by grade and quintile [Q] (2013)

<table>
<thead>
<tr>
<th>Grade</th>
<th>LANGUAGE</th>
<th>MATHEMATICS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HL F1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>7.6</td>
<td>5,7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>P</td>
<td>0.9</td>
<td>1,7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>P</td>
<td>6,6</td>
<td>0,5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>P</td>
<td>2</td>
<td>4,6</td>
</tr>
</tbody>
</table>

(Data derived from DBE, 2013)

The data on range across Q1 and Q5 show how markedly the equity gap widens up the grades. Consistent with earlier comments regarding the ‘fourth-grade slump’, overall the equity gaps are larger in IP than FP. The Grade 4 mathematics classrooms in the present study serve children from lower down South Africa’s SES ladder, and any disadvantages deriving from this circumstance are almost certainly compounded by the schools’ LoLT decisions.

**Some analysis around the LoLT issue**

The percentage of South Africa’s learners learning through their HL is low. In both research sites, although the HL of teachers and learners alike is isiXhosa, the LoLT is English, putting these learners amongst the 79.1% of South African Grade 4s using English as LoLT (DBE, 2010, pp. 13; 16). Motala and Dieltiens (2011, p. 11) note that South Africa’s 1997 Language in Education Policy “tends in practice to privilege English (and Afrikaans), despite a rhetoric of equality regarding the other nine official languages.” Figures in Table 4 highlight the scale of this privileging of English.
Table 4. Percentage of learners using English as LoLT (Grades 1-12) (2007)

<table>
<thead>
<tr>
<th>Grade</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.8</td>
<td>3.8</td>
<td>7.7</td>
</tr>
<tr>
<td></td>
<td>9.1</td>
<td>1.1</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>2</td>
<td>1.4</td>
</tr>
</tbody>
</table>

(Data derived from DBE, 2010, p. 16)

Research evidence shows a good correlation between “mother tongue education and scholastic achievement” (DBE, 2010, p. 5). The figures in Table 4 suggest, however, that such is the power of English that those making decisions about schools’ language policies may be turning a blind eye to some of the epistemic implications of an English LoLT whereby learners face the dual challenge of trying to master English, while at the same time learning through English. Mathematics, too, being a ‘language’ (Schleppergrrell, 2007) simply adds another layer of linguistic and conceptual complexity. Schleppergrrell commented that “a key challenge in mathematics teaching is to help students move from everyday, informal ways of construing knowledge into the technical and academic ways that are necessary for disciplinary learning” (2007, p. 140). Writing of her mathematics learning experiences, Setati Phakeng noted that her “greatest difficulty was learning in a language in which I was not fluent,” leading, she said, to heavy reliance on rote-learning (Setati Phakeng & Moschkovich, 2013, p. 121).

In terms of the language policies at both research sites, all the Grade 4s, although native speakers of isiXhosa, are - officially - learning mathematics in English. Both teachers acknowledge an element of bilingualism in their teaching. School A’s Grade 4s are in their first year of English as LoLT, having had mother tongue LoLT through foundation phase (Grades 1-3). In her initial interview, their teacher remarked: “We must teach them in English here. But I am doing them in both Xhosa and English.” Because School B has a straight-for-English policy, the Grade 4s there have had a longer and more extensive exposure to English. At neither school, however, are the Grade 4s yet fully proficient in English. They are thus unable to fully exploit it as a (semiotic) resource for learning. Rather than viewing this circumstance in deficit terms, the present study takes the more positive stance of viewing these children as ‘emergent bilinguals’ (Garcia & Kleifgen, 2010, p. 2), capable of functioning in their HL while building up their English proficiency.

Analysis of some recent curriculum revisions

South Africa’s Curriculum and Assessment Policy Statement [CAPS] describes mathematics as “a language that makes use of symbols and notations to describe numerical, geometric and graphical relationships” (DBE, 2011, p. 8). Amongst the specific mathematical skills that learners need, are the following: “correct use of the language of Mathematics”, “number vocabulary”, and “the ability to listen, communicate, think, reason logically and apply the mathematical knowledge gained” (DBE, 2011, pp. 8-9). All of these are language dependent, and invite reflection upon Cummins’s distinction between BICS (basic interpersonal communication skills) and CALP (cognitive academic language proficiency) (n.d., p. 1). Prior to CAPS, South Africa’s NCS specifications for the Languages Learning Area included a learning outcome (LO) explicitly requiring language teachers to attend to aspects of learners’ CALP development (Department of Education, 2002), but this appears to have been down-played in CAPS. I refer here to LO5: Learning and thinking: use of language to think and reason, and access, process and use information for learning. The diminution of this LO
carries the risk that language teachers – however inadvertently – may neglect giving systematic support to the development of aspects of learners’ meta-cognition and verbal processing. The ability to understand and use spoken language beyond the BICS level (Cummins, n.d.) is critical for literacy and numeracy development. One other element of diminution worth mentioning is that – paradoxically - in terms of language policy pre-1994, African children may in fact have been marginally better off relative to their preparation for English as LoLT. Pre-1994, in African schools English was introduced as LoLT from Grade 5 (a year later than is currently the case). Most African children would also have learned English as a subject for three years prior to its becoming the LoLT. Murray (2012) notes that “new policies and curricula have had the effect of delaying the introduction of English as a subject and accelerating its introduction as a LoLT, thus reducing the time to prepare for the transition” (p. 3).

A core goal of South Africa’s post-1994 educational reform is to help teachers move away from largely teacher-centred, behaviourist-dominated conceptions of teaching towards more learner-centred constructivist approaches. Sfard (1998) captures this distinction well in her discussion of what she termed the ‘acquisition’ and participation metaphors. Whereas the older (acquisition) metaphor conjures up images of “the human mind as a container to be filled with certain materials and about the learner as becoming an owner of these materials”, in the newer (participation) metaphor “learning a subject is … conceived of as a process of becoming a member of a certain community” (1998, pp. 5-6). Much remains to be done in helping teachers deepen their understanding of strategies for helping learners become more actively-engaged participants in “working on [their own] understanding” (Barnes, 2008, p. 3).

In relation to mathematics teaching and learning, Lerman (2000) notes how discussion around the absolutism/fallibilism dichotomy led to challenges to “the traditional mathematical pedagogy of transmission of facts” (p. 22), ultimately leading to a ‘social turn’ whereby ideas about “meaning, thinking, and reasoning” are viewed as much more ‘situated’ “products of social activity” (p. 23) (as opposed to being seen simply as the outcome of detached, and ostensibly objective logical reasoning). A similar ‘turn’ has occurred in literacy circles whereby it is recognised that conceptions of what literacy is, of how it is to be used, and of what value is to be placed on it, vary across social contexts. Heath’s classic Ways with words ethnography (1983) illuminates this point well. Her study demonstrated how the different patterns of language socialization children bring into the classroom can profoundly affect their educational attainments (1983, p. 349). The patterns children bring to the classroom may then be echoed in the pedagogical patterns of the classroom. Closer to home, Hoadley (2006) argued that “school and classroom processes potentially amplify differences between students, disadvantaging the working class” (p. 2). In a subsequent paper, focussing more specifically on mathematics pedagogies, she compared the teaching of mathematics in Grade 3 classrooms across the SES spectrum. Her analysis demonstrated how different teaching styles in the different social-class settings gave rise to “differential access to specialized school knowledge” (2007, p. 703). She concluded that children’s “potential for acquiring ... the specialized knowledge of mathematics, is seriously undermined in [by] the pedagogy” she observed in lower SES classrooms (2007, p. 704).

Some analysis around the importance of patterns of talk in classrooms

The decision to make talk the focus for the present study was fuelled by the recognition that – as aptly put by Douglas Barnes - “learning floats on a sea of talk” (cited by Simpson, Mercer and Majors, 2010, p. 1). These writers note that there has been a resurgence of interest in the value of classroom talk as a pedagogical device (2010, p. 1). A particular advantage of talk is
that, unlike writing, talk “is easy and impermanent. We can try out an idea and change it [my emphasis] even as we speak. Exploratory talk ... provides a ready tool for trying out different ways of thinking and understanding” (Barnes, 2010, p. 7).

Graven notes that encouraging children to communicate their mathematical thinking verbally (rather than having them either work things out on paper, or come up to laboriously try out solutions on the chalkboard) is a useful way of maintaining a brisk pace and helping children keep focussed on the task at hand. In many of the classrooms she has visited, however, she has noticed that teachers tend to emphasise listening at the expense of talking, and on occasions if children asked questions, they were scolded for not having listened attentively enough. She thus identifies the raising of primary mathematics teachers’ awareness of how crucial it is that children be given opportunities to engage verbally as one of NICLE’s key priorities. SANC’s support for the present study derives from this perception (personal communication, March 12, 2014).

Studies of patterns of talk in the classroom reveal substantial asymmetry in teacher: learner talk. Overwhelmingly, it is teachers who do the talking. This, as Wells notes, is at odds with the pattern of adult-child interactions revealed in his longitudinal study of differences in language at home and at school where, if anything, the asymmetry was in the other direction (1986, p. 86). The predominance of teacher-talk seems a particularly prevalent feature of many South African township schools. Hoadley, for instance, cites Chick’s finding of teachers “adopting authoritarian roles and doing most of the talking, with few pupil initiations, and with most of the pupil responses taking the form of group chorusing” (2012, p. 3). Chick’s work dates back to 1996; but classroom videos from Year One of the SANC project indicate that such patterns persist (Graven, 2012). It would seem that teacher talk is a deeply ingrained aspect of teachers’ habitus (after Bourdieu, 1974). Wells, in distinguishing between ‘monologic’ (teacher talk) and ‘dialogic’ (teacher/learner talk) classroom interactions, insists that “education requires both” (2007, p. 263). In his exploration of what he termed the ‘emerging pedagogy’ of the spoken word, Alexander too found monologic talk the prevailing mode, noting that, while “classrooms are places where a great deal of talking goes on, talk which in an effective and sustained way engages children cognitively and scaffolds their understanding is much less common than it should be” (2005, p. 2).

**Teachers’ language insights in relation to their numeracy teaching**

In interviews for the present study, both teachers identified the challenges their learners face with English: mastering it, plus having it as the LoLT for their mathematics lessons. School A’s language policy parallels that recommended in CAPS (mother tongue education through the foundation phase years, transitioning to English as the main LoLT in Grade 4). Although the current Grade 4 cohort started learning English (as subject) in Grade 2, Teacher A notes her learners’ struggle with it: “The English is not easy for them because in their foundation phases they were doing everything in Xhosa-mother tongue. But now in Grade 4, is that transition. Eh! It is not easy.” Despite School B’s earlier start with English as LoLT, Teacher B said that language remained a challenge. “Because,” she explained, “all these learners here at school are Xhosa-speaking learners, and maths is done in English. And maths also has its own language. ... that is a main problem.”

Most if not all the children at both schools come from relatively less affluent homes, although Teacher B did indicate that School B’s fees are significantly higher than those of other township schools: “Other schools, maybe the school fees are R50 for the whole year. Here I think it’s R140,00 per month.” School A’s learners appear to be especially vulnerable to the many difficulties attendant upon poverty. Unprompted, Teacher A remarked in the initial
interview that most of her learners had problems. “Some of these problems,” she said, “are social problems. Most of our kids are sick. They’re on treatment. So they don’t cope most of them.”

These remarks from the case study teachers clearly demonstrate the futility of tackling problems of low levels of numeracy achievement independently of attention to language and SES issues, or – indeed – to larger social issues.

Concluding comments

The contribution this paper makes is to provide a rationale for the need for research at the numeracy/literacy interface, and to bring together three socio-cultural and socio-linguistic theories of learning to conduct a triple-layered analysis of mathematics classroom talk in the context of low-SES and L2 teaching/ learning environments. The three theories cohere in that all acknowledge the centrality of language. At the same time each has the potential to provide insights from slightly different perspectives. The brief analysis provided here of the ANA results, curriculum policy, and teacher experiences points to the need for such a triple-layered analysis as a means of exploring synergistic opportunities at the numeracy/ literacy interface.

The two case study sites, being at once sufficiently similar and sufficiently different, together, promise to generate rich insights around the issue of mathematics classroom talk taking place predominantly through non-mother tongue, and in low-SES contexts. While it is seldom the nature of interpretive case studies that they allow for generalization, there is value in looking “carefully at individual cases ... not in the hope of proving anything, but rather in the hope of learning something” (Eysenck, cited by Flyvbjerg, 2006, p. 224). It is my hope that this case study investigation of teachers’ use of classroom talk in mathematics lessons may resonate in significant ways with others involved in similar circumstances. Cresswell and Miller (2000, p. 129) note that ‘thick description’ is one way of helping others assess the extent to which findings resonate with (or ‘relate’ to) other settings.

References


Sibanda, L. (2013). The linguistic challenges of the Grade 4 mathematics ANAs and the way in which teachers manage these demands in their preparation for the ANAs. Doctoral Research Proposal, Rhodes University, Education Department, Grahamstown.


**Acknowledgements**

I thank Professor Mellony Graven for her invaluable guidance on earlier drafts of this paper. This work of the SA Numeracy Chair, Rhodes University is supported by the FirstRand Foundation (with the RMB), Anglo American Chairman’s fund, the Department of Science and Technology and the National Research Foundation.
Applying a Linguistic Complexity Checklist and Formulae to the 2013 Grade 4 Mathematics National Assessments

Lucy Sibanda & Mellony Graven

1 Department of Education, Rhodes University, South Africa.
2 Department of Education, Rhodes University, South Africa.
1 lcysbnd@gmail.com, 2 m.graven@ru.ac.za

This article emerges from the first author’s broader PhD study that investigates the nature of the linguistic complexity of the Grade 4 Department of Education (DoE) Annual National Assessments (ANA) test items and how learners (with a poor command of the language of learning and teaching) and teachers experience them. This paper reports on the findings of a content analysis done on the 2013 mathematics ANA test items using Shaftel, Belton-Kocher, Glasnapp and Poggio (2006)’s linguistic complexity checklist and formula. Results point to some serious linguistic challenges of test items particularly in relation to: recurrent use of 7 or more letter words, homophones, prepositional phrases and specific mathematics vocabulary across the majority of questions. The study recommends a consideration of the linguistic complexity of test items, accompanied by trialling of the items with learners, by test designers prior to their use in national assessments. We argue that this consideration is especially important at the Grade 4 level where the majority of South African learners will only have had a few months of mathematics instruction in English before they write these assessments in English.

Introduction

The study is situated within the South African literacy and numeracy context, where international studies such as TIMSS (see Reddy, 2006); regional studies such as SACMEQ (see Taylor, 2009; Spaull, 2011) and national mathematics assessments such as the ANAs (DBE, 2012; 2013) reveal the underperformance of learners confirming Fleisch’s (2008) contention that primary education is in crisis especially in reading, language and mathematics. Additionally, inequality in performance is growing. Thus while in the 2003 TIMSS study SA was the lowest performing of 50 countries, equally of concern is that SA had the largest variation in scores with learners in African schools achieving scores half of those of historically White schools. Furthermore, mathematics scores for African7 schools decreased significantly from TIMSS 1999 to TIMSS 2003 which was not the case for other SA schools, pointing to increasing inequality of mathematics performance.

The Department of Basic Education (2011) attributes South African learners’ poor performance in numeracy benchmark tests to inadequate language capabilities since many learners did not understand what was expected of them during the assessments. In South Africa, most learners learn mathematics in English, a language that is not their Home Language (HL). It has been found that most learners who perform poorly in Grade 12 mathematics rarely use English at home or come from homes where English is rarely used (Simkins in Taylor, Muller & Vinjevold, 2003). In rural schools, learning and teaching occur in a context of limited English language infrastructure where “English is only heard, spoken, read and written in a formal school context” (Setati & Adler, 2000, p. 251).

7 Under apartheid four racially classified categories of schools existed: Black; Coloured, White and Asian. Previously Black schools are often referred to as African schools.
In 2011 the Department of Basic Education introduced the Annual National Assessments to be conducted in all government schools at Grades 1-6 and Grade 9 as part of their Foundations for Learning Campaign. The ANAs aim to expose teachers to better assessment practices, help districts to identify schools most needful of assistance, and inform parents about their children’s performance (DBE, 2011). The pivotal role of the ANAs requires the development of confidence in them as fair and valid measures of learners’ competence and performance with the levels test items neither too high nor too low.

In the Foundation Phase (FP) (where use of home language in classrooms is encouraged in the national language policy context), ANAs are provided in learners’ home languages as requested by schools, but in the Intermediate Phase (IP) (where the language policy demands a switch to either English or Afrikaans), the ANAs are set in English or Afrikaans. Thus the department’s website states:

The tests are administered in all the eleven official languages in the FP and in the two languages of teaching and learning in the IP and Senior Phase. Necessary adaptations are effected for learners who experience various kinds of learning disabilities to ensure that every learner has the opportunity to demonstrate what they know and can do in the assessment (DBE, 2014, no page).

The point about learning disabilities and necessary adaptations is interesting. While we consider the multilingualism of our South African learners as a proficiency the extent to which learners are disadvantaged by having to write the assessments in English rather than in their home language perhaps requires the same attention as those learners disadvantaged by learning disabilities to ‘ensure that every learner has the opportunity to demonstrate what they know and can do in the assessment’.

Results of the ANAs for the past three years are cause for concern. The 2012 and 2013 reports for the ANAs (DBE, 2012, 2013) reveal that learners performed poorly in mathematics across grades. In the FP, learners performed better but as they proceeded to IP, the levels of achievement decreased significantly. The 2013 ANA results for Mathematics, for example show learner results decreasing from an average of 60% in Grade 1 to 14% in Grade 9. Important, however, is the large drop that occurs in results from Grade 3 to Grade 4 (i.e. from an average of 53% to 37% for 2013) (DBE, 2013). This begs the question as to the role language plays in this large drop in performance from the FP to IP. The teaching of First Additional Language (FAL) from Grade 1 was made compulsory in 2012 by the Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011). However, the current Grade 3s and 4s did their Grades 1 and 2 under the National Curriculum Statement (NCS) dispensation when teaching in the FAL was not compulsory. If some of these learners used isiXhosa (or other South African languages) as language of teaching and learning (LoLT) in Grade 1 and 2 and only started using English in Grade 3, then it would be naive to expect them to have already acquired the basic vocabulary in English which they need to communicate and learn in that language.

This makes the analysis of the linguistic complexity of the Grade 4 ANAs imperative seeing that the grade marks the transition between the FP and IP. In South Africa Grade 4 is a critical stage where many learners experience four significant transitions from the FP. The first transition from Grade 3 to Grade 4 is from using isiXhosa (in the Eastern Cape where the study is being done) to using English as the LoLT in Grade 4. The second transition is from reading mostly narrative, story-like texts whose language closely approximates ordinary language of everyday social interaction in the FP, to reading expository texts with more content-dense vocabulary in Grade 4 (Chall, Jacobs, & Baldwin, 1990). The third transition
is the movement from ‘learning to read’ to ‘reading to learn’ (DoE, 2008). In the FP, learners are trying to develop the skill and art of reading but, when they come to Grade 4, they are expected to read different content subjects and learn from what they read. The mechanics of reading, which underpin learning to read, are supposedly developed in the home language (HL) in the FP and used in English in Grade 4 to access information from texts. The fourth transition is the movement from more concrete thinking in the FP to more abstract thinking in the IP. Mathematics abstraction is particularly critical for progress in the IP.

**Theoretical perspective, methods and analytic tools**

This study is guided by an assumption that language is central in the learning of mathematics. It is framed by a socio-cultural view of language and learning, with Vygotsky’s (1976) influential work informing the theory of language and learning. Hallidayan (1978) language theory, was also used, which cohered well with Vygotsky’s theory because for both, language is central to learning.

The broader PhD study from which this paper emerges researches two key questions. The first explores the linguistic complexity of the Grade 4 2013 ANA document (i.e. test/documentary analysis). The second question investigates, through a case study approach of two township mathematics classrooms, the way in which the participating Grade 4 learners and teachers experience the linguistic challenges of the ANAs. This paper reports only on the findings of the content analysis on the 2013 mathematics ANA test items using Shaftel et al.’s (2006) linguistic complexity checklist and formula. Shaftel et al (2006)’s checklist was selected after a thorough literature review, which indicated that was sufficiently comprehensive and took into account key complexity issues raised by mathematics education and language literature. A brief review of some of the broader literature that informs Shaftel et al. (2006)’s complexity tool and formula is given below before describing Shaftel et al.’s checklist and formulae that provides the analytical tool for this paper.

According to Bergqvist, Dyrvold and Osterholm (2012) mathematics is linguistic in nature because it has words, symbols, sentences and grammatical structures which are essentially part of the language. These linguistic features serve to describe mathematical concepts, which cannot be described in everyday language. For Halliday (1975, p. 65) mathematics register is ‘a set of meanings that belong to the language of mathematics and that a language must express if it is used for mathematical purposes.’ According to Halliday (1993), the difficulty of mathematics also lies in the grammar of the language used and not only with the vocabulary. He identifies some features that have tremendous effect on the performance of English as second language learners. These include long phrases in questions, complex sentences, syntactic ambiguity, special mathematical expressions, lexical density and many more. The grammatical density of sentences engendered by the linguistic features described above present linguistic challenges that confound young learners.

Another source of complexity is the dissonance between ordinary language and mathematical language. According to O’Halloran (2005, p.75) ‘the major process type found in mathematical language appears to be the relational process.’ This process is usually absent in English and when learners encounter it in mathematics they are challenged.

To compound the limited exposure to the English as the LoLT, mathematical language has been found to be complex even for English HL speakers learning mathematics in English (Halliday, 1989). The level of complexity for second language learners can only be greater as questions that are cognitively undemanding to English native speakers may be quite exacting to second language learners learning mathematics in English (Cummins and Swain, 1986).
According to Schleppegrell (2007, p. 140), “Learning the language of a new discipline is part of learning the new discipline; in fact, the language and learning cannot be separated.” Abedi (2006) notes that when assessments have complex language, this negatively affects the performance of learners and the performance gap between English language learners and HL speakers of English is increased. For Abedi, assessments where the linguistic component engenders unwarranted complexity to the mathematical component are unfair and invalid. Abedi (2006) also argues that standardised achievement tests that are prepared for English language learners but take no consideration of their language proficiency pose more challenges for the learners and cannot portray what learners really know.

For this paper we draw on Shaftel et al.’s (2006) linguistic complexity checklist and formula as an analytic tool for the content analysis of the 2013 Grade 4 mathematics test items. The larger analysis of test items using the broader Systemic Functional Linguistics that was conducted is beyond the scope of this paper. Thus we only expand here on Shaftel et al.’s (2006) analytic framework.

Shaftel, et al. (2006) investigated the influence of the language characteristics of mathematics assessments given to English language learners in English on Grade 4, 7 and 10 learners. Shaftel et al. (2006) analysed individual test items in a multiple-choice format that were presented as word problems, though the number of words per item differed. The learners’ performance was determined by the item difficulty as well as the ability to answer the question correctly. Items were coded according to their linguistic complexity, taking into consideration the “total number of words, sentences, and clauses in each item; syntactic features such as complex verbs, passive voice, pronoun use and vocabulary in terms of both mathematics vocabulary and ambiguous words” (Shaftel et al., 2006, p. 11).

The results for Shaftel et al. (2006) study revealed that the mathematical and linguistic features of the test items measured, had an impact on learner performance, “with a moderate-to-large effect at Grade 4, a medium effect at Grade 7, and a smaller effect at Grade 10” (p. 120). At Grade 4, prepositions, ambiguous words, complex verbs (verbs with three or more words), pronouns, and mathematics vocabulary showed unique effects on item difficulty. The greater the number of linguistic elements per item, the more difficult the item.

For this study, test items are defined as each item for which a learner got some marks. Unlike in Shaftel et al.’s (2006) study that looked only at multiple-choice items our analysis includes the ANA multiple-choice and other word problems not in multiple-choice form.

Shaftel et al.’s Linguistic Complexity Checklist Index was developed as an analytic tool to analyse the linguistic complexity of items. We use this tool because it is specifically designed for assessing mathematical test items. Four levels of language have been established and these are: basic level, word level, sentence level and paragraph level. Shaftel et al. (2006) list some individual language features that they considered to be challenging. These are:

A. Basic level: Number of words in an item
B. Word level: words of 7 letters or more; Relative pronouns (e.g. that, whom, whose); Slang / ambiguous / multiple meaning or idiomatic words (e.g. change, set); Homophones (e.g. two/too, prize/price); Homonyms (e.g. there, their, they’re); Specific mathematics vocabulary (e.g. pentagon, symmetry)
C. Sentence level: Prepositional phrases (e.g. beginning with, from, by, at); Infinitive verb phrases (to make, to sell); Pronouns (e.g. his, her, they); Passive voice (were sold, were rounded off); Complex verbs of 3 words or more (e.g. could have been);
Complex sentences (e.g. with subject and predicate); Conditional constructions (e.g. if…then); Comparative constructions (e.g. less than, greater than)

D. Paragraph level: references to specific cultural events.

The Linguistic Complexity Index (LCI) is then calculated as: \( \text{LCI} = \frac{\text{Sum A + Sum B + Sum C + Sum D}}{\text{Number of sentences}} \). For example, \( \text{LCI} = \frac{\text{Sum of words} + \text{Sum of pronouns, ambiguous words, homophones etc} + \text{Sum of prepositional phrases, passive voice etc} + \text{Sum of specific cultural events}}{\text{Number of sentences}} \).

**Analysis of the 2013 ANA test items**

The linguistic features of each item were evaluated using the Linguistic Complexity Checklist. In each item, the number instances of use of linguistic features were counted, as shown in the table below, added and the result was divided by the number of sentences, see bottom row of table for the LCI of each item. For the purpose of this analysis, an item subsumes sub questions. So for example, item 4 comprises two questions namely 4.1 and 4.2, with the instruction ‘Complete each of the following number patterns:’ applicable to both questions. Therefore, the 18 items analysed contain 31 questions. Each question is analysed individually using the LCI features and formula. For those sub questions where the instruction is given at the start of the item the instruction is analysed together with the first sub question only. This means for example, item 4, the instruction ‘Complete each of the following number patterns:’ is only analysed together with the first pattern in 4.1 and not again for 4.2. The reason for this is that learners are likely to read the instruction part and then go on to answer the first question followed by subsequent questions without going back to read the instruction for each sub question.

This resulted in the following questions having a 0 linguistic complexity index: 3.2, 4.2, 6.2, 6.3, 6.4 and 16.2 (i.e. they had no language). Table 1 below shows the frequency of use of language features in the 2013 ANAs.

**Table 7. Frequency of use of language features in all 2013 ANA questions**

<table>
<thead>
<tr>
<th>Question</th>
<th>A-Number of words</th>
<th>B-No. of words with 7 letters or more</th>
<th>C-No. of homonyms</th>
<th>D-No. of passive sentences</th>
<th>E-No. of complex verbs</th>
<th>F-No. of infinitive verbs</th>
<th>G-No. of specific mathematical vocabulary</th>
<th>H-No. of prepositional phrases</th>
<th>I-No. of conditional constructions</th>
<th>J-No. of references to specific cultural events</th>
<th>K-No. of sentences</th>
<th>LCI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>8</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>19 9.5</td>
</tr>
<tr>
<td>1.2</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>18 18</td>
</tr>
<tr>
<td>1.3</td>
<td>13</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>25 25</td>
</tr>
<tr>
<td>1.4</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>14 14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1.5</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.6</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.1</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4.1</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5.1</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5.2</td>
<td>12</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>6.1</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9.1</td>
<td>19.5</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9.2</td>
<td>16</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>9.5</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12.1</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12.2</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>7.6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15.1</td>
<td>10</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15.2</td>
<td>10</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15.3</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15.4</td>
<td>9</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16.1</td>
<td>17</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>16</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18.1</td>
<td>21</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>18.2</td>
<td>8</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18.3</td>
<td>14</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>9</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: questions 3.2; 4.2; 6.2; 6.3; and 6.4 all have an LCI of 0
Discussion of findings

Word level
At the word level, the number of words with plus or minus 7 letters, number of pronouns, number of ambiguous words and number of homophones in each item were considered to increase complexity.

The number of words with plus or minus 7 letters.
Thirty out of the thirty-six questions contained words with 7 or more letters. The questions that had the highest number of words with 7 or more letters were questions 9.1 (with 7 words), question 15.1 (with 6 words) and questions 7 and 10 (with 5 words). For example, question 9.1 asked, “Write down the flight number of a flight which will depart for its destination before midday.” These represented 83% of the total 36 questions which demonstrates the linguistic challenge in terms of item length for the vast majority of questions. This is of concern given Bergqvist, Dyrvold and Osterholm’s (2012) observation that this language feature is the major source of linguistic complexity. Thus that words with 7 or more letters featured so much in the items shows the potential to hinder the comprehension of many test items.

Number of pronouns
Seventeen of the thirty-six questions (47%) contained pronouns most of which were interrogative pronouns like ‘what’, ‘which’, and demonstrative pronouns like ‘that’. Other pronouns that were marginally used were indefinite pronouns like ‘each’, ‘much’ and subjective pronouns like ‘she’, ‘it’, and objective pronouns like ‘her’ and ‘its’. Questions 9.2 and 15.4 had the highest number of pronouns (3). Pronouns might be expected to cause confusion for less skilled English language learners because ‘they introduce a (possibly ambiguous) reference to another sentence element’ (Shaftel et al., 2006, p. 121). An example is question 15.4 “Mom shared a cake equally amongst Mary and her 3 friends.” For a less skilled English language learner, it may be confusing whether the pronoun her is referring to Mary’s friends or mom’s friends. Although they may bring difficulty to mathematical texts, pronouns are essential in sentence constructions as they serve to indicate possession and to form questions among other uses. That pronouns featured in slightly above half the items shows the potential to militate against the comprehension of significant number of test items.

Number of ambiguous words
According to Halliday (1989), syntactic ambiguity is the presence of two or more possible meanings within a single word or sentence. This is common in mathematical texts. For the present study, ambiguous words were those with multiple meanings where assignment of the unintended meaning compromised the comprehension of the item’s demands and inevitably the response given. For example, in question 7, ‘The difference between 1 613 and 859 is seven hundred and fifty-four.’ In this case, difference is not used in the everyday meaning which means ‘dissimilar or unlike.’ Here it refers to the answer you get after subtracting a number. From this example we note that ambiguous words may bring complexity and can be confusing to learners who are not proficient in the English language used to learn mathematics. Nine questions (25%) contained ambiguous words. Other examples of ambiguous words were ‘factor’, ‘multiple’ and ‘hands’ in questions 1.6, 1.4 and 8 respectively. Although ambiguous words could potentially confound the learners, they were not as prevalent as the other two features discussed above which reduced their potential to impact learners’ test performance negatively.
Number of homophones

Homophones are two or more words that have the same sound or spelling but differ in meaning. These words can make reading tricky as not knowing the definition of a particular homophone can change the meaning of what is read, thus affecting comprehension. Altogether 27 (75%) questions contained homophones. Question 9.1 had the highest number of homophones (8). Examples are write/right, buy/by, of/off and board/bored. For the last example given, the sentence reads, ‘Look at the departures board at the airport’ – the word ‘board’ if read as meaning ‘bored’ would change the meaning of the sentence. Questions 1.3, 5.1, 9.2 and 10 all had five homophones each. Homophones are another major source of ambiguity, and the ambiguity of language could hinder and cause confusion in understanding of test items. The fact that homophones appeared in the vast majority of the questions attest to their potential widespread effect on the learners’ test performance.

Sentence level

At sentence level the linguistic features that were analysed were number the of passive sentences, number of complex verbs, number of infinitive verbs, number of specific mathematical vocabulary, number of prepositional phrases and number of conditional constructions.

Number of passive sentences

In passive sentences, the sentence begins with the object rather than the subject, which is highly unlike the everyday use of language. For example question 1.2: ‘The number 6 555 rounded off to the nearest 100 is….’ A learner may find this question difficult to understand because of the passive voice ‘rounded off’ used. The question could be more easily understood if it was asked in passive voice like ‘Round off 6 555 to the nearest 100’. The more common voice construction in English is the active voice, not the passive voice, and thus passive voice constructions can be especially insidious, for failure to understand them correctly can actually lead to a misinterpretation of vital information (Tanko, 2010). According to Hinkel (2002, p.1), learning and teaching the ‘meanings, uses, and functions of the passive voice represents one of the thorniest problems in second language grammar instruction’, and many second language learners of English appear to have difficulty with passive constructions. Although complex to unravel particularly for second language learners with limited English language proficiency, passive constructions were marginally employed in the test items. Only 2 questions (5.5%) were in passive form. These were questions 1.2, 11 and 14. Each of these questions had one passive sentence.

Complex verbs

Complex verb phrases in this study are phrases with at least two verbs. This suggests the use of multiple or difficult verb tenses (Shaftel et al., 2006). In general, complex verb phrases consist of one or more auxiliary verbs plus a main (lexical) verb. Four complex verb phrases in 4 questions (11.1%) were found. They were found in questions 3.1, 5.2, 9.1 and 9.2. An example, question 9.1 and 9.2 asks learners to write down flight numbers ‘which will depart’ and in question 5.2, ‘buys and sells’. Though representing linguistic difficulty, complex verbs were infrequently used in the test items.

Number of infinitive verb phrases

An infinitive phrase is the infinitive form of a verb plus any complements and modifiers. Infinitive phrases are without a doubt the most complicated of all verbs. They can be used as adverbs, adjectives, and nouns. Because infinitives begin with the word ‘to’ they are
occasionally misidentified as prepositional phrases. Nine infinitive verb phrases were found (25%). Question 15.1 had two infinitive verb phrases. An example is ‘Use the fraction wall to calculate $1/4 + 2/4$’. In the given example, learners are likely to confuse the infinitive verb with prepositional phrases and this compromises the comprehension of the questions. The other questions had one infinitive verb phrase each. Although these affected only one quarter of the questions, they represent a substantial effect relative to other features at the syntactic level.

**Number of specific mathematics vocabulary**

Specific mathematics vocabulary was found across 23 questions (64%) of the test items. This is to some extent expected in a mathematics assessment since mathematics is a language. Examples include ‘ratio’ (in question 1.3), ‘multiple’ (in question 1.4) ‘factor’ (in question 1.6), ‘number patterns’ (question 4.1), ‘number sentence’ (question 7) and many others. Mathematical reading is dense, each vocabulary word is conceptually-packed, full of specific mathematics vocabulary which children are not often exposed to in their homes and social environments (Murray, 2004), and without understanding of specific vocabulary, many learners struggle to understand concepts (Lee, 2007). For example, question 16.1 as shown in Figure 1 below.

<table>
<thead>
<tr>
<th>Hexagon</th>
<th>Pentagon</th>
<th>Quadrilateral</th>
<th>Triangle</th>
</tr>
</thead>
</table>

From the above frame choose the word to name each of the 2-D shapes.

**Figure 11. Question 16.1**

That math specific vocabulary appeared in the vast majority of the questions may have affected learner performance while adding to the linguistic complexity. However, this is somewhat unavoidable as understanding mathematical vocabulary is imperative to mathematical problem solving. Wolf and Leon (2009) report that the overall amount of academic vocabulary in word problem items was most predictive of item difficulty for English learners in a study they carried. While to some extent unavoidable we must take into account that the vast majority of South African Grade 4 learners have only just switched from mother tongue instruction in the FP and thus have only had a few months of instruction of mathematics in English. Thus we would argue that given this reality wherever possible, where difficult mathematical vocabulary can be replaced with simpler ones, this would be preferable in assessments of learners having only recently switched to learning English mathematics vocabulary.

**Number of prepositional phrases**

A prepositional phrase is a word group that begins with a preposition. A preposition is a joining word that links a noun to another word in a sentence. 26 questions (72%) contained prepositional phrases. Questions 10.5 and 9.5 contained the highest number of these phrases (5) and question 1.3 had four. Examples include, question 10: ‘look at the grid below and write down the position of the picture’. Prepositional phrase was one of the linguistic feature with high frequency. Prepositional phrases potentially confound English language learners because they mark the existence of an additional phrase in the sentence and hence another concept to be understood (Shaftel et al., 2006). They are, however, necessary when describing how nouns relate to one another. That prepositional phrases featured so much in the items shows the potential to hinder the comprehension of many test items.
Number of conditional constructions

Conditional sentences are statements discussing known factors or hypothetical situations and their consequences. They are conditional because the validity of the subject of the sentence is conditional on the existence of certain circumstances, which in the case of this question, may be understood from the context. Failure to get a correct answer for the first part of the sentence means failure to get the answer for the second question. Question 5 is the only question with the conditional construction feature: ‘How much does Mrs Mazibe make if she buys and sells 10 apples?’ Of all the features at the syntactic level, conditional constructions were the least manifest.

Paragraph level

Paragraph level complexity is considered when there is reference to cultural events. Analysis of the questions, however, shows no added complexity in this respect (see table above). Shaftel et al.’s, (2006) contention that complexities at the word level are less inhibiting than those at the syntactic level and that complexities at the paragraph level are most inhibiting, if true, implies that the absence of linguistic complexities at the paragraph level is a welcome relief to the other noted complexities above. The figure below summarises the frequencies of complexities across categories and shows that linguistic complexities were most manifest at the word level.

Looking at the language use in the 2013 ANAs, the analysis of the linguistic features revealed that for each question analysed, a number of language features occurred and some appeared more frequently than others. When the language features that occurred in the items are arranged, starting from the most frequently used to the least frequently used, the features are as follows: words with seven or more letters (88), homophones (72), prepositional phrases (55), specific mathematical vocabulary (41), pronouns (26), ambiguous words (11), infinitive
phrases (9), complex verbs/passive voice (3), conditional constructions (1) and references to cultural events (0). As noted before, the greater the total number of linguistic features, the more difficult the question. In this case, question 16.1 may be considered the most difficult of all the questions because it has the greatest number of linguistic features: ‘Hexagon/Pentagon/ Quadrilateral/ Triangle/ From the above frame choose the word to name each of the 2-D shapes’.

Table 2 below presents a summary of the indicators of question complexity derived from Table 1. The questions are in their descending order, from the highest challenging linguistic complex question to the lowest challenging linguistic complex question. The summary is in terms of the 11 word and syntactic level features that were the focus of analysis and the formula for the Linguistic Complexity Index (LCI) which is (Number of words + Sum B + Sum C + Sum D) ÷ Number of sentences is applied. For the questions that had one sentence, the total number of the linguistic features was the same as the LCI. Therefore, for single sentence questions, the more the linguistic features the higher the LCI. For those questions that had two sentences, the LCI was half the total linguistic features. There was only one question with three sentences and so its LCI was one third of the total of its linguistic complexity features.

Table 2. Summary of complexity of individual questions ranked according to LCI

<table>
<thead>
<tr>
<th>Question</th>
<th>No. of features present out of the 11 types of features</th>
<th>Aggregate No. of features</th>
<th>Linguistic Complexity index</th>
<th>Types of Features in Question</th>
<th>Aggregate No. of features</th>
<th>Linguistic Complexity index</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.1</td>
<td>7</td>
<td>33</td>
<td>33</td>
<td>18.2</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>30</td>
<td>30</td>
<td>10</td>
<td>6</td>
<td>29.5</td>
</tr>
<tr>
<td>9.2</td>
<td>6</td>
<td>30</td>
<td>30</td>
<td>1.4</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>29</td>
<td>29</td>
<td>15.1</td>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>18.3</td>
<td>7</td>
<td>27</td>
<td>27</td>
<td>1.6</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>25</td>
<td>25</td>
<td>7</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>9.1</td>
<td>7</td>
<td>46.5</td>
<td>23.3</td>
<td>6.1</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>15.2</td>
<td>6</td>
<td>22</td>
<td>22</td>
<td>5.1</td>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>20</td>
<td>20</td>
<td>15.4</td>
<td>6</td>
<td>23</td>
</tr>
<tr>
<td>18.1</td>
<td>5</td>
<td>38</td>
<td>19</td>
<td>1.5</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>5.2</td>
<td>5</td>
<td>18</td>
<td>18</td>
<td>1.1</td>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>1.2</td>
<td>6</td>
<td>18</td>
<td>18</td>
<td>12.1</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>19</td>
<td>6</td>
<td>18</td>
<td>18</td>
<td>14</td>
<td>7</td>
<td>20.6</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>17</td>
<td>17</td>
<td>12.2</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3.1</td>
<td>6</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>6</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Column 1 above indicates the number of all possible 11 features present in the questions as this adds to the nature of question complexity. When the complexity of questions is ranked in terms of the number of different features manifest, question 8 is the most complex: ‘Draw the hands of the given clock face to show that the time is a quarter past eight.’

However when applying the LCI formulae the three most complex questions are those shown in Figure 3 below.

16.1 ‘Hexagon/Pentagon/ Quadrilateral/ Triangle/ From the above frame choose the word to name each of the 2-D shapes’

17. Complete the table:

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>NAME OF OBJECT</th>
<th>SHAPE(S) OF THE FACES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rectangles</td>
</tr>
<tr>
<td></td>
<td>Triangular prism</td>
<td>Triangles and</td>
</tr>
</tbody>
</table>

9.2 ‘Write down the flight number of a flight which will depart for its destination after midday.’

Figure 13. Three most complex questions according to the LCI formulae

On the other hand perhaps the poor performance could be explained in terms of the limited 3D experience learners are exposed in the Grade 4 (and earlier), and especially in this case also the difficulty of interpreting a 2D representation as a 3D object. This limited experience would go hand in hand with limited exposure to the vocabulary of naming pictures of three dimensional objects.

Conclusion

Being recently introduced assessments, not much research has been done on the ANAs. In a press article, Henning and Dampier (2012) argue that there is need for this research especially in South Africa where the majority of learners grapple with learning in a second language. The importance of the research will also have implications for other countries where learners write assessments in L2 or L3. A study by Graven & Venkatakrishnan (2013) indicated that while teachers supported the introduction of the ANAs because they are standardized and provide guidance on what is expected, they noted several concerns such as the complexity of language used.
The study was designed to explore the linguistic complexity of the Grade 4 mathematics ANAs posed in English but written by the majority of learners who are not proficient in English. We note, for the majority of learners, English has only been the LoLT of mathematics instruction for approximately 6 months before Gr 4 learners write these ANAs. Given this reality, we argue that especially in the early grades of learning in English, it is essential that test designers work as carefully as possible to minimise language complexity of test items. Failure to do so will inevitably unfairly advantage the minority of mother tongue English learners.

It is hoped that the empirical findings of the linguistic challenges of the ANAs will inform educationists, especially those involved in the design of national assessments, on the nature of language challenges that learners face when writing the mathematics ANAs, and pay greater attention to the language proficiency of the learners. Secondly, it is hoped that the in-depth case study that follows this analysis in the form of learner and teacher experiences of these ANA questions will inform teachers as to ways to support learners in meeting the language demands in the preparation for the ANAs.

References


Acknowledgements

The work of the SA Numeracy Chair, Rhodes University is supported by the FirstRand Foundation (with the RMB), Anglo American Chairman’s fund, the Department of Science and Technology and the National Research Foundation.
In this paper I share experiences from my recent three-year doctoral research journey specifically with regard to tensions that I encountered. I share how a pilot club at the same time influenced the design of the subsequent research study but also brought to light a number of theoretical and methodological tensions. I discuss how and why the research questions necessitated a dual focus on mathematical learning using the complementary lenses of acquisition and participation and illuminate methodological issues that arose from working with the questions. A key contribution of this paper is the focus on the dual nature of the study and how this was resolved both theoretically and methodologically. The tensions I encountered during the course of my research tell a story of my own learning process and they highlight reflective ‘praxis’ as a powerful part of the research process.

Introduction

Working from a broad Vygotskian perspective of learning and development, my research had a dual focus and investigated how Grade 3 learners’ mathematical proficiency progressed (or not) whilst participating in after school maths clubs over the course of a year, and explored how the mediation offered in the clubs enabled or constrained the emergence of zones of proximal development (ZPD) and thus learning for the club learners.

This paper points to experiences from a pilot club and the influence this pilot had on the subsequent design of the research study, specifically with regard to the questions and the data collection methods / instruments. I share how and why the research questions caused this dual focus to occur and illuminate some of the methodological issues that arose from working with the questions. A key contribution of this paper is the focus on the dual nature of the study and how this was resolved both theoretically and methodologically.

Context of the study and empirical field

As a member of the SANC project, I have the unique opportunity to participate in a number of maths clubs as both club mentor and as a researcher. Furthermore, I am the Maths Club co-ordinator and have been specifically tasked with the design and related facilitator training of the Maths Club programme for the SANC project. My work within the SANC project is focussed on both development and research in the field of numeracy.

In development terms the SANC project aims to improve the quality of teaching of in-service teachers at primary level and to improve learner performance in primary schools as a result of quality teaching and learning. The research remit is to grow an area of research which looks towards finding sustainable solutions to the many numeracy education challenges faced in our area. As part of the developmental work, the SANC project began in 2011 and has worked with 14 schools in the greater Grahamstown area, Eastern Cape, South Africa. The teacher development programme has worked with approximately 45 numeracy teachers (ranging from Grade 0 to 6) since the project started. These teachers have participated in regular workshops focused on issues and challenges in numeracy teaching.
In our project schools, we work with teachers and directly with learners. Learner activities are a key part of the SANC project developmental activities. We facilitate the development of learner mathematical proficiency by running learner-directed and learner-oriented mathematics activities as well as creating an ethos of ‘mathematics is fun’ in schools. Many teachers find themselves faced with the challenge that most of their learners do not have the necessary mathematical foundations to be learning at the grade level in which they are placed. As a possible way to address some of these challenges, the SANC project implemented after school mathematics clubs as a more focussed and regular learner intervention (Graven, 2011b, 2012). Within the SANC project, the clubs serve two purposes: firstly, they are a place where we can directly influence what happens with learners and secondly, they provide us with an empirical research field in which we can interact directly with the learners and thus be insiders to the learning process.

The empirical field for my research was two such after school maths clubs run within two of the SANC project schools. The after school clubs were conceptualised as informal learning spaces focused on developing a supportive learning community where learners can develop their mathematical proficiency, make sense of their mathematics and where they could engage and actively participate in mathematical activities. Individual, pair and small group interactions with mentors were the dominant practices with few whole class interactions. The clubs were intentionally designed to contrast some of the more formal aspects observed in the classrooms of the SANC project participating schools (Graven & Stott, 2012; Graven, 2011a). Of note is that some of the intended practices promoted in the clubs are those learner-centred practices promoted in the official curriculum documents (Department of Basic Education, 2011) and which Hoadley (2012) notes are absent in South African classrooms.

In the latter half of 2011, Graven and myself piloted a maths club in a local school. Guided by the practices we wished to promote in the clubs we went into the pilot club with a grounded approach and open minds as to how we would structure the club and it’s activities. The pilot club influenced the subsequent research study in three key ways. Firstly, it influenced the design of the two case study clubs (the empirical field) used for my research study. Secondly, it influenced the re-framing of research questions and hence the subsequent data collection and finally it revealed the ZPD as an analytic tool for the subsequent research study. This paper focuses on the dual nature of the study as reflected in the research questions and on the methodological tensions arising from the changes made to the research design and methodology. The issues regarding the nature and design of the SANC project clubs following the pilot have been elaborated elsewhere (see Graven & Stott, 2012; Stott & Graven, 2013b).

Working within an interpretive research paradigm, following the pilot, this longitudinal case study research aimed to explore the mathematical proficiency of learners in two clubs and to examine the nature of the mediation evident in the clubs. Specifically the research questions were:

1. How do learners’ mathematical proficiency levels evolve (if at all) over the period of participation in the maths club?
2. What is the nature of the mediation that enables or constrains the emergence of a ZPD in the club learners?

At first glance these two questions seem to be in conflict with each other. The next section of the paper illuminates how I navigated the dual nature of these questions.
Navigating the dual nature of the research study

This study took a Vygotskian perspective to development and learning. According to John-Steiner and Mahn (1996) Vygotsky conceptualised development as the transformation of socially shared activities into internalised processes in his "general genetic law of cultural development" arguing that "higher mental functioning appears first on the "intermental" and then on the "intramental" plane" (Wertsch & Kazak, 2005, p. 3).

Every function in the child’s cultural development appears twice: first, on the social level [intermental], and later, on the individual level [intramental]; first, between people ... and then inside the child. This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher [mental] functions originate as actual relations between human individuals” (Vygotsky, 1978, p.57).

According to Daniels (2008) this general genetic law of cultural development “introduces the notion of some form of relationship between something which is defined as ‘social’ and something which is defined as ‘individual’” (p. 12) and this raised some important methodological questions for my study, which I address later. For example, there is some debate with regards to the how the individual is seen from a methodological point of view (see Sawyer, 2002 for example). Is the individual separable from the context or environment or must the individual be studied within the situated practice?

Additionally, Holzman (1997) argued that “to Vygotsky, learning / instruction and development are a dialectical unity in which learning leads development” (p. 57). This unity of learning/instruction-leading-development develops as a whole. Learning cannot exist without development and development cannot exist without learning (Holzman, 1997). Levykh (2008) argued that this dialectical approach “stands in opposition to the mainstream Western educational views that are mainly grounded in somewhat ‘linear’ Piagetian thinking” (p. 89). In other words, the process of development is not a direct and natural process, but rather “indirect, artificial, mediated (governed) by cultural laws of teaching-learning and, in contrast to Piaget proceeds not toward socialisation, but toward converting social relations into mental functions” (Levykh, 2008, p. 89). Vygotsky described the dialectical nature of learning and development thus:

learning awakens a variety of internal-development processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. … learning is not development; however, properly organised learning results in mental development and sets in motion a variety of developmental processes that would be impossible apart from learning. Thus learning is a necessary and universal aspect of the process of developing culturally organised, specifically human, psychological functions (Vygotsky, 1978, p. 90).

These characteristics of learning and development highlighted key aspects for my study. Learning cannot exist without development and development cannot exist without learning. The “process of development is indirect and mediated by cultural laws of teaching-learning” (Levykh, 2008, p. 89) since learning and development are interlinked in this way for this study. As a researcher I needed to investigate how the learners in my study interacted with people in their environment and how mediation took place to encourage learning.

Sfard’s (1998) much-cited article on the two metaphors of learning identified and described the differences between two metaphors for learning. ‘Learning as acquisition’ implies that learning is the acquisition of something that is then stored in the individual. Learning as acquisition theories can be regarded broadly as mentalist in their orientation, with the
emphasis on the individual building up cognitive structures (Sfard, 1998). In contrast, the ‘learning as participation’ metaphor considers learning as a process of becoming a member of a certain community, which entails the “ability to communicate in the language of this community and act according to its particular norms” (Sfard, 1998, p. 6).

While some educators argue for the need for a paradigm shift away from (or even rejection of) acquisition perspectives in favour of participation, Sfard (1998) argued that these metaphors are not alternatives but that each provides different insights into the nature of learning. Hence, she argued:

an adequate combination of the acquisition and participation metaphors would bring to the fore the advantages of each of them, while keeping their respective drawbacks at bay. Conversely, giving full exclusivity to one conceptual framework would be hazardous (p. 11).

In my study, I purposely worked with both these perspectives using a complementary approach to the notions of acquisition and participation by drawing on Sfard’s ‘metaphorical mappings’ (1998, p.7). In later work, Sfard (2001) draws our attention to the idea that the participationist researcher will focus on the growth of mutual understanding and coordination between the learner and the rest of the community and the focus will turn to the activity itself and to its changing, interactional aspects. This was an important consideration for my study, as I explored how learners’ mathematical proficiencies evolved in relation to their participation in the maths clubs and the nature of the interactions with a focus on mediation, in the context of mathematical club activities.

Often these two perspectives are seen as being in opposition to each other. However, working within the broad sociocultural paradigm I describe here, the tensions between the two notions of acquisition and participation are nothing unusual. In my study I saw the two notions as forming a yin/yang type of fit, which complemented rather than conflicted with each other.

This complementary approach was a key part of my research study and was reflected and interwoven into all aspects of it. The two main research questions reflected the complementarity between the perspectives of acquisition (primary perspective foregrounded for question one) and participation (primary perspective foregrounded for question two) and as a result, so then did the methodology, findings, analysis and discussion of these questions.

**Methodological concerns with this dual approach**

With these complementary perspectives in mind and the idea that Vygotsky stressed the need to not concentrate only on the product of development but on the process of change (1978), both of these ideas influenced how I subsequently designed the data collection and chose the data collection instrument for the study.

Thus, question one foregrounded the acquisition perspective and looked at the measurement of possible learning for each club learner. The data collection instruments used in this study allowed me to measure this possible learning using diagnostic and formative methods. The data yielded individual data for each learner and was a mixture of both qualitative and quantitative data. Question two on the other hand foregrounded the participationist view and I collected video data that helped me to observe the process of how this learning was facilitated. This data was qualitative as I focussed on the activities and interactional aspects of learning in the context of the two clubs. This dual nature of the data collection instruments is shown diagrammatically in Figure 1. The left side of the diagram indicates the
acquisitionist aspects of the data collection whilst the right side reflects the participationist perspective.

![Figure 14. Dual nature of data collection instruments for this study](image)

Smagorinsky (1995) argues that when sociocultural researchers conduct research on developmental processes, they “become part of that setting and thus become mediating factors” (p. 201) in the learning they are hoping to research. He argues that this does not ‘contaminate’ the research environment; rather the researcher becomes an “additional mediational means in a learner's development”. He continues to say that even the selection of the type of assessment means that the researcher enters the learning environment with “assumptions that a particular means of assessment is capable of determining learning” (p. 203). Thus he argues that within the Vygotskian perspective of development, the instruments of data elicitation are mediational rather than neutral. He elucidated thus:

> to assume that the study of learning can take place outside the bubble of the social environment of learning is to misconceptualize the role of mediation in human development and to underestimate the effects of the introduction of any research tools into the learning environment (p.204).

Additionally, Smagorinsky argued (1995) that if the socially constructed data is to count as evidence for making claims in research, then there is a relationship between the assumptions about the optimal end point of development and the assumptions about the data that serve as evidence of progress towards that point. Therefore any assessment instrument “embodies the researcher’s sense of an appropriate developmental path for people to follow, and produces data that identifies people’s progress ... according to the direction of the path” (p. 200). In other words, the research process produces culturally shaped evidence of development towards a specific end point.

This insight about assessment instruments was important for this study. As the researcher, I chose to use the Learning Framework in Number (LFIN) (Wright, Martland, Stafford, & Stanger, 2006; Wright, 2003, 2013) to represent the developmental path of mathematical proficiency for the learners in my research study to address the first research question. In addition, I used an instrument which embodied the concepts of the LFIN to collect data on the club learners’ progress. Similarly, for question two, I used a data collection method (task-based interviews) that highlighted the importance of talk and dialogue between participants.
Thus the interview instrument was introduced into the learning environment and was mediational in nature.

**Navigating the changing nature of research instruments**

The timely administration of diagnostic assessment tasks in the pilot club enabled a powerful data-driven approach to activity selection to emerge from the club. The assessment instrument I drew on for the pilot club was the Askew, Brown, Rhodes et al. (1997) instrument which was orally administered to the club learners. Learners recorded their answers on individual scripts. However, I struggled with a way to track learners on-going progress in mathematical proficiency with the instrument as it stood. I was able see if a learner had answered a question correctly but was unable to see how they had answered the question and whether the methods they were using to arrive at an answer were efficient. Following this tension, I decided to draw on tools from Wright, Martland, Stafford and Stanger’s (2006) Maths Recovery programme for data collection and analysis for the subsequent research. I used the one-to-one interview as a data collection instrument and the Learning Framework in Number (LFIN) as an analysis framework for the subsequent case study clubs. These interviews took place out of club time by arrangement with the club learners’ teachers. This interview and the LFIN enabled me to administer detailed one-to-one interviews with club learners, to note how they arrived at various answers, profiling learners at particular stages / levels in the LFIN and allowed me to track detailed progression over time and thereby provide data for my first research question.

**Navigating ethical tensions with learner assessments**

Once ethical permission had been gained from the university, Eastern Cape Department of Education, the schools and parents, the collection of data via one-to-one interviews was straightforward. However this was not the case with a second set of instruments I used to track learner progress in the clubs.

In the pilot, Graven and I introduced a series of timed fluency activities to encourage learner fluency in basic facts (drawing on Askew’s (2012) basic facts). Initially, these activities were intended to simply be part of the mental maths warm-up activities promoted. However we realised that they provided useful research data for monitoring learner progression. A clear advantage of these fluency activities was that they were fast and took roughly six minutes to administer, giving us quick access to learners’ fluency levels in the basic facts. I thus decided to give them to my case study club learners in 2012 on a more regular basis as a way of supplementing data collection and for quick evaluation of learners’ progress. While these assessments in this form were useful, an ethical tension arose for me as timed activities are by design time pressured and thus can be stress inducing for learners. A review of literature reveals a sizable body of work that puts forward an argument against timed activities in mathematics (see for example Boaler, 2012; Burns, 2007; Gilliland, 2001). In our work within the SANC project we have noted that any kind of assessment can produce anxiety in learners, not just timed test. I have reported on the structure and effectiveness of these fluency activities and have addressed the ethical concerns elsewhere (Stott & Graven, 2013a).

**Navigating the challenge of collecting and transcribing data in authentic contexts**

For my second research question I planned to use video as the data collection method and I originally intended to collect video data in every club session, which I attempted to do in the first half of 2012. However, as the data collection period continued, this approach proved to be challenging. It transpired that I was collecting a lot of data but it was not particularly useful.
Looking at the early video data collected, I realised that it would not be suitable for analysis purposes. The first obstacle was that as the club mentor I could not collect video data at the same time as being a participant in the club. On realising this, I asked a number of colleagues to assist me in recording video data during club sessions. However, upon viewing the data recorded in this way, I encountered a key obstacle: lack of focus on learners faces, gestures, speech, activities or events of any significant duration. There were numerous reasons for this. Many of my club sessions took place on the carpet, thus much video showed tops of heads and little else. Additionally, learners in the clubs often spoke softly and with the background noise of other learners, I could not make out from the video what was said. Also, learners were highly mobile making videoing difficult. I realised that this video provided data with regard to the structure of club sessions, the socio-mathematical norms we were promoting and the pace of the sessions in the clubs (Research journal entry, 3 July 2012) but did not provide data that zoomed into the mediation or emergence of ZPDs. This was a central problem for my research given my second research question was focused on mediation.

To find a way forward, I selected a number of video excerpts from this early data set and shared them with Prof Lerman. We discussed the issues I had encountered and he agreed that another approach would be necessary. I talked this problem through with my supervisor and after some preliminary reading on task-based interviews, I decided that I would use task-based interviews as a data gathering tool to collect focussed and data rich video data for the second research question with pairs of learners in a noise free environment. Task-based interviews are generally designed so that the interviewees not only interact with the interviewer and with each other but also with a task that is carefully designed for the purposes of the interview (Maher & Sigley, 2013). Maher and Sigley (2013) highlight that a carefully constructed task is a key component of the task-based interview. They further suggest that as the interviewees are engaged in mathematical activity, the researcher is able to observe their actions and record them with video recordings. In my case, one mentor facilitated the interview and another recorded the video. These recordings, along with transcripts of the recordings, interviewees work and post interview reflections provided the data for analysis of the second research question.

I had some concerns that these task-based interviews were contrived as learning situations, but Prof Steve Lerman pointed out that all learning situations in formal classrooms are ultimately contrived. When conducting the interviews, I emphasised to the learners that these interviews were similar to being in the club environment and that during the interview, we would participate and interact the same way as we did during normal club sessions (I often worked with pairs of learners and moved from one pair to another). The only difference was that it was quieter and there were only two learners at a time, so we were able to record their discussions. I also carried out the interviews in the same venues used for the clubs, in order to add to the learners’ comfort levels. The following extract from a transcript with two girls participating in the task-based interview illustrates how I introduced the interview:

Can I just explain what we’re doing? Alright. You know I try and video the club sometimes and I find it really hard because I have to run around with all the groups and they’re all doing different things and also I can’t hear properly what’s going on.

8 I worked with Prof Lerman at the SAARMSTE research school in June 2012 on my video data and we had a discussion about how I was to resolve the issues I was having with collecting video data.
So what we decided to do is we’ve done a little mini club with the boys and we’re going to do a little mini club with you two. Okay? So that we can hear and we can see and get some proper video because the stuff that I do in the club normally doesn’t work very well.

All right, so all the club things that we know, about talking to each other, arguing with each other, giving your opinion, trying, working together. All those things that we do in the club. I want you to pretend this is just a club, but it’s just the two of you. (Transcript of task-based interview video recording, 19th October 2012)

The main focus of the task-based interviews was to facilitate the learners in undertaking various carefully selected tasks with the aim of eliciting learner talk and thus providing data which focused on the nature of mediation in the clubs and how this may enable or constrain the emergence and sustainment of ZPDs in club learners. More specifically I used the video data to examine how interview participants caught each other’s attention and the overall nature of mediation used by the mentors.

Navigating the issue of playing multiple roles in the research process

As mentioned, I play multiple roles in the SANC project: those of researcher, club facilitator and club coordinator. I thus wore multiple hats during the research study. At the start of the research process it was possible that these multiple roles could have caused tension. To that end I used my reflective journal to document as much about how the roles impacted on each other and on the process of research.

Throughout my research study I kept a detailed research journal. Following the pilot, this became a significant data collection tool. Through the course of my study I made a point of writing in the journal frequently and it became a habit to do so after weekly club sessions, after reading literature, after working on data, after supervisory sessions, when grappling with a theoretical or methodological tension, when writing research papers and conference presentations and so on. I also used the journal to sketch the many diagrams I used in my study as these diagrams helped me to make sense of many aspects of my work. The entries were invaluable when it came to remembering the stories, the personal learning process, decisions I made and why and how aspects of the research evolved. Ultimately the entries enabled a reflexive praxis to emerge during my study.

For me ‘praxis’ is a way of doing things or a way of translating theoretical ideas into action. This gave me a new way of looking at my roles and how those roles interacted with each other. For Graven (2004), the dual role gave her a number of advantages: it enabled a form of action-reflection practice, gave form to her research and the process, the on-going reflection was stimulated by her research and her own learning was maximised by the on-going reflection. These insights encouraged me to be aware of how multiple roles might bring a powerful praxis to my research project. By being aware of these insights before I started this research, I was able to make the most of the opportunities presented by these differing and challenging roles.

Smagorinsky (1995) argues that from a Vygotskian perspective, data generated through the research process is a social construct developed through the relationship of the researcher, research participants, the research context and the methods of data collection. He further points out that researchers need to reflect on how their involvement in the research process affects teaching, learning, and the evaluation of both. Drawing on these comments, I argue that in my case, the relationship between the researcher and the research context (the clubs within the SANC project) was a highly complex one. Nonetheless it was one that allowed me to reflect not only on how the research process affected teaching and learning in the clubs but
also on how the research process affected me as researcher and the context in which I operate. In the following paragraphs I illustrate how my triple role enabled an on-going, powerful reflexive practice to develop between the club learning programme design, mentoring in clubs and my research with reference to Figure 2, which illustrates this triple role diagrammatically.

Figure 15. Diagrammatic representation of my triple role in maths clubs

In my role as club co-ordinator and club programme designer, I had the opportunity to translate a theoretical or methodological idea from the research literature into action. It is not within the scope of this paper to detail more than one example, therefore I briefly provide an example where all three roles influenced each other. Mellony Graven’s and my experiences in the pilot brought to light the entwined and dialectical nature of the data collection and design processes and the significance of the post-club reflection sessions as a powerful data collection instrument for planning the club sessions. Furthermore, through the pilot we identified and shaped the zone of proximal development for the purposes of our club as the critical design concept for each club session for each learner. In addition, the pilot influenced the broader study leading to an increased focus in my research questions and thus the theoretical and conceptual frameworks and the methodological design. This reflective praxis illuminates the relationships between theory and practice and how the dialogue between the two elements informs each. We used empirical evidence to move from an initial multifaceted design to a much simpler, more learner-centred design and these findings informed the data collection process and the club session design for the subsequent research study.

Concluding remarks

My motivation for sharing these experiences was to illuminate that the path of undertaking a research study is not straightforward and without tensions and issues. Educational research takes place in real world contexts, with real people. Issues that arise have to be confronted, navigated and resolved and I believe that the process of sharing how this is done in a research context is an important aspect of academic citizenship. The tensions I encountered during the course of my research tell a story of my own learning process and they highlight reflective ‘praxis’ as a powerful part of the process.
References


**Acknowledgement**

The work of the SA Numeracy Chair, Rhodes University is supported by the FirstRand Foundation (with the RMB), Anglo American Chairman’s fund, the Department of Science and Technology and the National Research Foundation. I thank the Chair, Mellony Graven, who was also my supervisor, for her guidance with regard to my research and this paper Prof Steve Lerman for informative and productive conversations about Vygotsky’s work and my video data.
Teachers’ Pedagogical Content Knowledge in the Teaching of Grade 3 Mathematics Concerning Equivalent Fractions in Some Rural Schools of Limpopo Province

Kgalushi Themane¹ & Kakoma Luneta²

¹ Department of Education Studies, University of Limpopo, South Africa
² Department of Childhood Education, University of Johannesburg, South Africa

¹mariathemane@gmail.com, ²kluneta@uj.ac.za

This article presents the results of the description of teachers’ pedagogical content knowledge (PCK) of mathematics concerning the teaching of equivalent fractions in grade 3. The study followed a qualitative research approach where a case study design was adopted. Five grade 3 teachers were sampled through a purposive sampling strategy. Data were collected through interviews, observations and document analysis. The study found that teachers lacked PCK in the teaching of equivalent fractions. This is manifested by their lack of confidence in teaching fractions and the inability to teach for conceptual understanding. Firstly, if attempts are to be made to improve the performance of learners in the FP, there should be concerted efforts to improve the PCK levels of teachers. Secondly, there is a need for the development of a vocabulary word bank that could deal adequately with common errors and misconceptions committed by both teachers and learners.

Introduction

Research indicates that South African grades 1 to 3 learners perform poorly in reading and mathematics (Progress in International Reading Literacy Study (PIRLS), 2006 (Mullis, 2007); The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) (Moloi & Strauss, 2005); Department of Basic Education, 2011; Trends in International Mathematics and Science Study (TIMMS), (Mullis, 2007). Several factors are attributed to this problem, For example, the quality of teachers. Studies show that teachers’ content knowledge has an influence on learner performance (Fennema, Carpenter, Jacobs, Franke & Levi, 1998; (Naiser, Wright & Capraro, 2004). This is even more critical in subjects like mathematics; where teachers seem to struggle more. This problem seems to be even is more pronounced among grades 1 to 3 teachers. This appears to be more so in the teaching of fractions.

Research (Newton, 2008; le Roux, Olivier, & Murray, 2004; Moss & Case, 1999) indicates that teaching mathematics in primary schools, and in particular equivalent fractions, seems to present a challenge to many teachers. Several studies (Wu, 1999) found many bottlenecks in the teaching of fractions in primary schools. Studies from the Rational Number Project (RNP) indicate that teachers struggle with lessons related to the teaching of fractions, which include fraction operations, decimal concepts, order, equivalence and operations with decimals (Cramer, Post, & del Mas, 2002; Ball & Mewborn, 2001). Their struggle against fractions calls into question their pedagogical content knowledge (PCK).

There is a limited research on the PCK of teachers in South Africa, especially in impoverished provinces like Limpopo Province; where among other problems, the Annual National Assessment (ANA) results show the lowest as shown in table 1 below.
Table 1. Showing ANA results in Mathematics per province in % for 2011, 2012 and 2013 (Department of Basic Education, 2011, 2012 and 2013).

<table>
<thead>
<tr>
<th>Province</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastern Cape</td>
<td>35</td>
<td>40.5</td>
<td>54.9</td>
</tr>
<tr>
<td>Free State</td>
<td>26</td>
<td>44.7</td>
<td>63.2</td>
</tr>
<tr>
<td>Gauteng</td>
<td>30</td>
<td>46.9</td>
<td>69.6</td>
</tr>
<tr>
<td>KwaZulu Natal</td>
<td>31</td>
<td>42.2</td>
<td>64</td>
</tr>
<tr>
<td>Limpopo</td>
<td>20</td>
<td>34.4</td>
<td>44.4</td>
</tr>
<tr>
<td>Mpumalanga</td>
<td>19</td>
<td>35.6</td>
<td>50.2</td>
</tr>
<tr>
<td>Northern Cape</td>
<td>21</td>
<td>37.9</td>
<td>54</td>
</tr>
<tr>
<td>North West</td>
<td>21</td>
<td>34.1</td>
<td>51.9</td>
</tr>
<tr>
<td>Western Cape</td>
<td>36</td>
<td>47.4</td>
<td>66</td>
</tr>
</tbody>
</table>


Teachers seem to struggle in three areas: (1) knowing facts, concepts and theories around fractions, (2) development of explanatory frameworks in order to organize and connect ideas, and (3) espouse rules and evidences in the teaching of fractions. Poor PCK in these areas is associated with poor learner performance (Luneta & Makonye, 2011). The National Council for Teachers of Mathematics (NCTM) (2000) points out that the teachers’ knowledge in these three areas affects the way children perform in all core tasks of learning fractions. Thus the present study sought to investigate the PCK of teachers in the teaching of equivalent fractions in these three areas.

PCK can broadly be defined as the knowledge of the subject matter, student learning, development, and the teaching methods associated with effective teaching (Ball, Thames & Phelps, 2008). Thus understanding the teachers’ PCK on fractions is important for at least four reasons. One, there has been an outcry about the teachers’ content knowledge being poor in recent years (Moloi, & Strauss 2005; Blömeke, Suhl, & Kaiser, 2011; Marchionda, 2006). Two, in South Africa, such knowledge might prove useful to teacher education institutions which need to restructure their programmes in line with new Minimum Requirements for Teacher Education Qualifications (MRTEQ) (Department of Education, 2011). Three, such knowledge could be useful to policy makers who are hard pressed to get reading and counting levels to the acceptable levels in the country. Four, an understanding of fractions is critical in laying a foundation for other forms of learning mathematics later in life (Van de Walle, Karp, & Bay-Williams, 2010).

Therefore the purpose of the study was to describe the grade 3 teachers’ PCK in the teaching of equivalent fractions. To do that we sought to answer the question: What pedagogical content knowledge do rural teachers posses to teach equivalent fractions to grade 3 learners effectively?
Research Methodology

Research design

A qualitative research approach was employed to describe the teachers’ PCK in the teaching of grade 3 fractions. Within the qualitative approach, we adopted a case study design. We found a case study design helpful to describe the phenomenon better.

Sampling

A purposive sampling strategy was used to recruit five teachers to participate in the study. The features of interest were based on two criteria, one, that the teachers were to be teaching mathematics in grade 3. Two, the teachers were to have had five years teaching experience at the time of the study. The two criteria were important to consider as such type of participants would be better positioned to assist in answering the research question.

The study was conducted in two primary schools of Mankweng circuit in the Capricorn District in Limpopo Province. Mankweng circuit is situated 30 km from the city of Polokwane (the former Pietersburg). Mankweng circuit consists of 20 primary schools of which 2 were selected for the study.

Data Collection

Data were collected through three methods, interviews, observations and document analysis. Interviews were meant to find out from the teachers themselves what the level of their PCK was like at the time of the study. Observations were meant to see strategies used in their classrooms. Document analysis was used to read classwork books, home work tasks, and teachers’ workbooks.
## Results

### Table 1. Presenting summarised data from interviews

<table>
<thead>
<tr>
<th>Theme</th>
<th>Respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher’s knowledge of fractions</td>
<td><strong>Teacher 1</strong></td>
</tr>
<tr>
<td>Learners’ previous knowledge of fractions</td>
<td><strong>Teacher 1</strong></td>
</tr>
<tr>
<td>Helping learners differentiate fractions</td>
<td><strong>Teacher 1</strong></td>
</tr>
<tr>
<td>Demonstration of equivalent fractions</td>
<td><strong>Teacher 1</strong></td>
</tr>
<tr>
<td>Use of classroom materials</td>
<td><strong>Teacher 1</strong></td>
</tr>
<tr>
<td>Supporting and helping learners knowledge</td>
<td><strong>Teacher 1</strong></td>
</tr>
<tr>
<td>Make learning exciting and teaching</td>
<td><strong>Teacher 1</strong></td>
</tr>
</tbody>
</table>
**Results from interviews**

Table 1 above presents a summary of results from interviews with the teachers. The results are divided into seven themes which were developed during the data analysis process. The results can further be divided into three main categories: (a) information related to the teachers’ knowledge of fractions, (b) how they teach fractions, and (c) their experiences in teaching fractions.

With regard to their knowledge of fractions, the teachers in the study indicated that their knowledge of fractions was rather limited, hence their lack of confidence. This was even confounded by the fact that they found their learners also with a very weak grounding on fractions.

On as to how they taught fractions, we got an impression that the teachers were trying their best to make teaching interesting by using variety of strategies. They used manipulatives, and other teaching and learning support materials such as workbooks. They involved learners in their classrooms, and supported them in a variety of ways. Their experiences in teaching fractions showed a general lack of interest in teaching fractions because they found it difficult to teach. For example, teacher 2 remarked: “I am not confident to teach fractions”.

**Results from observations**

We condensed the themes into three categories, which are: (a) teaching fractions in the context of the learner, (b) attention to learners, and (c) the teacher’s time management of the classroom. By doing this we wanted to see how competent the teachers were in teaching fractions.

Also, it was observed that where the classes small the teachers paid attention to individual learners. But, where classes were large (72 and 80) (for teachers 3 and 4) the teachers ignored the learners, and could not pay individual attention. For example, teacher 4 could not give tasks because she had exhausted all her time on explanation of basic concepts. Of the four teachers observed, only two used their time effectively. The rest taught only without giving tasks at the end of their lessons.

Besides the problem of over-crowding, the two teachers also struggled with the use of Sepedi terminology. Although the workbooks and other learner support materials were written in Sepedi, the teachers appeared to be unfamiliar with most concepts. The learners also struggled with Sepedi terms even though it is their mother tongue. This observations were also articulated during the interviews. On this point teacher 3 said: “Sepedi se hloka mantsa a go lekana, kudu equivalent fractions” (Sepedi lacks sufficient words to explain, especially equivalent fractions).

**Results from document analysis**

We found that lessons were prepared in mother tongue, Sepedi. We also found that the learners’ classwork and homework followed the pacesetter from the policy document, step by step. We also found that workbooks were used to assess learners.

**Discussions**

The purpose of this study was to describe teachers’ PCK of mathematics concerning the teaching of grade 3 equivalent fractions. The study followed a qualitative research approach where a case study design was adopted. The study found that: teachers lacked PCK in the teaching of fractions, which is evidenced by incompetent teaching. Subsequently they teach for procedural knowledge rather than conceptual understanding.
Our findings are consistent with others elsewhere. Drews (2005), Naiser, Wright Capraro (2004), Smith, diSessa and Roschelle (1993), and Siegler, Fazio, Bailey and Zhou (2012) found that teachers were not fluent confident and they struggled with lessons related to the teaching of fractions. Venkat and Adler, (2013) cite that South African teachers lacked sufficient mathematics skills to teach with confidence. Gainsburg (2012) and Zulu (2013) also found that mathematics teachers in the FP lacked confidence to teach for conceptual understanding because of the language barrier. They argue that because of this lack of PCK, teachers lack confidence in them, and then resort to procedural knowledge.

When we asked the teachers what their content knowledge of the subject was, they indicated that they had a shallow content knowledge of fractions. For example, teacher 1 admitted: “My knowledge of teaching fractions is rather weak because I was not taught on how to teach fractions”. This confirms what Gainsburg’s (2012) assertion that teachers are not confident to practice what they learned from university or college.

The lack of confidence has a number of ramifications, which include, teachers not being fluent when teaching fractions (Drews, 2005). Resultant to this, teachers tend to be incompetent to teach correctly and efficiently and resort to procedural teaching.

Flowing from their inability to teach, learners committed varied errors, careless errors, and application errors, conceptual and procedural errors. Studies argue that poor performance in mathematics could be correlated with learners’ misconceptions and errors, and aggravated by teaching in English (Adler & Setati, 2001; Luneta & Makonye, 2011; Mdaka, 2012).

However, in his study the teachers were not keen to teach in Sepedi. They encountered difficulties in the use of mother tongue (Setati & Adler, 2001; Zulu, 2013). This included lack of appropriate terminology. Instead they resorted to code-switching. Whereas our findings are not generalizable, mother tongue teaching remains a thorny issue (Foley, 2008; Kazima (2008). The matter needs further attention (Ball, Thames, & Phelps, 2008; Zulu, 2013). These findings thus have serious implications for the PCK of teachers.

There are at least three implications from the findings of this study. Firstly, if attempts are to be made to improve the performance of learners in the FP, there should be concerted efforts to improve the PCK levels of teachers. Secondly, there is a need for the development of a vocabulary word bank that could deal adequately with common errors and misconceptions committed by both teachers and learners. These could be used as resource book for teachers, especially those in impoverished learning environments such as those in our study. Thirdly, in order to improve the competence levels of teachers in FP, there is a need for more short courses. Such courses could be delivered in various forms such as one week workshops, mentor system, and on-line courses - open resources (Botha & Reddy, 2011).

References


Acknowledgements

The European Union (EU) through the Department of Higher Education and Training (DHET) is duly acknowledged for providing funds for the research
Representation of the Equal Relationship in the Development of Mathematical Thinking: A Case of Grade One’s

Zingiswa Mybert Monica Jojo
University of South Africa
jojozmm@unisa.ac.za

This paper presents a practical demonstration and presentation observed whilst a pre-service teacher was teaching a mathematics lesson on ‘doubling a number’ (Ukuphinda kabili), ‘to thirty two grade one learners in a primary school in kwaZulu Natal of South Africa. The purpose of the study was to explore learners’ understanding of the doubling concept. The lesson was presented in Zulu, the learners’ home language in line with the Curriculum Assessment Policy Statement (CAPS) which requires all phase one (grade 1-3) learners to be taught mathematics and other subjects except English in their vernacular in South African public schools. The lesson engaged learners in games and interactions involving replicating a given number twice using stones or bottle tops, combining them and then counting and registering their total. The symbolic representations though and the use of equal sign were misleading on the chalkboard. Interviews with the teacher after the lesson revealed that the teacher assigned no particular meaning to the equal sign used in the number sentences. Also the teacher revealed that he was aware that at grade one level the multiplication sign could not be used and did not know how to represent a duplicate of a value without a sign so as to get double the number.

Introduction

Learners need language in order to develop mathematical concepts. Language development is always important in the use of numeracy, particularly mathematical language. This can be justified by the inclusion of the basic concepts of colour, shape, size and others associated with mathematics in the ‘Thinking and Reasoning’ of the Language Learning Area. Numeracy is “…the ability to process, communicate and interpret numerical information in a variety of contexts (Askew, Brown, Rhodes, Williams, & Johnson, 1997:25). This implies that numeracy intersects with number sense, a concept that incorporates both understanding and using mathematics. Howden (1989:11) describes number sense as ‘as a good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualising them in a variety of contexts, and relating them in ways not limited by traditional algorithms. Number sense is vital for learners to use when building up an understanding of computational strategies. The challenge is that number sense cannot be taught to learners but they acquire it by being exposed to various activities that allow them to construct knowledge for themselves.

Very often learners in grade one, are taught to count and to know ‘facts off by heart’ such that they can recite them. It is in this grade where a foundation for development of mathematics has to be built. This suggests that some groundwork upon which learners can reflect as the basic ideas found in mathematics has to be laid and utilised so that it can be developed in latter grades. Reciting informs recalling with thorough practice but cannot guarantee that the learners understand any of the concepts, what they are doing, or that they will be able to use these facts in different contexts. Teachers need to design activities aimed at helping learners to develop a sense of number. Researchers, Greeno (1991), Reys (1991), and Sowder and Schappelle (1994) extend the meaning of number sense to include well
understood number meanings, well developed multiple relationships among numbers, a level of “comfort”, a “friendliness” with numbers, a relating of number to one’s own experiences from which number relationships are readily perceived, and knowing the relative effect of operating on numbers. This, Howden (1989) describes as an exploration and interpretation of number beyond algorithmic contexts as an element of number sense. Often teachers need to ask and encourage the learners to reflect on what they are doing and afford them a chance to talk about it to enhance the development of a sense of number.

Pre-service teachers are always encouraged to keep learners active and mentally engaged in the mathematics lessons. Constructivists argue that all learners use the ideas they currently have to form or create new ones. Constructivism is not a way of teaching but rather it is a theory about how the human mind learns. Children must be mentally active for learning to take place. In the classroom I observed, grade one learners were provided with opportunities to construct their own knowledge during processes of articulating, defending, evaluating, and reflecting upon their understandings and shared practices on doubling a number. The actions involved picking a certain number of items, repeating the action with the same number of items, combining them and then counting how many items one had altogether. Pictorial and symbol illustrations on the chalkboard only followed the practical part at a later stage respectively.

**Literature Review**

Essien (2009) argues that the equal sign should first be introduced using appropriate pictorial representations and artefacts before the introduction of the plus and minus signs. This was the strategy used meticulously by the teacher observed. Various artefacts in the form of stones, bottle corks, buttons and match sticks were used in action whilst the learners observed first how a number of objects can be replicated in number and then added and counted together by all learners. These were followed by pictorial representations on the chalkboard indicating ‘ukuphinda kabili’ as in the pictorial in Fig 1. In the caps document of the English version, this topic appears as ‘adding doubles’. Thus the operation is clearly addition of the same number twice. The Zulu version instructs the learners to ‘repeat (phinda) twice (kabili) whatever action the learners was doing. The controversy could be that it is obvious that after addition, the learners need to count the combination of the doubles and give an answer. When giving the number of objects in the combination, the learner has to internalise the equal sign as a relational object. Repeating twice has to stress a combination of addition which would then be equal to a double. This understanding is not obvious for a grade one learner, it has to be taught. This was reiterated when Essien (2009) suggested that knowledge on how to organise the learners’ first encounter with a particular concept is crucial to the type of internalisation that occurs in the learners.

Various researchers (Behr, Erlwanger, & Nichols, 1980; Carpenter, Franke, & Levi, 2003; Demonty & Vlassis, 1999; Essien & Setati, 2006; Falkner, Levi, & Carpenter, 1999; Kieran, 1981; Knuth, Stephens, McNeil, & Alibali, 2006) and others have long recognised that learners tend to misunderstand the equal sign as an operator, or as a symbol inviting them to “do something”, to “find the answer”, rather than as a relational symbol signifying equivalence or quantitative sameness. Rather many learners at primary and early secondary levels understood the equal sign as either a do-something symbol that automatically invite them to write the answer and/or a unidirectional symbol. Essien (2009) though argues for the equal sign to be given the same status and attention as the addition and subtraction signs both in the textbooks and the curriculum. The *Principles and Standards* also notes that the common learners’ understanding of the equal sign at this stage (foundation phase) should be
more accurate than the limited understanding of the equal sign as signifying “the answer is coming”. Learners need to understand that the equal sign “indicates a relationship that the quantities on each side are equivalent” (NCTM, 2000, p. 94). The CAPS document fails to stress the use and understanding of equal sign as was with the previous curricula, Outcomes Base Education (OBE) and Revised National Curriculum Statement (RNCS) in South Africa. The CAPS seems to take for granted the fact that the equal sign, (unlike the plus sign or minus sign), needs to be highlighted explicitly in texts and pedagogy in order for learners to develop a more sophisticated understanding of the equal sign (DOE; 2011).

McNeil, Rittle-Johnson, Hattikudur, and Petersen (2010) assert that as children develop, many come to a relational conception of the equals sign as indicating numerical sameness, and are accepting of a wider variety of equation types. However, this happens to varying extents and even those that develop a sophisticated understanding of the equals sign readily revert to operational views (seen as a do something signal) of symbolic mathematics. They further presented evidence that in addition to the operational and relational conceptions of the equal sign, there is a distinctive substitutive conception. The substitutive conception of the equal sign is another level where learners need to understand that the two sides in an equation or a number sentence can be replaced by each other since they are equal. They then classified the following meanings on understanding equal signs.

Table 1. Equals signs definitions presented

<table>
<thead>
<tr>
<th>The equals sign means…</th>
<th>Conception</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>the two amounts are the same</td>
</tr>
<tr>
<td>R2</td>
<td>that something is equal to another thing</td>
</tr>
<tr>
<td>R3</td>
<td>that both sides have the same value</td>
</tr>
<tr>
<td>O1</td>
<td>the total</td>
</tr>
<tr>
<td>O2</td>
<td>work out the result</td>
</tr>
<tr>
<td>O3</td>
<td>the answer to the problem</td>
</tr>
<tr>
<td>S1</td>
<td>the two sides can be exchanged</td>
</tr>
<tr>
<td>S2</td>
<td>the right-side can be swapped for the left-side</td>
</tr>
<tr>
<td>S3</td>
<td>that one side can replace the other</td>
</tr>
</tbody>
</table>

Haylock (2008) asserts that the essence of the problem with the equal sign is that the concept of equals is such a complex network of ideas and experiences. He further notes that there is not just one form of words that goes with the symbol (=) but that there is a range of language and situations to which the symbol may become attached, including both the ideas of transformation and equivalence. It is a symbol which most teachers articulate their anxieties about its meaning. Haylock further emphasises that the phrase ‘is the same as’ is particularly significant as the underlying equivalence in statements in arithmetic that use the equals sign.

When the child puts out two sets of three bottle corks, forms their union and counts the new set to discover that there is now a set of six bottle corks, it is a bit obscure to suggest that this is an experience of ‘two sets of three, is the same as six’. The child has actually transformed the two sets of three into a set of six. The child’s attention therefore is focused on the transformation that has taken place. This being so, it seems perfectly natural, and surely
appropriate, to use the language ‘two three’s make six’ to describe the transformation the child has done. The teachers in this study noted that she regarded the symbols as instructions to do something. In other words, the equals sign tells you to apply some sort of transformation. Thus in practice, the equal sign represents equivalence and transformation.

**Theoretical Framework**

The equal sign meanings described above are analysed using the variation theory in this article. Leung (2012) is of the opinion that teaching and learning of mathematics is about providing learners opportunities to experience mathematics and to create (new) mathematical experiences. He further argues that although memorizing a formula, executing an algorithm, writing down a string of mathematical symbols, proving a proposition, or recognizing a pattern, are all related to mathematics they are critical enough to ensure the occurrence of a genuine mathematiccal experience. There needs to be a connection between the learner and the ‘object of learning’ in mathematics. This involves pedagogy and use of tools or manipulatives that become potential mediators for mathematical experience (Maschietto & Trouche, 2010). To this Leung (2010) describes mathematical experience as “the discernment of invariant pattern concerning numbers and/or shapes and the re-production or re-presentation of that pattern.” He therefore proposed the variation theory where he defined variation as what changes, what stays constant and what the underlying rule is, in any phenomenon.

Variation is defined by its critical features that must be discerned in order to constitute the meaning aimed for in a lesson (Marton & Tsui, 2004). The teacher then has to decide on a pedagogic approach that accommodates a pattern of variation as a useful tool in structuring teaching to help the learners to construct relevant mental constructs for the concept to be learnt or object of learning. Marton (2009) proposed four kinds of awareness brought about by different patterns of variation. These include:

- **Contrast** which presupposes that for one to know what a concept is, he/she has to discern and know what it is not, (Leung, 2012). Ling (2012) observed that teachers have a tendency of putting emphasis on what a concept is (examples), instead of what it is not or how the concept differs from related ones (non-examples).

- **Separation** assumes that all concepts have a multitude of features, each of which give rise to different understandings of the concept. For example doubling a number is the same as adding the same number twice or just twice the number. Ling (2012) therefore suggests that it is necessary for teachers to consider learning as a function of how learners’ attention is selectively drawn to the critical aspects of the concept. He is of the view that certain aspects of the concept should be varied while others are kept constant to help learners to discern new aspects of the concept and construct new meanings that might have not been apparent before.

- **Generalisation** according to Leung and Chik & Marton (2012) refers to the verification and conjecture making activity that checks out the validity of a separation. Variation of separation and generalisation of features of a concept are sometimes not distinct from each other.

- **Fusion** is the simultaneous discernment of all the critical features of a concept and a relationship between them which allows a learner to make connections gained in past and present interactions. For more elaborations on the variation theory see Mhlolo (2013, p11). The diagram below gives a summary of the variation theory.
**Methodology**

The main question in this study was, ‘To what extent could the grade ‘one’ learners understand the doubling concept?’ In trying to answer this question the study followed an exploratory design where data was collected through observations of the pre-service teacher presenting a mathematics lesson to a grade one class of thirty two learners. The school was a no-fee school, under-resourced and in an impoverished rural setting. The observations in class were followed by interviews with the teacher to get clarity on the pedagogy dynamics used in class and later some three learners on their understanding on the lesson. The exploratory design was used to determine the understanding displayed by the learners regarding number sense with respect to the constructivists view. The study also explored the views and roles of the teacher, learners and the challenges learners had in understanding ‘ukuphinda kabili’.

This was a qualitative study where 32 grade one learners were divided into eight groups of four and were presented with learning aids like bottle corks, stones, ten 2 square cm² colorful paper strips, buttons and match sticks. Two of the eight groups had similar items. Instructions started with a racing game. There were four games and each learner was the leader of a particular game. The groups were numbered one up to eight with colorful stickers. The instructions were clear and were orally given to the learners in their mother tongue. They were to pick a certain number of items, isolate them from the group and then had to repeat the action once more. This the teacher referred to as ‘ukuphinda kabili’. A group would get a score by counting the total number of items combined and giving a correct response. The second exercise involved a diagrammatic representation of the ‘ukuphinda kabili’ by the teacher on the chalkboard. Lastly the teacher represented the processes using numerical symbols on the chalkboard.
Analysis

The game phase aroused much interest to the learners. They enjoyed the game. They understood the instructions with ease since it was presented in their mother tongue. The teacher also demonstrated physically for each group the nature of the activity involved in this exercise. She isolated two items, repeated the action, combined the first two objects with the other ones. She then counted out the combined group of items to get a total of four. She also explained in vernacular that ‘Uma uphinda izinto ezimbili kabili, ufumana ezine (When doubling two items twice, we get four). Learners were provided with opportunities to construct, interpret and, acquire mathematical knowledge in communication with their peers in their different groups. The learners were also active, worked collaboratively in groups, and engaged in negotiation and consensus building on the meaning of mathematical ideas using number sense. This matches nicely with what doing mathematics is about, for a major aim of mathematical activity is to separate out invariant patterns while different mathematical entities are varying, and subsequently to generalize, classify, categorize, symbolize, axiomatize and operationalize these patterns. Contrast was not used in this exercise since learners were not exposed to non-examples. Terms like even and odd were also not built in or mentioned in this exercise.

The exercise above was operational and was successful in making the learners to understand the doubling process at the concrete stage. Learners were for example asked to put one bottle cork next to another and to count and say how many there are altogether. The teacher instructed learners to place an item on the desk, and then place another one next to it and to ask the learners how many items there are altogether. The idea was to show, for example, that “three and three makes six” and to show the learners that this process is called doubling, (phinda kabili’). This was then followed by the pictorial representations of items to be doubled, and the illustrations were drawn by the teacher on the chalkboard (see fig 1).

Figure 1. Illustration of three repeated twice

These pictorial diagrams aimed at enabling learners see and understand the significance of the equal sign in the doubling process. Here the equal sign was operational and relational serving level of O3 and R3 respectively. The equal sign was not represented as a symbol on the diagrammatic picture, but rather as an arrow. There was also an elaborate explanation on activities around doubling and the outcomes after doubling. Activities included the counting of the items in a figure, recording and counting items after combination in a bigger figure.
Results

In the lesson observed, contrasting examples were not adequately suggested so that learners could engage with non-examples. For example, they were cautioned that if you have five items, one needs to bring the same number of items to double them. ‘Uma uphinda izinto ezintathu, ngekhe uze ne zimbili’ (When doubling three items, you cannot bring two items. ‘Kuzoshoda enye’ (you will be one short). Contrasting issue here was for example when one brings three bottle corks and three match sticks, does this mean that ‘siphinda kabili’ (are we doubling the number or must it be the same items always)? Although the teacher demonstrated the doubling process, he failed to lead the learners to generalisation and thus fusion could not take place. Emphasis was just on repetition of the doubling of numbers. This is supported by Marton, Wen and Wong (2005) who pointed out that the likelihood of being able to recall something is higher if the learners hear or see something several times than if they do not. Nonetheless the activities were not aimed at explicitly entrenching in learners the significance of the equal sign.

The teacher’s generalisation and consolidation of the lesson were misleading. From the pictorial representations on the chalkboard, the concept of the equal sign (as signifying an equivalent relation) was mis-represented by the teacher when using symbols. He introduced the equal sign, without explicitly putting meaning to it. This representation of the equal sign as “the same as” or “makes” or ‘is equal to’ just conceived the equal sign as simply a tool for writing the answer. It did not play the role in relating the quantitative sameness between objects on the left-hand-side and objects on the right-hand-side. Essien (2001) notes the importance of the equal sign in mathematics as one of the most used signs and notations in mathematics, but reckons that it is a symbol that is very easily misunderstood by learners. He therefore argued that the equal sign needs to be accorded the same status as the plus and minus signs in the curriculum.

Essien (2001) also encourages the use of pictorial representation of objects familiar to the context of learners of that age bracket to introduce the concept of the equal sign. He further cautions teachers to avoid using the equal sign between two objects (e.g., a door = a rectangle) as this does not represent a relationship of equality between numbers and therefore, does not focus on the significance of the equal sign. This the researcher witnessed during observation when the teacher made a symbol representation of the exercises performed practically and pictorially on the chalkboard.

‘Ake sibhale ke manje konke loku esikade sikwenza ngamanani ebhodini (Let us now write all that we have been doing on the chalkboard in numbers on the chalkboard.). The captured picture in Figure 2 represents the four problems given by the teacher for learners to give answers to the outcome after doubling a number. One of the questions asked by the learners was whether they needed to double the sequential 1; 2; 3; and 4 or finish up the sentence after the equal sign? This in itself was much confusing for the grade one’s even though their ‘equal sign’ concept lacked construction of meaning. The teacher made some false illustrations as examples to be followed by the learners in completing the sentences. She completed the first one as 4 = 8 and instructed the learners to follow suit with the others.
The above representations are delusional. The teacher has attached no meaning to the equal sign. It was neither operational, nor relational. It was also not important that one side can replace the other or that both sides have the same value. The teacher focussed only on the fact that the learners are able to double the given number of items. In this way, the learners were not given a chance to contrast to know and discern what doubling is and what it is not. In terms of action and diagrammatic representations, they were able to separate and discern the critical characteristics of a doubling and could differentiate it from a single action. The misleading symbolical representation where the equal sign is used to instruct ‘double’ uprooted any other true meaning that could be assigned to this symbol. The meaning of the equal sign was compromised when the learners had to double a number, and to represent their actions in symbolical representations.

Interviews conducted with the teacher indicated that he didn’t know how to represent doubling using a mathematical symbol since the grade one South African syllabus did not allow children at that stage to be introduced to multiplication by two or use of any multiplication sign for that matter. She also indicated that the textbook they used also registered this error. Doing mathematics and writing it in this lesson were two separate entities and even though the teacher had assisted the learners to do mathematics, he could not help them conceptualise it. It was also clear from this lesson that it is difficult to teach the concept of doubling effectively without relying on the concept of multiplication by two. The researcher therefore recommends that there is a need to explore other ways of teaching the doubling concept effectively without relying on symbolic representation like the multiplication sign.

References


Science

Long Papers
Mentoring Physical Science Subject Advisors on Acid-Base Titration

Washington T. Dudu
School for Teacher Education and Training, Faculty of Education, North West University, Mafikeng Campus, Private Bag X2046, Mmabatho, 2735, Republic of South Africa
24879436@nwu.ac.za OR wtdudu@gmail.com

This paper reports on mentoring as a form of professional support for South African Physical Sciences subject advisors coping with curriculum reforms. The study focused on the subject advisors’ content knowledge and their competency in facilitating the teaching of a new topic in a revised physical sciences high school curriculum. Using a case study method, the study investigated one case of mentoring. The case explored a mentoring relationship between an experienced Physical science teacher educator and 16 experienced Physical science subject advisors. This study referred to them as the ‘bedrock’ of the profession basing on their job description. They either make the South African Physical science teachers swim through or drown in the challenges presented by the reforms in the Physical Science curriculum since they are the teachers’ mentors. The study revealed that subject advisors expressed uncertainly as to their content knowledge and titration skills of this topic. In spite of the case study following the cascade model consisting of short one-shot workshops of in-service training, still a common practice in South Africa, the findings suggest that mentoring, although complex does provide a viable means through which professional development efforts can be consolidated. Implications and recommendations for further study are suggested.

Background and rationale

The South African curriculum has undergone a metamorphosis of revisions since 1994. In trying to address the imbalances in the education system, a new outcome-based curriculum (Curriculum 2005) (Department of Education [DoE], 1997) promulgating a student-centred constructivist approach to teaching was introduced in 1998. The introduction and implementation of Curriculum 2005 saw quite a number of changes in the Physical Science curriculum (Ramnarian & Fortus, 2013). The changes back then saw the removal of some topics such as the teaching and learning of titration and basic polymerization under the Chemistry section. According to Chisholm (2005), a review of Curriculum 2005 in 2000 directed the way forward to the design of the National Curriculum Statement (NCS). Ramnarian and Fortus (2013) highlight that in physical sciences, curriculum planners were therefore tasked with developing a curriculum with content that would be meaningful, accessible and relevant to all students. This resulted in restructuring of the existing content and the addition of new topics. Many topics from the previous curriculum were retained, but were redirected and reworked so that their utility value and relevance were emphasised. The NCS introduced together with the Outcomes-Based Education philosophy in 2005 has been recently revisited. The aim was to produce national Curriculum and Assessment Policy Statements (CAPS) as a “refined and repackaged” version of the original documents, and not create new curricula (Vinjevold, 2012). The introduction of CAPS saw the re-introduction of some topics such as the teaching and learning of titration and basic polymerization under the Chemistry section which had been scrapped off during the implementation of C2005.

Studies in other countries have reported that when new topics were introduced to the curriculum, teachers often experienced uncertainty about their content knowledge and this
influenced the implementation of curriculum reform (Henze, Van Driel & Verloop, 2008). According to Roehrig & Kruse (2005:421), “the role of teachers’ content knowledge in the implementation of reform-based curriculum needs to be addressed.” In the South African context, Roehrig and Kruse’s assertion needs to be taken seriously. Experienced teachers received their qualifications at teacher training colleges during the apartheid era. These colleges, especially those that were situated in the homelands, had deficiencies in the teaching of specific content knowledge, leaving their students with knowledge gaps (Rusznyak, 2008). Some of these teachers went on to upgrade themselves professionally by taking in-service courses at universities after the closure of teachers’ colleges, graduating with qualifications like Advanced Certificate in Education (ACE). According to Ramnarian and Fortus (2013:3), the ACE qualification was a way in which the government gave a firm mandate to train more teachers and provide additional training for those already in the service and as a way of addressing the shortage of qualified teachers to teach physical sciences and other subjects. Despite the upgrading, most teachers still have knowledge gaps since in-service courses at universities such as the ACE focus more on pedagogy rather than the content (Hwenha, 2013).

Teachers who recently graduated from universities after the closure of teachers’ colleges are not spared either of having knowledge gaps. They go through a curriculum designed in a manner not likely to equip the students with enough content knowledge (Hwenha, 2013). A ministerial task team established to investigate teaching in mathematics, science and technology published a ‘damning report’ in 2014 about South Africa’s education system. It noted that universities were not training teachers adequately, and district officers were largely unable to provide adequate support to teachers and there was a shortage of qualified teachers (SAPA, 2014). In other words, the students are churned out half-baked. Most of the teachers are under- and some even unqualified in the areas of Science. The severity of the challenge has been evident in the learners’ achievement outcomes, which are low in a subject such as Physical science as evidenced by its low pass rate. In one way or the other, the practising teachers have to be workshopped in these new topics if changes advocated by reform visions on the curriculum such as the introduction of new topics are to be a success. As Spillane (1999) puts it, virtuous implementations of such reform visions ultimately rely on teachers. To assist in lessening the burden on teachers, they [teachers] have to be mentored especially when it comes to the teaching of new concepts. However, mentoring has to start with subject advisors because they are the ones who play a mentoring role to the teachers.

**Conceptualizing mentoring**

Mentoring is ‘off line help by one person to another in making significant transitions in knowledge, work or thinking’ (Meggison & Clutterbuck, 1995, p13). This activity is an important part of learning by adults because it represents a holistic and highly individualized approach to learning in an experiential fashion (Lai, 2010). According to Meggison and Clutterbuck (1995), mentoring is very complex and varies from one situation to another. As such, it is interpreted in different ways by different people. Thus, it is important that the purpose and intentions of mentoring in a particular context are made explicit. Making use of mentoring relationships as a way to enhance professional development activities is not a new idea. However, mentoring has increased in popularity as a means through which a teacher experiencing shortcomings in his/her practice can be supported by a skilled and experienced colleague (Lai, 2010). There is no agreed definitive conceptualisation of mentoring (Lai, 2010). A possible reason for this is that mentoring in teacher education involves complex personal interactions ‘conducted under different circumstances in different schools in which
it cannot be rigidly defined’ (Wildman, Maggliaro, Niles & Niles 1992:212). The lack of consensus as to what constitutes mentoring may be the result of researchers attempting to focus on different dimensions of mentoring (Lai, 2010).

Ramnarian and Ramaila (2012:256) acknowledge that a review of literature undertaken by Lai shows that “mentoring has been conceptualised in light of its relational, developmental and contextual dimensions”. Lai (2010:3) points out that ‘the groupings are based on differing degrees of emphasis given to each of the three dimensions by particular conceptualizations, and they are not intended to be exclusive’. The relational dimension of mentoring refers to the relationship between the mentor and the mentee. According to Franke and Dahlgren (1996), the mentor and mentee collaborate as partners to solve problems of practice within a community of practice. Communities of practice are characterised by members who are informally bound by what they do together, from engaging in discussions to solving difficult problems, and by what they have learned through their mutual engagement in these activities (Wenger, 1998). Thus the delineation between the novice and the expert becomes blurred (Franke & Dahlgren, 1996)

To Lai (2010:3), the developmental dimension of mentoring focuses on “mentoring functions and behaviours aimed at promoting the professional and/or personal development of both the mentor and mentee.” The process occurs in a sequence. Firstly, the developmental level of the mentee is recognized by the mentor who then shifts the novice towards a higher level of development (Vygotsky, 1978). “This does invoke the notion of scaffolding that emerged from socio-constructivist views of learning, particularly Vygotsky’s (1978) notion of learning in the zone of proximal development (ZPD)” (Ramnarian & Ramaila, 2012: 257). This zone is the difference between a person’s actual level of development, as determined by activities that can be performed without assistance, and the potential level of development as determined by performance of tasks under the guidance of a more capable person who guides a learner through the ZPD towards a new level of development in a gradual process of scaffolding (Van der Valk & De Jong, 2009). Second, over time there has to be a gradual withdrawal of support to enable the mentee to become more autonomous in his or her practice. In this mentor-novice relationship, the interactions with the more capable or experienced others are critical in order for novice to acquire knowledge beyond the independent level of exploration (Vygotsky, 1978).

The contextual dimension focuses on a complex web of cultural and situational features unique to each mentoring setting (Lai, 2010), and recognises the powerful influence of the school organization and culture on teaching and learning. A good example of this dimension according to Lai (2010:446) is the view of “induction as an enculturation process” advanced by Feiman-Nemser (2003). Along the same line, Feiman-Nemser suggested that mentoring should be linked to a vision of good teaching, guided by an understanding of teacher learning, and supported by a professional culture that favours collaboration and culture. The conceptualization of mentored learning to learn as enculturation implies that mentoring is about helping mentees fit into the organisation and culture of a particular community of practice. Mentoring is a multi-faceted and complex concept, and the researcher takes a position that it is best understood from the three perspectives outlined. The researcher hopes to uncover some of this complexity in the single case of mentoring researched by framing the work according to these dimensions.

**Acid-Base titration- the phenomenon under study**

Acid-base titrations are common experiments carried out by students in introductory chemistry classes. The most frequently conducted titrations involve the neutralization of
strong acids with strong bases, with students being required to calculate the concentration of unknowns using this method. Acid-base chemistry has posed student and teacher difficulties and the causes have been ascribed to the existence of many alternative conceptions or misconceptions (Schmidt, 1995), a poor understanding of the particulate nature of matter (Smith & Metz, 1996), difficulties with the use of different models used in acid–base chemistry (Kousathana, Demerouti & Tsaparlis, 2005) and confusion between acid–base terminology and everyday words (Schmidt, 1995). To ensure that subject advisors have a good grasp of content knowledge and practical skills on acid-base titration which they can relay to practising teachers, this study investigates South African Physical Science subject advisors’ understandings on their content-related knowledge competency of this topic. In particular, focus was on terms; acid, base, neutralization, endpoint, equivalence point, calculations and the conceptualization of the whole process of acid-base titration.

Research Question

The following research question was formulated: How does mentoring enable Physical Science subject advisors to overcome some of the challenges posed by curriculum reform in particular, the acid-base titration topic?

Conceptual framework

The study was grounded in research related to two constructs: teachers’ knowledge (Shulman, 1987), which in order for them[teachers] to be effective in the classroom should be well-developed; and conceptual change (Vosniadou, Ioannides, Dimitrakopoulou & Papademetriou, 2001), which describes learning as a process that requires the significant reorganization of existing knowledge structures and not just their enrichment.

Shulman (1987) proposed seven categories of teacher professional knowledge, two of which are content-related (i.e. content knowledge and pedagogical content knowledge). The other categories refer to general pedagogy, students and their characteristics, educational contexts, and educational purposes (Shulman, 1987). Focus of this paper is on subject advisors’ content-related knowledge on acid-base titration, so the study elucidates only the category of content knowledge. To Grossman, Wilson and Shulman (1989), content knowledge refers to knowledge of the substantive and syntactic structures of a discipline where substantive knowledge refers to the global structures or principles of conceptual organisation of a discipline. Constructs such as knowledge of facts, concepts, and principles within a content area and knowledge of the relationships between them are included in substantive knowledge (Ramnarian & Fortus, 2013). According to Shulman (1987:9), syntactic knowledge encompasses knowledge of the “historical and philosophical scholarship on the nature of knowledge” in a discipline. Since the focus of the paper is on topic specific- acid-base titration, syntactical knowledge as a construct is excluded. The content knowledge mentioned within the context of this study therefore refers primarily to the substantive structure of the subject.

The conceptual change theoretical framework focuses on knowledge acquisition in specific domains. Vosniadou et al., (2001) assert that this learning of science differs in important ways from the usual empiricist approach, which says science learning is mostly a matter of enrichment and improving existing conceptual structures. These structures are built on the basis of experiences that are initially concrete and limited. The conceptual change approach has its own critics. To Vosniadou et al. (2001), some researchers have criticized the conceptual change approach on the grounds that earlier beliefs do not disappear when the currently accepted scientific explanations are understood. This disappearance of earlier
representations is not, however, a necessary requirement of the conceptual change approach. The conceptual change approach forces the creation of new, qualitatively different representations. The old representations may continue or may disappear. This is a question for empirical research to determine which is beyond the scope of this study.

Methodology
The study followed a generic qualitative case study design, as the researcher wanted to understand the nature, dynamics and complexity of mentoring at a selected training centre as a case of mentoring of subject advisors (Cohen, Manion & Morrison, 2002). The case study research method was deemed appropriate for its in-depth particular research and effective to attain a desired goal in a short time by emphasizing detailed contextual analysis of a limited number of events or conditions and their relationships (Çepni, 2003).

Sample
The sample consisted of 16 Physical Science subject advisors (7 males and 9 females) drawn from the whole of the North-West Province of South Africa. Each participant represented an Area Project Office (APO) and was a mentor to all Physical Science teachers under his/her cluster who teach at FET band, that is, Grades 10-12. The participants were selected purposefully as they depicted a single case where subject advisors of the Physical Sciences were involved in a mentoring relationship. The participants were considered as a single case during the mentoring process and not as multiple cases. This was then the bounded system within which the study was conducted. The location of the training centre was convenient as both researcher and the participants were booked at the same place hence accessible in terms of contact time and after-hours consultation.

Research Methods
Data were collected through performing the actual prescribed experiment (acid-base titration), observations during the activity, interviews, listening to discussions between the subject advisors and written responses to post activity questions. The experiment (prescribed experiment number 3) is exactly the same as the one the teachers in the province perform with their learners in the classroom in Term 2 according to the recent CAPS document. The interviews, discussion and classroom observations were all audio-recorded and later transcribed. In total, 3 interviews were conducted with each lasting approximately 30 minutes. Observations focused on the manner in which subject advisors elicited their solution preparation and titration techniques throughout the experiment, and listened to pair and class discussions that varied between 15 to 30 minutes. The discussions were held after individual completion of the worksheet questions. The data were analysed using the Atlas.ti software. The analysis began with an open coding of the data by assigning codes to segments of the text. As suggested by Henning, Van Rensburg and Smit (2004:132), this was followed by axial coding where “the parts of the data identified and separated in open coding back together in new ways to make connections between categories or the codes” were then put. Various dimensions of mentoring, content knowledge particularly substantive structure of the subject, pedagogical content knowledge and conceptual change constructs guided this process. The codes were grouped into code families, which to a large extent corresponded with mentoring dimensions and the respective constructs within which the study is framed. The researcher and two colleagues (members of a research team where the researcher is based) sought to establish reliability in this process of coding and grouping codes into families by doing the coding independently. An 89% percentage of agreement was obtained in this process of data analysis. This paper reports on challenges that subject advisors faced
during the mentoring process based on the themes that emerged as result of this analysis. Since this is a single case, first to be reported is the profiling of the performed activities, and then the mentoring relationship between the mentor and the mentees is described in terms of its relational, developmental and contextual dimensions. However, when doing this, all is related to content knowledge, pedagogical content knowledge and conceptual change constructs. No real names are used this study. All names used are pseudonyms.

Results

Profiling the performed activities

Before performing the activities, the mentor gave a power point presentation on acid-base titration concepts, namely; definition of acids and bases, conjugate acid-base pairs, ampholyte, neutralization, endpoint, indicator and hydrolysis, then moved on to the experiment. The experiment was titled ‘titration of oxalic acid against sodium hydroxide base’. It was an experiment in two parts. First was preparation of the standard solution (hydrated oxalic acid- H$_2$C$_2$O$_4$. 2H$_2$O). The subject advisors were then asked to calculate the concentration of oxalic acid solution they had prepared. The worksheet also asked them to explain what they understand by a standard solution. Three additional questions were asked by the mentor to aid on the skills, comprehension and conceptualization of the titration process.

Second part involved the titration of the prepared oxalic acid solution against sodium hydroxide. The participants were asked to record any colour changes they observed after adding 3-5 drops of phenolphthalein indicator to the oxalic acid. They were also required to note down the colour change they observed at endpoint during titration. Questions on the worksheet included asking the participants to write down the balanced chemical equation for the neutralisation reaction that had taken place. The participants were also asked to calculate the unknown concentration of sodium hydroxide used in the titration to neutralise oxalic acid. Finally on the worksheet, the participants were asked to motivate why they thought phenolphthalein is the best indicator for this experiment. Eight (8) additional questions were posed by the mentor to aid on the skills, comprehension and conceptualization of the titration process.

Relational dimension: A mutually dynamic beneficial relationship that is built when the mentor and mentee collaborate as partners to solve problems of practice within a community of practice

The relationship between the mentor (researcher) and the mentees (subject advisors) was one that was built on benevolence, engagement and nurturing among other traits. In responding to the first additional question on why one had to be extremely careful when adding the last few drops to the volumetric flask? All participants elicited that they did not know the definition of a standard solution. Some just thought that if a solution was labelled 1.0M, then it was standard. Of the 16 participants, no one stated that a standard solution is one whose concentration is accurately known. When asked during discussion, what they would recommend to teachers if they had over shoot the mark, some suggested teachers should just proceed with the experiment. Mr Seoke said, “I would recommend them to continue with the experiment” Again this showed that the definition of a standard solution was not well understood by some of the participants. The other question where the mentor had to exude empathy is the additional question focusing on why the conical flask, rather than a beaker, was used in the experiment. All 16 participants went blank. No individual got it correct. The mentor then asked all the participants to put some acid or base in a beaker in front of them.
and try to swirl it in a mixing fashion. Some spilled the solutions and the mentor asked them to repeat the process using a conical flask. To their amazement, all shouted “for easy mixing without spilling”, much to the respect of the mentor. This showed that participants lacked knowledge of facts and principles and relationships between these constructs.

Developmental dimension: Developing PCK through reflective discussions and exploiting each other’s’ strengths in overcoming deficiencies in their practice

When asked an additional question related to the first part of the activity; when a solution has been made up, why is it necessary to mix the contents thoroughly? What feature of the volumetric flask makes this particularly necessary? Participants did not do justice by giving convincing answers to this question. This question further revealed that the participants did not know the relevance and usefulness of certain apparatus. Of the 16 participants, not even one managed to mention the narrow neck of the volumetric flask as a feature necessary for obtaining a homogenous mixture. This shows how conceptually challenged the participants were regarding this concept.

The other additional question which posed problems is one which dealt with titration procedures. Participants were asked to give just one reason for carrying out each of the following procedures; (a) the sides of the conical flask were washed down with deionised water, (b) the conical flask was frequently swirled or shaken. Nine (9) of the participants got the answers to this two-part question correct and 7 participants got it totally wrong. One participant, Mrs Paulus wrote, “Deionised water must not be added to wash down the sides of the conical flask as this will change the concentration of the resultant solution”. During interviewing, when asked to elaborate on her written response, Mrs Paulus said,

“During the preparation of the standard solution of oxalic acid, we made sure that we did not exceed the calibration mark when adding water to the volumetric flask so that our concentration becomes accurate. In the same manner if we add water, I thought it would affect the concentration of the resultant solution.”

This is a clear misconception where the terms ‘moles’ and ‘concentration’ and the titration concept as a whole are ill-conceived. During the discussion session, Mrs Paulus argued vehemently for her point only to say after 15 minutes of debate, “I did not do this during my training, now I understand.” One participant, Mr Madito answered part (b) by saying the conical flask was frequently swirled or shaken so that the reaction can take place. When asked to clarify his written response, he said “shaking enables the reactants to react.” Another aspect which was worrisome observed by the mentor is how the subject advisors performed the titration. Since they were doing the experiment in pairs, one was opening the tap of the burette whilst the other was shaking the conical flask. The mentor did not demonstrate initially as he wanted to see how much of the good practices the participants harboured. Having ascertained the developmental level of the participants, the mentor mentee and mentee-mentee relationships were used during the discussion session to shift the mentees toward a higher level of development. In doing so, subject advisors translate subject content knowledge into useful forms of representations of ideas in the form of demonstrations, powerful analogies, explanations, illustrations and examples to facilitate comprehension by teachers they mentor.

Contextual dimension: A community of practice conducive to mentoring confined to subject advisors’ areas of influence

All participants got the third additional question relating to the first part of the experiment correct. The question was, why is a funnel used in transferring the solute from the watch glass to the volumetric flask? However, problems were encountered in answering additional
questions related to the second part of the experiment which is titration process. The question which gave problems to participants states: In using a burette, why is it important to: (a) rinse it with a little of the solution it is going to contain; (b) clamp it vertically; (c) have the part below the tap full. Most (10) of the participant’s answers were not precise. Mrs Methikge’s responses to parts (a) and (b) were, “to ensure that the burette is clean” and “to make it full” respectively. During interviewing, she was asked; would the burette not be clean by cleansing it with water only? She responded, “It will be clean but not very clean.” Issues of dilution were missed here. During the discussion session, the participants were first asked to help each other consolidating the mentee-mentee relationship before the mentor popped in to assist with strategies of good teaching, guided by an understanding of teacher learning promoting the mentor-mentee relationship. Participants assisted each other well during discussion and it could be seen they need more of this peer-to-peer environment. The mentor reiterated collaborations and formation of communities of practice so as for them [participants] to assist teachers with knowledge acquisition in specific domains in their respective districts.

Discussion of findings

The findings of this study show that, mentoring has provided a viable and sustainable means through which content knowledge, pedagogical content knowledge and conceptual change efforts were consolidated in this single case. The dynamic nature of the mentoring relationship, from a relational perspective became more egalitarian and the initial delineation between the roles of mentor and mentees was becoming increasingly blurred. Initially, the mentees had a strong dependency on the mentor in seeking advice on how to overcome the challenges they were experiencing as they prepared to go and mentor teachers in their area project offices regarding acid-base titration. They listened attentively and took notes solemnly as the mentor continued with his presentation during the initial period. As the day progressed, the mentoring roles changed as the mentor-mentee relationship evolved. The initially one sided relationship changed and mentees started to assist each other with explanations through use of models and examples during discussion assisting their colleagues. The mentee-mentee relationship got stronger making the mentor-mentee relationship even more blurred, as the participants who had grasped the concepts took control and the relationship resembled that of a collaborative mentor-mentor relationship. The mentor became more of a critical friend. This finding is in agreement with Ramnarian and Ramaila’s (2012) findings which showed that with time one of the novice teachers with his increasing confidence and competence started to engage more collaboratively with his mentor. The implication of this for mentoring is that it cannot be conceived of as a static relationship, but is dynamic by nature instead. It was also apparent that at any given stage in this relationship there was empathy and respect. The participants laid out their concerns about their lack of conceptual understanding on some of the processes and skills without reluctance. As much as the participants were conceptually challenged with aspects which seemed easy basing on their ‘bedrock’ nature to the profession, the mentor had to exude empathy as there had been reciprocal mutual trust. On a professional level the subject advisors developed capacity in addressing the challenges in their practice at the end of the day.

From a developmental perspective, it was apparent that development was taking place in both the professional and personal domains of the subject advisors. Regarding the mentor-mentee relationship, Mrs Paulus, for example, in the professional domain showed misconceptions where she used the terms ‘moles’ and ‘concentration’ interchangeably in arguing against addition of deionised water to the sides of the conical flask during titration. After the mentor tried several times to explain the difference between these two terms, she partially understood
and was not convinced. It was now left to the mentee-mentee relationship to try and bridge the gap. Two participants stood up to the task and used models to explain to their colleague so as to address the shortcomings. At the end of the day, Mrs Paulus became more confident in her own ability and the initial tentativeness that she experienced when responding to additional questions posed after the activity was no longer evident. A possible explanation for this might be that through a reflective process and the advice she received, Mrs Paulus was able to initiate steps in improving her comprehension of these concepts. In particular, she was able to develop her content knowledge and using the model explanation rendered by her colleagues. At the end of the workshops, she openly said her content knowledge had developed and she would explore strategies on how best to make scientific knowledge more accessible to teachers under her tutelage. The analogy given on the use of microscopic representations by one of her colleagues during titration is one example which resulted in participants acknowledging conceptual change in the whole process of mentoring on this topic. Through collaboration the subject advisors were able to improve their conceptual understanding and acquired knowledge beyond their independent level of exploration of this topic.

The findings with regard to the contextual dimension of mentoring suggest that a more concerted effort needs to be made in creating conditions that would facilitate a community of practice in which mentoring may thrive. Peer-peer collaborations need to be promoted. In this regard subject advisors need to be instrumental in initiating communities of practice so that teachers engage collaboratively on curriculum issues that are significant for the context in which they teach. As they have been identified as the ‘bedrock’ of the profession, they need to make formal arrangements and move from school to school mentoring Physical science teachers as per their job description. They can even assist the teachers in small groups in their respective districts. In the process, the subject advisors should also encourage formation of communities of practice by the teachers so that the teachers may end up providing support to other teachers who are in need of professional development in the absence of the subject advisors. This might result in teachers developing a culture of working together for a good cause linked to a vision of good teaching, guided by an understanding of teacher learning, and supported by a professional culture. Although, these results differ from some published studies (Lai, 2010), they are consistent with those of Ramnarian and Ramaila (2012) with South African teachers.

Conclusion

The findings of this study have provoked further questions on mentoring that should be addressed in subsequent studies. Research questions that could be asked include: What factors limit the success of mentoring relationships and what can be done to overcome them? To what extent does mentoring eliminate misconceptions? What type of impact does mentoring have on the classroom practice of mentored teachers? What level of support in a mentoring professional development is ideal? What factors support mentoring relationships between subject advisors and teachers? In crafting the way forward it is recommended that further research be undertaken on mentoring in science education and that other methodologies for inquiry be employed to investigate mentoring beyond single cases. In addition, more case studies of mentoring across diverse settings that is synonymous with the South African educational landscape would validate the viability of mentoring as a widespread form of professional support for teachers. As Achinstein and Athanases (2006:12) point out, ‘mentors are not born, but made, and are in a continuing process of becoming’ and ‘making mentors is a deliberate act’. Deliberate effort is therefore needed to produce new
mentors from teachers to be mentored by the subject advisors and to sustain their learning over time.

References


Smith K.J. & Metz P.A. (1996), Evaluating student understanding of solution chemistry through
The Handbook of Research on Science Education, Volume II has recently been released. Building on the foundation set in Volume I, this is a landmark synthesis of research in the field. Volume II is a comprehensive, state-of-the-art new volume highlighting new and emerging research perspectives. The 87 contributors, all experts in their research areas, represent the international and gender diversity in the science education research community. The volume is organized around six themes: theory and methods of science education research; science learning; culture, gender, and society and science learning; science teaching; curriculum and assessment in science; science teacher education. Each chapter presents an integrative review of the research on the topic it addresses, pulling together the existing research, working to understand the historical trends and patterns in that body of scholarship, describing how the issue is conceptualized within the literature, how methods and theories have shaped the outcomes of the research, and where the strengths, weaknesses, and gaps are in the literature.

Editors – Norman G. Lederman and Sandra K. Abell

Science Education evolved as a separate discipline in the 1960s. Prior to this time, scientists with an interest in education primarily completed research on the teaching and learning of science. Although the field is relatively new, research in science education is a vibrant area and it continues to thrive throughout the world. The SAARMSTE organization and meeting is just one example of how the field has expanded over the years.

The Handbook of Research on Science Education, Volume II has recently been released. Building on the foundation set in Volume I, this is a landmark synthesis of research in the field. Volume II is a comprehensive, state-of-the-art new volume highlighting new and emerging research perspectives. The 87 contributors, all experts in their research areas, represent the international and gender diversity in the science education research community. The volume is organized around six themes: theory and methods of science education research; science learning; culture, gender, and society and science learning; science teaching; curriculum and assessment in science; science teacher education. Each chapter presents an integrative review of the research on the topic it addresses, pulling together the existing research, working to understand the historical trends and patterns in that body of scholarship, describing how the issue is conceptualized within the literature, how methods and theories have shaped the outcomes of the research, and where the strengths, weaknesses, and gaps are in the literature. The organization of the Handbook and its chapter authors are summarized below:

Handbook Sections

I. Theory and Methods of Science Education Research
   Section Editor: David F. Treagust

II. Science Learning
Section Editor: Richard Lehrer

III. Diversity and Equity in Science Learning
Section Editor: Cory A. Buxton and Okhee Lee

IV. Science Teaching
Section Editor: Jan H. van Driel

V. Curriculum and Assessment in Science
Section Editor: Paul Black

VI. Science Teacher Education
Section Editor: J. John Loughran

Section I. Theory and Methods of Science Education Research

1. Paradigms in Science Education Research
   David F. Treagust, Mihye Won, and Reinders Duit
2. Quantitative Research Designs and Approaches
   Hans E. Fischer, William J. Boone, and Knut Neumann
3. Contemporary Qualitative Research: Toward an Integral Research Perspective
   Peter Charles Taylor

Section II. Science Learning

4. Student Conceptions and Conceptual Change: Three Overlapping Phases of Research
   Tamer G. Amin, Carol L. Smith, and Marianne Wiser
5. Attitudes, Identity, and Aspirations Toward Science
   Russell Tytler
6. Classroom Learning Environments: Historical and Contemporary Perspectives
   Barry J. Fraser
7. Learning Science Outside of School
   Léonie J. Rennie
8. Teaching Learning Progressions: An International Perspective
   Per-Olof Wickman

Section III. Diversity and Equity in Science Learning

9. Unpacking and Critically Synthesizing the Literature on Race and Ethnicity in Science Education
   Eileen Carlton Parsons
10. Gender Matters: Building on the Past, Recognizing the Present, and Looking Toward the Future
    Kathryn Scantlebury
11. English Learners in Science Education
    Cory A. Buxton and Okhee Lee
12. Special Needs and Talents in Science Learning
    J. Randy McGinnis and Sami Kahn
    Angela Calabrese Barton, Edna Tan, and Tara O’Neill
14. Rural Science Education: New Ideas, Redirections, and Broadened Definitions
   J. Steve Oliver and Georgia W. Hodges

15. Culturally Responsive Science Education for Indigenous and Ethnic Minority Students
   Elizabeth McKinley and Mark J.S. Gan

Section IV. Science Teaching

16. General Instructional Methods and Strategies
   David F. Treagust and Chi-Yan Tsui

17. Discourse Practices in Science Learning and Teaching
   Gregory J. Kelly

18. Promises and Challenges of Using Learning Technologies to Promote Student Learning of Science
   Joseph S. Krajcik and Kongju Mun

19. Elementary Science Teaching
   Kathleen J. Roth

20. Interdisciplinary Science Teaching
   Charlene M. Czerniak and Carla C. Johnson

   Reuven Lazarowitz

22. Teaching Physics
   Reinders Duit, Horst Schecker, Dietmar Hötteecke, and Hans Niedderer

23. The Many Faces of High School Chemistry
   Onno De Jong and Keith S. Taber

24. Earth System Science Education
   Nir Orion and Julie Libarkin

25. Environmental Education
   Justin Dillon

26. From Inquiry to Scientific Practices in the Science Classroom
   Barbara A. Crawford

Section V. Curriculum and Assessment in Science

27. Scientific Literacy, Science Literacy, and Science Education
   Douglas A. Roberts and Rodger W. Bybee

28. The History of Science Curriculum Reform in the United States
   George E. DeBoer

29. Scientific Practices and Inquiry in the Science Classroom
   Jonathan Osborne

30. Research on Teaching and Learning of Nature of Science
   Norman G. Lederman and Judith S. Lederman

31. The Evolving Landscape Related to Assessment of Nature of Science
Chapter 4. Student Conceptions and Conceptual Change: Three Overlapping Phases of Research

This chapter reviews the science education literature on student conceptions and conceptual change, recognizing three overlapping phases of research. In the first phase, researchers rejected a domain general view of concept development and focused on characterizing the content of student conceptions in specific domains. During the second phase, research investigated the multiple contributors to the process of conceptual change – namely, ontology, epistemology, models and modeling, and social interaction. In the current third
phase, researchers are increasingly adopting systemic perspectives, considering multiple interacting knowledge elements at multiple levels of analysis.

**Major findings**

- It is important to present concepts in contexts of their use
- The importance of a complex, non-verbal component to concept formation is necessary
- In addition to mental models to ground understanding of scientific concepts, other representational resources are needed
- Identify the set of conceptual metaphors that construe a given scientific concept
- Future research needs to examine the interaction between student language and other forms of symbolization and inscription
- Curriculum should foreground relations among concepts rather than focusing among concepts rather than focusing on concepts in isolation as is typical in traditional instructional units
- Rather than overloading students with too many factual details, curricula should target the development of higher-level forms of knowledge that might offer needed guidance

**Challenges of Designing Instruction from More Systemic Perspectives**

- Need for curriculum to support multiple paths and to devise methods to respond differentially to the needs of students
- For each main idea in the standard, curriculum developers need to identify a set of learning performances by combining that idea with important practices (e.g., defining terms, creating models or explanations, designing investigations, making arguments based on evidence).
- By being organized around coherent learning goals, the project-based curriculum focused on the kinds of “sparse knowledge” (e.g., organizing models and general principles) that may be most helpful in preparing students for further learning.

**Implications**

- Research results from specific Learning Progression (LP) projects are just beginning to be reported and it will, of course, take some time to develop, revise and test conjectures about productive stepping stones in different domains, as well as to assess the overall value of the LP approach.
- But already one result is clear: elementary school children are capable of developing much more sophisticated models and understandings than is observed with traditional instruction
- An exciting next step will be not only to continue to clarify our understanding of knowledge growth in the elementary school years, but also to explore how learning in the middle and high school years is affected by this foundational preparation as well as what the long term payoffs might be.
- Piaget’s domain-general view of conceptual development in terms of changes in logico-mathematical structures has now been replaced by a systemic view of concepts
and conceptual change involving complex interactions between various forms of knowledge: propositionally expressed beliefs of various kinds (domain specific, ontological, and epistemological) and iconic representations that help ground understanding in perception and action (image-schemas, imagery and mental models).

- These knowledge elements are distributed across internal and external representations and processes of change involve processes internal to individual learners’ minds and interactions with others (including more knowledgeable individuals and peers).

- Future research will need to take on the challenge of improving our understanding of this complexity and fostering conceptual change through instruction that takes this understanding into account.

**Chapter 7. Learning Science Outside of School**

Opportunities for learning science outside of school make a significant contribution to science education. Today’s youth use a variety of out-of-school resources and activities to learn science, both purposefully and incidentally. This chapter presents a critical review of recent research into the nature of the learning outcomes from three clusters of out-of-school resources: museums and other institutions with an educational focus, community-organized activities targeting families, and media.

The contribution to learning and participation in STEM attributable to informal science education legitimizes the continuation of research in this field. The progress made over the last decades has marked the maturity of the field and bestowed belief in the significance of its findings, but there are still gaps and opportunities for consolidation and cooperation among the various providers.

**Implications for Research Into Learning Outside of School**

- An attribute of the out-of-school activities that result in science learning relates to the context that makes engagement meaningful.

- Learning from any science-related experience is most likely when youth’s engagement is purposeful and prolonged or revisited, such as by discussing with family and friends – in person or online – enabling the experience to be remembered, rehearsed, and reflected upon.

- If learning outside of school is to be linked with the science curriculum in school, students need sufficient structure to prepare them for active but enjoyable participation in the experience, and consolidating activities back in the classroom.

- Need for efficient collaboration between teachers and museum educators and/or docents to support student learning

- Another attribute relates to meaningful communication. The required transposition of science content is difficult and highlights the need for clear definitions of purpose and audience and evaluation of the effectiveness of communication

- A recurring thread through research in almost all out-of-school settings is the need for an effective means to assess outcomes. Establishing impact may have implications for funding, for example, to continue support for an activity, or to choose between alternative methods of delivering a program. Implications for Research Into Learning Outside of School
Need for research to focus on the linking of or collaboration between different kinds of venue for informal science education or even among institutions of the same kind.

A stronger and common identity should be developed among providers of informal learning experiences.

Informal educators need to conceptualize that they are part of a complex, interconnected system and become more reflective, and evidence-based in its practice, helping it to acquire more sophisticated models of science, the value of science, how science is learned and what makes science interesting and engaging.

Need for a research agenda to highlight the closing of gaps that might assist in reaching the underserved groups in the informal sector.

**Implications for Practice and Policy**

1. Creating a “third space” for science education that integrates the informal sector into the day-to-day workings of the formal learning environment.

2. Technology incorporation into museum environments would strengthen the role of science centers in their communities, helping them to occupy a special niche in the expanding STEM learning ecology.

3. Need for investigation of future role of technology incorporation into museum environments.

4. Learning in science outside of school is unlikely to be under a unified theoretical base but a variety of approaches to research in the field. It could be broadly divided into two themes:
   
i. Learner as individual (Knowledge gaining or acquisition)
   
ii. Learner as a community (Participation in building of knowledge)

**Chapter 15. Culturally Responsive Science Education for Indigenous and Ethnic Minority Students**

This chapter describes, through reviewing empirical studies, the importance of pedagogical relationship studies in science classrooms and how this impacts on shaping science learning for indigenous and minority students. This chapter presents an argument for the need to move towards a cultural perspective that is situated in the science classroom and focuses on teacher-student relationships, such as culturally responsive pedagogy and taking into consideration the indigenous and minority community’s worldviews into the science curriculum.

- The underrepresentation of indigenous and some ethnic minority students in secondary science education is a major social and economic disadvantage for these communities and a major challenge for science educators in industrial countries.

- The reason for this is that the lack of participation and achievement by these communities is perceived as being particularly urgent as they strive for a highly skilled workforce specifically in science-based subjects to build their knowledge-based economies.
The field of indigenous and ethnic minority science education research is fraught with conundrums, tensions and contradictions because of the specific contexts necessary to work with the communities, and the complexity that such contexts present.

No clear consensus on how to identify ethnic minority and indigenous students

The point is not about finding a consensus or a universal definition about what constitutes each of these groups, identity understandings will always be locally driven, but that the inclusion of the communities (over a wide range of influences) has been shown to be influential in student academic success.

Need to promote more equitable relationships and interactions between Indigenous communities and academia

Interactions between Indigenous communities and academia should consider the following four requirements: respect, relevance, reciprocity, and responsibility

Need to respect the cultural integrity of indigenous people, make learning relevant to their ways of knowing, offer reciprocal teaching and learning relationships that build upon the cultural background of the students and empowering them to exercise responsibility over their own lives.

These four Rs (respect, relevance, reciprocity, and responsibility) may act as discursive norms for building positive relationships and to better understand and embrace indigenous ways of knowing.

Increasing school connections with local communities will result in a greater involvement in science and science education

More research needs to be done in how partnerships between schools and local indigenous and/or ethnic minority communities are formed and work in the context of science learning

Accountability is very important for indigenous communities and has to be fully explored in relation to science education.

Teachers, students, and others in school settings establish new forms of participation that bring together the first space of school science with the second space of the home/culture to create a “third space” that is inclusive of both in the form of hybrid knowledges. This “third or hybrid” space draws attention to the different knowledge, discourses, and relationships that influence science learning, allowing for a collective construction of new knowledge, discourses, and identities.

Culturally responsive research in science education classroom serves two purposes, namely:

1. It brings a resolution to the question of how to respond to the situation of indigenous and ethnic minority students’ “underachievement” in science education without the need to engage in deep and meaningful ways with the excluded communities

2. It serves to domesticate the knowledge of others- to make indigenous knowledge “fit” prevailing views of science and bring it under the control of others

Culturally responsive science pedagogy (CRSP) presents the following research avenues:
1. Research is needed to identify ways to support teachers and students to better leverage the ‘funds of knowledge’ (from school and home environments) that each brings to the science classroom

   -“Third Space” research:

   - Resolution of tensions and challenges that arise when the indigenous resources are found to be discontinuous with the way science is defined and taught in the classroom

2. Developing teachers’ culturally responsive pedagogies must arise from the actions of an entire school system rather than a classroom teacher.

   - How to plan science content to be taught

   - How to involve the community in a collaborative learning environment

3. Role of Indigenous communities and developing their potential as partners of science and education not just to inform but also direct research programs within science education

4. Sustainability of these programs in school and promoting student success

Chapter 20. Interdisciplinary Science Teaching

This chapter provides an examination of interdisciplinary science teaching in a historical context, paying particular attention to recent findings in the field, as well as the emergence of integrated STEM (science, technology, engineering, and mathematics) education. Research included focuses on preservice and inservice settings, as well as impact on student achievement, student and teacher attitudes, and teacher quality.

- Integration is a pivotal component in the Framework for K-12 Science Education, which includes scientific and engineering practices, crosscutting concepts, and disciplinary core ideas. The science and engineering practices clearly encourage integration of science with engineering, technology, and literacy (e.g., discourse, reading and writing).

- The common method of schooling is organized in an artificial manner, as disciplinary content is taught in isolation as academic coursework in K-12 schools. The real world, in contrast, is integrated by nature, and an interdisciplinary approach provides authentic contexts for learning

More research needed on integration to:

- Verify the benefits of curriculum integration and determine whether the results can be used to inform school practice

- Investigate effective models of preparing teachers to deliver integrated instruction

- To fill the gap on empirical research of integrating engineering and science

- To explore the inclusion of engineering practice advocated in the NGSS in terms of integration, teacher preparation, and integrated curriculum
Some major concerns are:

- Lack of consensus regarding the definition of integration
- A clear definition would help science educators eliminate confusion when discussing curriculum and instructional approaches that endeavor to integrate curriculum
- A clear-cut theoretical framework would provide the stimulus for the design and completion of further research regarding the impact of integrated curriculum
- Educators should continue to search for good curriculum materials that provide sufficient, high-quality science and mathematics content
- More research is needed to explore the benefits of integrated curriculum materials and the use of mobile technologies to integrate the curriculum
- Problems regarding the structure of the school day need to be mitigated before integration becomes commonplace in schools (Block Scheduling)
- The pressure of high stakes standardized testing continues to be a limiting factor in implementing an integrated curriculum
- For integration to be accepted in a standards environment, either standardized tests need to measure knowledge and skills associated with learning in an integrated manner or integrated units should have assessments consistent with those in the standards and on high-stakes tests.

Challenges surrounding attempts to integrate across the curriculum

- Integration of mathematics and science
- Collaborative efforts among various disciplines and personnel
- Identifying what teachers are expected to teach and students are expected to learn through integrated curricula
- Need for communication about successes and failures of integration
- More focused attention about integration of curriculum and instruction.

Ch 27. Scientific Literacy, Science Literacy, and Science Education

This chapter updates and extends an earlier review and analysis of scientific literacy (SL) as a feature of school science education. Two “visions” of SL (Visions I and II) were presented in the earlier review, representing a competition about the interpretation of SL as a major goal for school science. The present chapter examines recent trends and develops a methodology for detecting apparent changes in the cachet being enjoyed currently by each vision.

Vision I – Science Literacy

The scientific literate person is one who:

- is aware that science, mathematics, and technology are interdependent enterprises with strengths and limitations,
- understands key concepts and principles of science,
is familiar with the natural world and recognizes both its diversity and unity, and
uses scientific knowledge and scientific ways of thinking for individual and social
purposes”

Vision II – Scientific Literacy

“We would expect a scientifically literate person to be able to:

- appreciate and understand the impact of science and technology on everyday life,
- take informed personal decisions about things that involve science, such as health, diet, use of energy resources,
- read and understand the essential points of media reports about matters that involve science,
- reflect critically on the information included in, and (often more important) omitted from, such reports, and
- take part confidently in discussions with others about issues involving science.”

<table>
<thead>
<tr>
<th>Table 1. SL Competencies 2000 to 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2000 Competencies for Scientific Literacy</strong></td>
</tr>
<tr>
<td>• Use Scientific Knowledge</td>
</tr>
<tr>
<td>• Identify Questions</td>
</tr>
<tr>
<td>• Draw Evidence-Based Conclusions</td>
</tr>
</tbody>
</table>

Conclusions

- Vision I programs run the risk of including situation-oriented materials only as a source of motivating students in lessons
- Vision II programs run the risk of paying insufficient attention to science
- Vision II (outward-looking toward science-related situations) is receiving less emphasis at this time than Vision I (inward-looking to scientific disciplines).
- Need to develop ways to science literacy and scientific literacy in science education programs that can successfully meet the needs of all students
- This is a systemic problem because neither vision will accommodate both functions
- The Twenty First Century Science project in England suggested that an overall, systemic solution can provide for the two major functions of school science
- This project could provide a template for other strategies, perhaps more suited to other educational systems
- The focus has to be on the science education needs of the students, not the ideological purity or attractiveness of a vision that fits the needs of only a fraction of the students to be served
**Chapter 42. Research on Science Teacher Knowledge**

This chapter begins with an overview of teacher knowledge research, briefly discussing some of the different strands in this research domain, and some of the models that have been used to frame this research. In the next part, findings of recent research on science teacher knowledge are discussed. Following this, some of the research approaches and instruments that have been developed and used in this research are reviewed.

Measuring teacher knowledge (and beliefs) is a recurring theme in research in science education. It seems that science teachers’ SMK is mostly investigated with standardized tests, such as questionnaires and achievement tests.

- With respect to eliciting science teachers’ PCK, the studies, which are described in this review show that there’s still a huge variety in methods and instruments used.

- The various methods can be represented along a spectrum that ranges from a focus on how science is actually taught in practice (*in situ*), to the use of standardized written tests with open ended or multiple-choice items.

- Whereas the former methods aim to capture ‘PCK-in-action’, the latter target ‘declarative PCK’

**Strategies that further the development of science teacher knowledge**

- Programs need to have a focus on student learning of science content
  - Science teachers, both prospective and experienced, can benefit from studying authentic student work, collecting data from students in their own classes, or analyzing videos of classroom situations, preferably taught by the teacher

- Programs need to include opportunities for science teachers to plan and design ways to teach certain science content and try at least some of these ways out in their own practice
  - Analyzing existing or innovative curricular materials may be part of this strategy
  - During these activities, support from, and collaboration with peers, mentors, as well as facilitators can be particularly useful, for instance, through observing each others’ lessons and discussing these afterwards.
  - To specifically promote the development of both SMK and PCK, the use of real world applications of science seems beneficial
  - Preferably, these strategies are combined, for example, in a project based approach where teachers do (collaborative) action research.

- Programs for science teacher education and professional development should aim at the development of science teacher knowledge in relation to the practice of teaching or learning to teach science

**Recommendations for Future Research**

- The relationship between SMK and PCK needs more attention, preferably in association with pedagogical knowledge
• Need for both qualitative and quantitative studies that relate science teacher knowledge to student learning

• Such studies should use a combination of instruments that capture both what is in a teacher’s mind and how teachers enact their understanding in their classrooms

• Need for research to be done on the role and expertise of the people who play a pivotal role in promoting the development of science teacher knowledge, science teacher educators and facilitators

More studies needed to:
• Document the expertise of science teacher educators and facilitators
• Investigate the interaction of science teacher educators and facilitators with the participants in their programs taking into account contextual aspects of these programs
• Increase the understanding of the knowledge development of preservice and inservice science teachers

References
The Nature of Interactions of the Components of Topic Specific Pedagogical Content Knowledge

Elizabeth Mavhunga
Marang Centre for Mathematics Science and Technology Education Research, Wits University, South Africa.
Elizabeth.Mavhunga@wits.ac.za

This study builds on earlier work done on Topic Specific Pedagogical Content Knowledge (TSPCK) which is defined as the knowledge from which transformation of the content of specific topics for teaching purposes emerges. TSPCK is considered to be Pedagogical Content Knowledge (PCK) at a topic level. The study investigates the natural interactions between components of TSPCK and identifies the teaching contexts in which they emerge. Such understanding would enable the teacher development community to increase opportunities for development of TSPCK in pre-service teacher education. The study was conducted as a comparison of two separate studies located in the physical science methodology courses with pre-service teachers. Both studies aimed at developing TSPCK in the topic of chemical equilibrium however through different means. In the one case, the development was through exposure to an explicit intervention on how to transform content for purposes of teaching and the other through transfer of learned competencies in pedagogical transformation in an intervention with a different topic, ‘particulate nature of matter’. The study followed a qualitative research approach. Primary data collected in the individual studies included a completed TSPCK instrument, a CoRe and responses to an open ended vignette on Chemical Equilibrium. Findings indicate that both studies reflected moments where the different components of TSPCK were interacting explicitly which we called ‘TSPCK episodes’. The interaction appeared mostly to be that of components of curricular saliency, representations and learner prior knowledge. The quality of the interactions as measured by the number of the components interacting was richer in the study with an explicit intervention. Teacher tasks requiring conceptualization of teaching strategies and related tasks served as platforms onto which sophisticated interactions emerged. Recommendations for implementation of TSPCK in pre-service programmes are made.

Introduction

Learning to teach is much more than learning the content knowledge of the specific subject of specialisation. In the last three decades, there has been an increased realization of the need to pay attention to how teachers need to understand the subjects they teach (Ball, Thames, & Phelps, 2008) and therefore the subject they are learning to teach in pre-service programmes (Nilsson, 2008). Ever since the major breakthrough by Lee Shulman (1986) in conceptualizing a special domain of teacher knowledge called Pedagogical Content Knowledge (PCK), the research community has been attracted to PCK in a variety of ways. As the interest grew, more models defining aspects of the nature of the construct infiltrated the literature as outlined in the reviews by Kind (2009) and more recently (Jing-Jing, 2014). According to these authors the bulk of the studies on PCK in the last twenty-five years are responding to the challenge of identifying and understanding the components of PCK. The PCK components however, vary from one model to another. Furthermore there seems to be a shift from a topic focus (Grossman, 1990) to a broader, general perspective of the science subject. Such a shift brings the risk of losing the very aspect that made PCK attractive, its
direct attention to ways of understanding the content knowledge from pedagogical perspectives (Ball et al., 2008 p.1). In a recent initiative the science research community reached consensus for the need to refine the understanding of PCK to refer mainly to PCK within a topic and called this construct ‘Topic Specific Professional Knowledge’ (Gess-Newsome & Carlson, 2013). This construct is similar in conceptualization to a construct that we have previously called Topic Specific PCK (TSPCK) (Mavhunga & Rollnick, 2013). The significance of this agreement is that it acknowledges the topic specific nature of PCK as well as places primary focus on ways of understanding content knowledge for purposes of teaching.

The purpose of this study is thus to develop an understanding of the nature of the interactions between the components TSPCK and the contexts in which they naturally emerge. Such an understanding would enable the teacher development community to increase opportunities for development of TSPCK (Nilsson, 2008). While similar research exists exploring interactions of PCK components in a model such as that of Park and Chen (2012), none have been done with a PCK model that has a primary emphasis on topic specific knowledge in its epistemological design. The questions asked in this study are: (i) What is the nature of the interactions between the components of TSPCK and (ii) What is the nature of the contexts in which the interactions naturally emerge?

**Background**

**An understanding of PCK from a content knowledge perspective**

“What happened to the content?” (Shulman, 1986, p5). This is the radical question asked by Shulman in 1986, expressing unhappiness about the general direction of research at the time. This question marked a turning point in science education research studies, from an exclusive focus on general aspects of teaching such as classroom management back to the teaching of content knowledge. Shulman argued for a new perspective on content knowledge that considers it as the “technical knowledge key to the profession of teaching” (Ball et al., 2008, p.3). He argued that when content knowledge is to be taught, it first needs to be understood then the s, “comprehended ideas must be transformed in some manner if they are to be taught” (Shulman, 1987, p. 16).

In mathematics, Shulman’s statement prompted research that asked specifically “What do teachers need to know and be able to do in order to teach effectively?” (Ball et al., 2008, p 394). Based on the analysis of work mathematics teachers do, a response to the question pointed to a set of specific tasks described as ‘mathematical knowledge needed to carry out the work of teaching mathematics” pg. 395. What surfaced from Ball and colleagues’ work was the evidence that teaching may require a ‘specialized form of pure subject matter’. Ball et al.’s argument for referring to such knowledge as ‘specialized’ includes the fact that the knowledge demonstrated by teachers in explaining operations was beyond mere algorithms commonly known to all professions using mathematics. Her work in this line of thought has gone further in identifying what such knowledge in mathematics may contain: e.g. error analysis; mathematical reasoning; etc., all referring to the awareness of the teacher’s content knowledge in relation to teaching.

In science education, following an empirical study of science teachers, Geddis and Wood (1997) argued that as a consequence of focusing on teaching as transformation of content knowledge, a variety of different kinds of knowledge are observed from which subject matter transformations emerge (p. 612). The different kinds of knowledge include five components: (i) students’ prior knowledge, including the preconceptions about a topic; (ii) what are the
most important concepts in the topic, their sequence and knowledge needed prior to teaching the topic – all these called ‘curricular saliency’;(iii) what makes the topic easy or difficult to understand; (iv) representations and (v) effective conceptual teaching strategies. While Geddis and Wood (1997) did not give an overall term or title for these different kinds of knowledge, the authors clearly distinguished between these from content knowledge, as well as emphasized their collective pedagogical transformative powers on content knowledge. Both assertions from the mathematical and science perspectives are concerned with specific ways of understanding content knowledge in preparation for teaching. From the science perspective, the five knowledge components by Geddis and Wood (1997) are located in the deeper understanding of the concepts of a specific topic. Responses from their considerations would yield different versions of knowledge of content specific to a given topic. The results of the successful transformation of a given topic using the amalgam would display evidence of PCK within that topic and a further advantage of improving the understanding of the content knowledge of the topic itself (Mavhunga, 2014). PCK defined in this manner has a primary focus on the ways of understanding content knowledge from a perspective of the profession of teaching, hence the similar move to mathematics to call this knowledge ‘Topic Specific Professional Knowledge’.

Understanding the interactions between the components of Topic Specific PCK

As discussed above, PCK defined from a perspective of content knowledge has constituent components that are different from models with a broader subject perspective. It is therefore of practical significance to clarify the components and understand the nature of their interactions. Previous research to understand the integration of PCK components has been conducted largely through several approaches. The first approach explores how a single component affects another component. For example, Cohen and Yarden’s research (2009) indicated that teachers’ lack of curricular knowledge of the topic of cells limited their use of instructional strategies. The other approach examines how a particular component is related to the whole construct of PCK and practice. For example, the studies conducted by Clermont, Krajcik, and Borko (1993). Another approach is the exploration of all the components in a model as in the study by Park and Chen (2012). The study used a pentagon model (Park & Oliver, 2008) comprised of five components: (a) Orientations toward Teaching Science (OTS), (b) Knowledge of Students’ Understanding in Science (KSU), (c) Knowledge of Science Curriculum (KSC), (d) Knowledge of Instructional Strategies and representations (KISR), and (e) Knowledge of Assessment of Science Learning (KAs). The study found that the Knowledge of Student Understanding (KSU) and Knowledge of Instructional Strategies and Representations (KISR) were the most frequently integrated with any other component. Also that the component of Knowledge of Science Curriculum (KSC) had a limited link with any other component. Different to the studies above is that in this study the TSPCK model is defined at a topic level of the PCK taxonomy (Veal & MaKinster, 1999) thus the constituent components are different to those in the listed studies which has generic science subject focus. In addition, this study seeks to capture the teaching contexts in which the interactions are naturally observed.

Method

The study took a format of a comparison of two separate studies located in the physical science methodology courses with chemistry pre-service teachers. Both studies had a common purpose of improving TSPCK in chemical equilibrium through learning of pedagogical transformation. In the first study, called study A for simplicity, the transformation of concepts of chemical equilibrium was learned explicitly in class as an
intervention (Mavhunga and Rollnick, 2013). In the second study, called study B, transformation of chemical equilibrium learned as a transfer of competencies learnt in an intervention that used the topic of particulate nature of matter (Mavhunga, 2014). Both these studies employed mixed methods research strategies. The background to the two studies and the method used in this comparison is provided below.

**Background to the sample**

Study A consisted of 16 pre-service teachers in their final year of study towards a teacher qualification - the BEd degree, with Physical Science as their major subject. Study B consisted of 38 chemistry pre-service teachers in their third (3) year of study towards the same qualification. This class was a mix of pre-service teachers majoring in physical science and non-continuing sub-majors majoring in mathematics. While the two classes differ by academic level and composition, they are however regarded as appropriate cases providing different scenarios where the interaction of the components could be studied. The background of the pre-service teachers is provided in Table 1. The nature of the treatment in both studies was similar other than the difference in the topic used in the intervention. The treatment consisted of an intervention that explicitly taught the five components of TSPCK reported to effect transformation of content knowledge as outlined above. In study A, the topic of chemical equilibrium was used as the topic discussed in the intervention explicitly, while the topic ‘Particulate nature of matter’ was used in the intervention of study B. In both cases the choice of the topics for the intervention were informed by familiarity as a result of a previous exposure in the content courses.

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample size</th>
<th>Nature of Treatment</th>
<th>Topic of treatment</th>
<th>Topic for transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16</td>
<td>Explicit discussion of components of TSPCK</td>
<td>Chemical Equilibrium</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>36</td>
<td>Explicit discussion of components of TSPCK</td>
<td>Particulate nature of matter</td>
<td>Chemical Equilibrium</td>
</tr>
</tbody>
</table>

In both cases the intervention happened over a period of six weeks, with three 50 minute sessions a week. The format of the sessions followed a similar structure where a specific TSPCK knowledge component is introduced, its meaning discussed and illustrated in the concepts of the topic. For example, the component of representations was explained to include the three levels of representations used in explaining phenomena in chemistry.

**Research Strategies for the comparison**

The research method used in the comparison was qualitative in line with the purpose of understanding the nature of the interactions of the components of a tacit construct such as TSPCK. The data consisted of the primary data from the two studies treated as two similar sources but with a different context. From study A, data used comprised of a combination of completed validated TSPCK instrument on chemical equilibrium (Mavhunga, 2012) as well as a Content Representation (CoRe) (Loughran, 2007) on the same topic. From study B, data
also consisted of completed TSPCK instrument on chemical equilibrium and a vignette of open ended teacher tasks on teaching chemical equilibrium. While the CoRe in study A and the vignette in study B provided rich qualitative descriptions the responses in the completed TSPCK instruments provided complementary data in both cases.

The structure of the tools used in the studies above to capture TSPCK have been described extensively in the respective publications and therefore not repeated here, however it is important to note that the TSPCK instruments designed to measure the construct were based on the five components of TSPCK, where each of the components is regarded as a test item with one or few sub-questions. The CoRe on chemical equilibrium consisted of prompts that have been adapted to elicit thinking and responses with respect to the five components of TSPCK. An extract of the adapted CoRe used in study A is show Figure 1 below. Each of the columns is used for listing the Big Idea for teaching the topic.

<table>
<thead>
<tr>
<th>Curriculum Saliency</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>What are the big ideas for the topic?</td>
<td></td>
</tr>
<tr>
<td>What do you intend the learners to know about this idea?</td>
<td></td>
</tr>
<tr>
<td>Why is it important for learners to know this big idea?</td>
<td></td>
</tr>
<tr>
<td>What concepts needs to be taught before teaching this big idea?</td>
<td></td>
</tr>
<tr>
<td>What else do you know about this idea (that you do not intend learners to know yet)?</td>
<td></td>
</tr>
<tr>
<td>What do you consider easy or difficult in teaching this big idea?</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 1.** An extract of the adapted CoRe used in study A

The vignette used in study B comprised two teacher tasks prompting natural open ended responses that reflect engagement and transformation of the topic through the five TSPCK knowledge components in thinking about its teaching. Similar to a study by Brovelli, Bölsterli, Rehm, & Wilhelm (2013), the first major task required the pre-service teachers to design an evaluation schedule and use it to evaluate a recorded lesson on chemical equilibrium looking for evidence of TSPCK and making suggestions for improvement where found lacking. The video recorded lessons were extracts from a lesson where a teacher introduced the concept of dynamic chemical equilibrium. Pre-service teachers were requested to capture their evaluations using ‘extracted episodes of evidence’ from the recorded lesson and accordingly provide reasons for their evaluations. Responses from this task shed light on whether pre-service teachers could recognize practical moments where TSPCK is displayed in the topic, and whether they could suggest improvements.

The second major task required the written identification of the big ideas of the topic of the recorded lesson along the lines of a Content Representation (CoRe) (Loughran, Berry, & Mulhall, 2004). The task further contained two sub-tasks, requiring pre-service teachers to map back the content of the recorded lesson to the listed big ideas and suggest, in a form of a lesson plan, the pedagogical transformation of the CK of the lesson which could conceptually and logically succeed the recorded lesson, see Figure 2 below.
For the comparison, a total of six top performing pre-service teachers were identified, three from each study. The reason was to choose pre-service teachers who are likely to demonstrate many opportunities for applying TSPCK so as to increase the chances of TSPCK Episodes from which the interactions could be studied. The analysis, similar to Park and Chen (2012) took a form of an In-Depth Analysis of explicit PCK. In this approach, two or more components of TSPCK demonstrated in a specific teacher task segment, are first identified and labelled as a ‘TSPCK Episode’ (pg. 928). Unlike in the study of the authors with practicing biology teachers where episodes were taken from classroom observations, the episodes in this study were lifted from pre-service teachers’ written work. Each identified TSPCK Episode was then described in detail in terms of (i) what the pre-service teacher had written, (ii) what components of PCK at play in the PCK Episode were, and (iii) the context in which the presence of the components emerged (referred to hereafter as ‘the platform’). Thus the focus of the analysis of the ‘TSPCK Episode’ was on the nature of the interactions of the components and the nature of the teacher tasks within which the display happened. The presence and connections between components found in a TSPCK Episode are indicated in the analytic device called a ‘TSPCK Map’, adapted from Park and Chen (2012). According to the authors a TSPCK Map describes connections between two or more components used in defining TSPCK. For purposes of simplicity, the components of TSPCK have been abbreviated as follows: Learner Prior Knowledge referring to common misconceptions = LP; Curricular Salience = CS; what is difficult to understand = WD; Representations = RP and Conceptual teaching strategies as CTS. Figure 4 below provides an example of a TSPCK Map that could represent a TSPCK Episode. A detailed description is given below to illustrate how the TSPCK maps were constructed.

![Figure 4. A Typical TSPCK MAP](image)

In this map, there are three components of TSPCK: curricular saliency (CS), Learner Prior Knowledge (LP) and Representations (RP). The lines between the components represent evidence of a link between the components. The direction of the arrows indicate the sequence in which components appear in a TSPCK episode. The platform at the base of the
identified components indicates the type of the teacher task in which the TSPCK Episode emerged. The TSPCK Episodes identified in study A and B, respectively are described below.

**Results**

Table 3 shows the quantity of TSPCK episodes found in the six pre-service teachers, three from study A and B, respectively. As mentioned earlier, pre-service teachers in both studies were engaged in a teacher planning task on chemical equilibrium. In study A, they learnt to transform concepts in the topic of chemical equilibrium explicitly in an intervention, while in study B a different cohort was engaged with chemical equilibrium as a result of transfer of transformation competencies learnt in an intervention with a different topic - particulate nature of matter.

**Table 3. A summary of TSPCK Episodes in study A and B, respectively**

<table>
<thead>
<tr>
<th>Study</th>
<th>Participant</th>
<th>Number of “TSPCK Episodes”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study A:</td>
<td>Nduna</td>
<td>3</td>
</tr>
<tr>
<td>Transformation of chemical equilibrium learnt explicitly</td>
<td>Khanya</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Sipho</td>
<td>4</td>
</tr>
<tr>
<td>Study B:</td>
<td>Lebo</td>
<td>3</td>
</tr>
<tr>
<td>Transformation of chemical equilibrium acquired as a transfer from learning in a different topic</td>
<td>Gladys</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Mimi</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3 shows that the pre-service teachers in the sample had 3-4 TSPCK Episodes in their written work, with slightly more episodes were identified in the explicit intervention based study, study A. The identified TSPCK Episodes are analysed closely below starting with those in Study A.

**TSPCK Episodes in study A**

An extract of one of the TSPCK Episodes of pre-service teacher Nduna is presented in Figure 5 below.
Figure 5. Nduna’s TSPCK Episode and a corresponding TSCK Map

The extract presents a suggestion for a conceptual teaching strategy. In the extract there is evidence of three components of TSPCK. The first identified is the component of representations, where conducting an experiment is suggested as a start. According to Gabel (1990) an experiment falls in the macroscopic level of representations. The experiment is about reversible reactions, an aspect that needs to be understood as a pre-concept to dynamic chemical equilibrium. The understanding of what pre-concepts are needed prior to teaching a particular concept is an aspect of curricular saliency, which among other things also requires identification of concepts that are most important in a topic (Geddis and Wood, 1997). In the same vein the pre-service teacher emphasizes what the concept of chemical equilibrium means by showing what it does not mean, thereby addressing a common misconception: “..colour observed just tells us about the proportions of the reactants to the products but does not necessarily mean equal concentrations”. The explicit emphasis in the same sentence of the meaning not to refer to equal concentration is evidence of an understanding of common misconception about the topic, an indicator of the presence of the component of learner prior knowledge. The presence of the two components (CS) and (LP) in the same sentence signals an interwoven interaction of the components, different to how reference was to the component of representations (RP) at the beginning of the TSPCK Episode which appeared to refer to a sequence. In the extract above, the description of the suggested strategy show a return to a reference to representations with an intention to secure learner understanding. The TSPCK Map constructed for the identified TSPCK Episode reflects the use of the TSPCK components as having a combination of both a sequence and an interwoven nature. It is further noted that the display of the TSPCK components happened within a teacher task that required suggestion of a conceptual teaching strategy, thereby serving as a context in which the interplay is displayed.

In another case, a TSPCK episode is captured in a context where pre-service teacher Khaya was providing a summary of the most important content knowledge in a lesson is shown in Figure 6 below.

Figure 6. Khanya’s TSPCK Episode and a corresponding TSCK Map

The extract contains a TSPCK Episode where several most important concepts of chemical equilibrium are listed. The important conditions for the occurrence of the state of equilibrium is explicitly described “Important condition for the existence of an equilibrium system is that the change which occurs must be reversible”. Such a description indicates knowledge of curricular saliency about the topic. It is further observed that in the same sentence gate keeping concept that is regarded as difficult to understand is identified as the simultaneous reversibility of the forward and reverse reactions. The mentioning of phrase ‘same rate’ signals an additional understanding by the pre-service teacher of what is most difficult to
understand (WD) by learners. Her summary further refers to the use of representations to illustrate the identified gate keeping concept. In addition, the reference specifies explicitly what feature of the representation displays the gate keeping concept, that is, “The reversibility of a change is represented by a double arrow”. In this TSPCK Episode the component of curricular saliency (CS) identified as the first starting component appear to have been interwoven with that of what is difficult to understand (WD), which is further interwoven with that of representations (RP). The reason is mainly based on the observation that the evidence of their existence is simultaneously seen in the same sentences in an effort to explicitly link each other. The observed interwoven nature of the TSPCK components is seen within a context of a teacher task that requires a summary of the most important aspects of content knowledge in a lesson. This confirms a reciprocal kind of a relationship between content knowledge and TSPCK (Mavhunga, 2014). Few more cases of TSPCK Episodes demonstrating an interwoven nature of the TSPCK components were identified for pre-service teacher Khanya, see another TSPCK Map in Figure 7

Clarity of terms: system, reactant vs. reagent, etc.
In open systems: reactions go to completion, one directional reaction

$$\text{H}_2\text{O}(l) \rightarrow \text{H}_2\text{O}(g)$$

In a closed system: evaporation occurred at the same time as condensation, the reaction is said to be reversible, represented by the double arrow:

$$\text{H}_2\text{O}(l) \rightleftharpoons \text{H}_2\text{O}(g)$$

Water $\leftrightarrow$ water vapour
Evaporation $\leftrightarrow$ condensation
Forward change $\leftrightarrow$ reverse change

There is a stage when the evaporation and condensation processes happen at the same rate [note: this is shown the volume of the liquid water seemed not to change anymore].

State of equilibrium is then reached in the reaction. [explain the dynamic nature of that equilibrium, in that the state of balance in which opposing processes, evaporation and condensation, occur at the same rate]

Then explain what happens at equilibrium using graphs of concentrations vs. time and the rate vs. time. This is to show that rates and not concentrations are equal.

Figure 7. Khanya’s richer TSCK Map

In Figure 7 the TSPCK Episode indicates the presence of curricular saliency (CS), recognition of what is difficult to understand (WD) and representations (RP) all present in the opening sentence “In a closed system: evaporation occurred at the same time as the
condensation, the reaction is said to be reversible, represented by the double arrow”. The extract demonstrates understanding of one of the prior concepts needed in teaching chemical equilibrium, namely, closed systems. Such a recognition is evidence of the knowledge of curricular saliency. The explicit reference to the forward and reverse reactions happening at the same time is recognition of a concept commonly found difficult to understand. The reference to a representation explicitly points to what is relevant in the representation. Another interweaving interplay is seen in the last two sentences where the use of representations is suggested for a purpose of dealing with a potential common learner misconception.

Similar TSPCK Maps to those above were constructed for pre-service teacher Sipho, but of particular interest were TSPCK Episodes that contained the fifth component of TSPCK, Conceptual teaching strategy (CTS) as it is generally considered more difficult as it subsumes the other components (Mavhunga and Rollnick, 2013). The extract in Figure 8 below reflects a TSPCK Episode in a context of teacher task providing a brief outline of the teaching strategy covering lesson intentions.

![Figure 8](image)

**Figure 8.** Sipho’s TSPCK Episode and a corresponding TSCK Map

An interesting observation is that the suggested conceptual teaching strategy is influenced by knowledge of a common learner misconception that the Le Chatelier’s principle is applicable in all situations. However, unlike in the previous TSPCK Episodes discussed, the misconception of concern is not explicitly mentioned in the extract but is implied. It is thus represented in the TSPCK Map as a dotted line. A similar observation was noted in one of the TSPCK Episodes of pre-service teacher Khanya, also addressing the same common misconception about the Le Chatelier’s principle (Figure 9 below). Her TSPCK Episode was located in a slightly different teacher task where a response to a misconception by a learner is sought from the pre-service teacher.
The analysis of the data collected from study B, a case where the competency to transform content knowledge in chemical equilibrium was acquired through transfer, showed slightly fewer TSPCK Episodes than in study A. Interestingly it is further noticed that the episodes mainly involve the curricular saliency (CS), representations (RP) and what is considered difficult to understand (WD) in a CS-RP/WD sequence, where a dash CS-RP means a sequence i.e. CS first then RP and a slash RP/WD means an interwoven interaction where simultaneity is suggested. The extracts were taken from tasks about developing a teaching strategy for the topic of chemical equilibrium. Figure 10 presents an identified TSPCK Episode extracted from pre-service teacher Lebo’s written work.

'I will start the lesson by recapping with learners what the concepts of endothermic and exothermic reactions mean and how they are illustrated in a chemical equation using change of enthalpy. I will then explain Le Châtelier’s principle and write on the board an example an equation at equilibrium and explain the reverse and the forward reaction as I write. I will do an experiment of the NO₂/N₂O₄ system illustrating effect of temperature at equilibrium. Following a discussion on the experiment in a manner that involves learners, I will draw on the board two microscopic representations of the NO₂/N₂O₄ molecules at equilibrium at different temperatures, linking to the symbolic representation in a form of chemical equation. I will play around (in a form of questions) with the concepts of endothermic and exothermic at the condition of a changing temperature when the system is at equilibrium.'
A striking feature of the extract is that the teaching strategies considered are based on the content knowledge of the topic (Ball et al., 2008). There is regard for learners’ prior knowledge in ensuring that the concepts of endothermic and exothermic are understood prior to the discussion about the effect of temperature at equilibrium. This is also an aspect of curricular saliency which is about knowing what concepts need to be in place prior to the introduction of the intended concept. The suggested strategy also demonstrates consideration for what is potentially difficult for learners to understand (WD). Evidence of this component is seen through the recognized need to mention the dynamic nature of chemical equilibrium in terms of the reversibility of the reactions as the representations are being drawn on the board. The explicit description of the timing of these actions reflects a sophisticated use of the components of TSPCK. Furthermore the simultaneous use of representations at different levels, in particular sub-microscopic representations, is a crucial element of explanations in chemistry (Gabel, 1990). There is further indication of recognition by the pre-service teacher to talk to the links between the representations. The issue of exothermic and endothermic reactions is repeated and re-emphasized at the end of the lessons, in the form of questions, with reference to the effect of temperature at equilibrium. This act of emphasizing what is core and important in an explanation is a crucial aspect of curricular saliency. Here we see consideration and pulling together of several TSPCK components interacting to create depth in a suggested teaching strategy (Park, 2011), making it uniquely rich for teaching. The constructed TSPCK Map is CS-RP/WD-CS as illustrated in Figure 10 above. In this case the episode reflects both a sequential and a simultaneous interplay.

Another evidence of the presence of the same components (CS, RP and WD) is seen in the extracted written work of pre-service teachers, Ntombi and Mimi in Figure 11 below.

![Figure 11. Ntombi’s and Mimi’s TSPCK Maps](image)

However, different in Ntombi’s TSPCK Episode is the presence of the component of Learner Prior Knowledge (LP) as she explicitly addresses a common learner misconception “the absence of the observable changes do not mean that the reactions have stopped”. This statement is made in the same discussion as the issue of reversibility which indicates awareness of the component of what is difficult to understand (WD), suggesting an interwoven existence of the component. The observed TSPCK Episode also emerged from a teacher task that is requires development of the conceptual teaching strategy.

The TSPCK Maps of the six pre-service teachers from studies A and B are summarized in Table 4 below.
Table 4. A summary of the nature of PCK Episodes in study A and B

<table>
<thead>
<tr>
<th>Study A: Transformation of chemical equilibrium learnt explicitly</th>
<th>Study B: Transformation of chemical equilibrium acquired through transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Participant</strong></td>
<td><strong>Number of “TSPCK Episodes”</strong></td>
</tr>
<tr>
<td>Nduna</td>
<td>3</td>
</tr>
<tr>
<td>Khanya</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Sipho</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Lebo</td>
<td>3</td>
</tr>
<tr>
<td>Gladys</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Discussion**

The first question explored in the study is the quest to understand the nature of the interactions between the components of TSPCK in chemical equilibrium. The findings indicate that in both the intervention based and the transfer studies, the interaction of the TSPCK components largely occurs across three components: the (i) curricular saliency; (ii) representations and (iii) learner prior knowledge or what is difficult to understand. Also the components appear to be largely interwoven with few cases where a sequence is observed. The interplay is such that both the interacting components and their links are explicitly identifiable, confirming the understanding in the literature that PCK is more than the sum of the individual components but also the extent in which they interact (e.g. Abell, 2008; Park and Chen, 2012). A unique observation made from study A is the evidence of two cases of interaction of the component of Conceptual Teaching Strategies (CTS) with other components. The observation is interesting as the CTS component is normally considered to subsume the other components of TSPCK in its nature. Of particular interest to note was that where the component of Conceptual teaching strategy (CTS) is explicitly present in a TSPCK episode, its interaction with the component of learner prior knowledge is implicit. The presence of the learner prior knowledge, in particular a misconception, is recognized only by the way the provided conceptual teaching strategy is constructed. The structuring of the
conceptual teaching strategy naturally display knowledge of CK in ways that could be interpreted as confronting a particular misconception. The implication is that one needs to recognize the targeted misconception in the TSPCK while it is not explicitly mentioned. According to Ball and colleagues (2008) such understanding about CK demonstrates specialized knowledge for teaching the topic important for effective teaching. That is understanding the potential cause of the errors/misconceptions and addressing it in teaching. Thus the findings confirm qualitatively that pre-service teachers can learn about pedagogical transformation of content knowledge of a specific chemistry topic and be able to transfer their learned competency for transformation of another topic using the TSPCK theoretical framework.

Furthermore, noting the higher quantity of TSPCK Episodes in the explicit intervention study (A) implies that while the explicit intervention places a heavy demand on teacher development programmes in terms of time, it however provide valuable exposure for chemistry pre-service teachers to engage in CK of the intervention in ways that increases opportunities for pedagogical transformation.

The second question examined in this study was the nature of the teaching context in which TSPCK Episodes are displayed. The platforms that enable the display of the interaction of the components appear to be teacher tasks where conceptual teaching strategies are being considered. Four salient teacher task were identified as contexts promoting TSPCK episodes: (i) recommendations for improving observed teaching strategies; (ii) response to learner misconception and (iii) teacher summary of most important knowledge in a lesson and (iv) sequencing of a lesson. The value of the identification of these teaching contexts and bringing them to the fore lies in influencing the choice of tasks used in the development of teaching knowledge for specific topics

**Conclusion**

The established understanding of the interwoven nature of the interaction of the components of TSPCK, confirms the complexity of teacher knowledge needed for teaching core chemistry topics. The understanding assist in channelling teacher development efforts towards understanding content knowledge from a sophisticated pedagogical perspective. It is further encouraging to observe the presence of pedagogical perspective in the TSPCK Episodes acquired through ‘self-engagement’ by pre-service teachers as they transferred pedagogical transformation competencies learned from a different topic. The observed transfer opens possibilities for incremental acquisition of the sophisticated topic specific content knowledge in core topics of a discipline. However, caution is advised concerning the limitations in the transfer case as the frequency and the degree of complexity of the interactions is richly developed in the intervention based case, where multiple sets of interplays are seen in a single ‘TSPCK Episode’ (e.g. Figure 5, in Khanya TSPCK Episode). Thus, the understanding of the teaching contexts that serve as platforms for the display of the interactions is critical for regard as one of the enablers in the matrix for the development of topic specific professional knowledge. It is therefore recommended that the implementation of PCK in teacher education programmes, take into consideration the topic specific content knowledge and the awareness of the pedagogical platforms which promote its emergence.

This project was kindly funded by Sasol Inzalo, Wits University and the NRF Thuthuka fund.
References


‘We use guided inquiry and open discovery in our lessons’: Investigating the Extent to Which this is True in the Practice of In-service Science Teachers in Malawi

Dorothy Cynthia Nampota
Department of Curriculum and Teaching Studies, University of Malawi, Malawi
dnampota@cc.ac.mw

Inquiry based instruction has been at the centre for debate and research in science education over the past decades. This is why in her attempts to improve the quality of secondary science education, the Malawi government institutionalised a teacher in-service training programme called Strengthening of Science and Mathematics in Secondary Education (SMASSE). The SMASSE programme centres on Activity, Student centred, Experimentation and Improvisation (ASEI) principles which relate very closely with inquiry based instruction. The study reported in this paper had two purposes; the first was to explore the pedagogical perceptions of in-service science teachers for given lesson contexts and the second was to explore the extent to which teacher practices follow the ASEI and therefore inquiry based instruction techniques after having attended the SMASSE trainings. A quantitative methodology employing questionnaires and lesson observation schedules was used in the study. The Pedagogy of Science Inquiry Teaching Test (POSITT) questionnaire (developed at Western Michigan University by Schuster and Cobern (2011) and a lesson observation schedule for SMASSE programme was used to collect data from 514 science teachers and 18 lessons respectively. The findings show that while teacher perceptions were that they would use guided inquiry and open discovery pedagogy in their teaching of various lesson vignettes, they were given, their practices remained didactic. The implications of the findings on both pre-service and in-service training are discussed. Implications on teacher monitoring of teaching are also discussed.

Introduction

Malawi has for many years faced myriad challenges with regards to student participation and performance in science and mathematics at levels of education. It is understood that the reasons for such a situation include insufficient number of qualified teachers; lack of laboratories, equipment and chemicals; attitudes of both teachers and students towards these subjects; and, poor teaching methodologies (Mbano, 2003, Nampota, 2010). On the part of teaching methodologies, didactic teaching has remained the pedagogical mainstay of many classrooms in Malawi (Government of Malawi, 2003). Teacher practices have been characterised by teacher “chalk and talk” even in situations where the teachers are qualified and there are laboratories and chemicals for experiments. Although not unique to Malawi, such practices have been associated with rote learning and do not result in proper comprehension of concepts.

In an attempt to improve quality of secondary science and mathematics education, the Government of Malawi has implemented a number of initiatives including construction of additional laboratories and training of teachers. One notable initiative with teacher training is the implementation of an in-service teacher training programme popularly known as Strengthening of Science and Mathematics in Secondary Education (SMASSE). With support
from the Japanese Government, the programme was first piloted from 2004 to 2008 in one of the six education divisions in the country (South East Education Division) and later rolled out nationwide in a phased approach. The programme has been institutionalised within the Ministry of Education and trains all secondary school mathematics and science teachers at least once a year. The focus of the SMASSE programme is to provide in-service training to science and mathematics teachers to enhance content knowledge (especially for the under-qualified teachers) and to improve their pedagogy by making it more student-centred. SMASSE focuses on four pedagogical principles of “Activity, Student-centred, Experiment and Improvisation” (ASEI). Teachers are encouraged to use activities in their lessons, ensure that the lessons are students centred, use a variety of resources including those that are locally found, and where applicable conduct experiments. As a way to encourage teachers to continuously seek improvement for their pedagogy, SMASSE also introduces four additional principles of Plan, Do, See and Improve (PDSI) to the teachers. Through these principles, teachers are encouraged to ensure that they have planned for the lessons adequately (Plan), teach the lesson (Do), and make observations in the progress of the lesson (See) to identify areas for improvement both within the lesson and in subsequent lessons as a reflective practitioner (Improve). To this effect both the ASEI and PDSI principles form the core of SMASSE in-service trainings.

A review of literature shows that there is a close relationship between the ASEI principles, which hinge on teacher practices in a lesson, to inquiry based instruction which has been a focus for debate and research worldwide in science education. Inquiry based instruction has its roots in the cognitive theories advocated by psychologists such as Jean Piaget and Lev Vygotsky. Piaget’s theory of cognitive development had two strands: stage-wise development and construction. While stage-wise development entailed that children’s thinking changes quantitatively as a child progresses from one stage to another, construction meant that children construct their own meanings or understanding through interaction with social and physical environment. The construction perspective to learning has been the subject for debate and research in science education since the 1980’s (Champagne, Gunstone, & Klopfer 1983; Driver, 1989). Children are believed to be active learners who construct knowledge and understanding as they interact with the physical and social worlds and use language (Piaget, 1970; Ausubel et al, 1978). As a result, teaching is expected to involve provision of necessary experiences for the students to construct meanings. Similarly Vygotsky (1978) distinguished scientific and spontaneous or everyday concepts. While spontaneous concepts were said to consist of everyday and frequently used knowledge, scientific concepts were seen to consist of formalised knowledge, which was learnt from more able individuals in formal situations such as school. Vygotsky saw teaching as involving two stages, exposition to the formal knowledge on the one hand and practice in using that knowledge to make it everyday knowledge on the other. In the 1990s, when there was much research on pupils’ understanding of science concepts and teaching strategies to bring about conceptual change, most of the proposed strategies adopted Vygotsyian ideas in some form and included elicitation and exploration or clarification of previous knowledge, introduction of new ideas, practice using new ideas and reflection on changes in understanding (Driver, 1989). These constructivist teaching strategies emphasised social interactions in form of small group work.

The link between constructivist teaching strategies and inquiry based instruction can best be explained by the inquiry based instruction framework developed by Minner, Lvy and Century (2009). In the framework inquiry based instruction has three key aspects: (1) the presence of science content (2) student engagement with science content, and (3) student responsibility for learning, student active thinking, or student motivation within at least one component of
instruction – question, design, data, conclusion or communication. While the science content refers to all science subjects that could be taught, student engagement includes all the various ways in which students could interact with phenomena. Students responsibility for learning is more in line with constructivism where the learner is expected to ‘participate in making decisions about how and what to learn, identify where they and others need help in the learning process and ask for that help and, or contribute to the advancement of group knowledge’(p.5). Students active thinking involves students using logic, thinking creatively, building on prior knowledge and /or making deductions and student motivation concerns instruction that builds on and develops students’ curiosity, enthusiasm and concentration.

Two of the three components of inquiry based instructions augur well with the SMASSE ASEI principles – student engagement with science content and student responsibility, active thinking and motivation. Table 1 shows the relationship between indicators for ASEI principles of SMASSE and inquiry based instruction.

Table 1. The relationship between ASEI and inquiry based instruction.

<table>
<thead>
<tr>
<th>SMASSE Principle</th>
<th>SMASSE lesson Indicators</th>
<th>Inquiry based instruction aspects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity</td>
<td>Activities incorporated in lessons, students engaged in the activities, activities enhance student understanding and students draw relationships between activities and theories</td>
<td>Student engagement with science content</td>
</tr>
<tr>
<td></td>
<td>Students draw relationships between activities and theories</td>
<td>Students thinking</td>
</tr>
<tr>
<td></td>
<td>Activities arouse student interest</td>
<td>Student motivation</td>
</tr>
<tr>
<td>Student centred</td>
<td>Students do something to show to the class, students give prior experiences, students make own predictions, students discuss differences in own predictions, student verify predictions on their own, students present own theories, teacher summarises lesson, students evaluate lessons.</td>
<td>Students responsibility for learning</td>
</tr>
<tr>
<td></td>
<td>Students thinking</td>
<td>Students thinking</td>
</tr>
<tr>
<td>Experimentation</td>
<td>Experiments conducted, teacher relates experiments to theory</td>
<td>Student engagement with science content</td>
</tr>
<tr>
<td></td>
<td>Students deduce theories from experiments</td>
<td>Students thinking</td>
</tr>
<tr>
<td>Improvisation</td>
<td>Teacher uses improvised equipment and other locally available materials, teacher simplifies activities for efficient resource use.</td>
<td>Student engagement with science content</td>
</tr>
</tbody>
</table>

Knowing that SMASSE principles are in line with inquiry based instruction, and that most science teachers have gone through several SMASSE training sessions, it was of interest in this study to find out the extent to which science teachers in Malawi have inquiry based pedagogical orientations and whether or not these are practised in science lessons.

**Study purpose**

The purpose of the study was to find out the pedagogical perceptions and practices of in-service secondary school science teachers after several years of SMASSE trainings.
**Research questions**

What pedagogical perceptions do in-service secondary school science teachers have for teaching their subjects?

What are the practices of in-service science teachers when teaching their subjects?

What is the effect of gender on the practices of the teachers?

**Framework for assessing teacher pedagogy**

In order to assess teacher pedagogy, it is important to understand the various pedagogies available in science education and their epistemological orientations. Didactic pedagogy as implied in the foregoing discussion concerns the teacher taking centre stage in the instructional process and the student as a passive recipient. Open discovery pedagogy on the other extreme end is where the students are given full autonomy to discover things the way they feel fit. The students identify own problems, investigate them using a methodology of their choice and find solutions. Lying in between are the two extreme pedagogies however is ‘direct interactive,’ and ‘guided inquiry’. While direct interactive is an improvement from direct didactic in that students are involved in some activity, guided inquiry is where the students are expected to investigate science under the guidance of the teacher. Figure 1 illustrates the continuum of pedagogical orientations.

Two epistemologies are reflected in the orientations. The ‘direct didactic’ and ‘direct interactive’ orientations emphasize an epistemological orientation towards science as a known product while ‘guided inquiry’ and ‘open discovery’ emphasize inquiry and consider science as always evolving. Direct didactic usually entails teacher-centred lessons, while direct interactive is an improvement towards student centred pedagogy. As such, student autonomy increases as teachers change their pedagogy progressively from direct didactic to open discovery. The degree of teacher control decreases as pedagogy moves towards open discovery. Considering the ASEI principles of SMASSE programme, the focus is on teacher use of guided inquiry although aspects of direct interactive and open discovery pedagogy cannot be ruled out. Inclusion of activities and experiments enhances student engagement with the content while the student centred principle enhances students’ responsibility for learning and therefore lying in between guided inquiry and open discovery. Open discovery is however an extreme pedagogy which may not suit junior students at secondary school level. As such, it needs to be complemented with guided inquiry and direct interactive pedagogy. The combination of these pedagogies in the science lessons in Malawi were of interest in this study.

![Figure 1. A continuum of teachers’ pedagogical orientations](image-url)
Methodology

The study reported in this paper is part of a bigger study that used both quantitative and qualitative research methods. While on the one hand a survey was necessary to get more widespread teacher pedagogical perceptions warranting use of quantitative methods, getting in-depth knowledge and explanations on the perceptions and practices of the teachers on the other hand required use of qualitative methods. The quantitative methods thus sought to explore the pedagogical perceptions of the teachers and their practices through use of questionnaires and classroom observation schedules respectively while Focus Group Discussions and individual face-to-face interviews were used to collect more in-depth qualitative data that were used to explain the quantitative findings. The findings reported in this paper however focus only on the quantitative part of the study that used questionnaires and classroom observation schedules.

The sample

In-service science teachers from government and grant aided secondary schools from all the six education divisions across the country participated in the study. Since not all districts and schools could participate in the study due to resource constraints, one district in each of the six education divisions was purposively selected. To do so, districts that had all the three types of government secondary schools: conventional, community day and grant aided were selected as summarised in Table 2.

Table 2. Sampled districts by education division.

<table>
<thead>
<tr>
<th>Education Division</th>
<th>Selected Districts</th>
<th>No. of schools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern</td>
<td>Nkhatabay</td>
<td>23*</td>
</tr>
<tr>
<td>Central East</td>
<td>Kasungu</td>
<td>25</td>
</tr>
<tr>
<td>Central West</td>
<td>Dedza</td>
<td>25</td>
</tr>
<tr>
<td>South East</td>
<td>Zomba rural</td>
<td>24*</td>
</tr>
<tr>
<td>South West</td>
<td>Blantyre urban</td>
<td>25</td>
</tr>
<tr>
<td>Shire Highlands</td>
<td>Chiradzulu</td>
<td>23*</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>145</td>
</tr>
</tbody>
</table>

*Total number of schools in the district is less than 25

In each of the selected districts, a maximum of 25 schools were randomly selected for the study. Since Nkhatabay, Zomba rural and Chiradzulu districts had less than 25 secondary schools, all schools in these districts participated. For the other three districts, the sampling was conducted as follows. Firstly, all grant aided secondary schools and all conventional secondary schools formed the sample\(^1\). However, since there were more Community Day Secondary Schools (CDSS)\(^2\) in the districts, a table of random numbers was used to sample the required number of community day secondary schools so as to reach the total target.

---

\(^1\) The districts have a maximum of three grant aided secondary schools (co-owned by churches) and a maximum of five conventional secondary schools (fully government owned). Both of the school types are usually boarding schools although a few conventional schools are day.

\(^2\) CDSS are government secondary schools that are co-owned with communities and usually have limited resources for teaching and learning. They are all day secondary schools.
number of 25 schools for each district. The total number of sampled schools for the study was 145.

To conduct classroom observations on teacher’s practices, the study team purposively sampled three schools from each of the six districts (one conventional, one community day and one grant aided from each district) giving a total of 18 schools.

Methods and instruments

Questionnaires

Teachers’ pedagogical perceptions were assessed using a questionnaire which was adapted from the Pedagogy of Science Inquiry Teaching Test (POSITT) questionnaire (developed at Western Michigan University by Schuster and Cobern (2011). The instrument is a scenario-based assessment tool, requiring respondents to apply their understanding of inquiry teaching to specific topic teaching vignettes, which represent typical lesson design and classroom situations that teachers encounter. The items are therefore posed not in generalities about inquiry but in terms of particular science topics in real teaching contexts. The scenarios presented in the items elicit responses through questions that are posed on choosing a preferred teaching approach, evaluations of a teacher’s actions so far, suggestions for what a teacher should do next, or ways of handling a question or classroom event. Suitable options are offered and these span a range of different approaches, each corresponding to a particular pedagogical orientation. The spectrums of pedagogical orientations that have been defined for POSITT include ‘direct didactic’ to ‘direct interactive’ to ‘guided inquiry’ to ‘open discovery’. An example of a scenario is given below.

Light reflection

Ms. Banda is teaching her Form 2 students the law of reflection: when a ray of light strikes a mirrored surface, it leaves at the same angle as when it arrived. Ms. Banda has to decide how she will teach the lesson.

Thinking about your own teaching, which of the following is the most similar way on how you would teach the lesson?

A. I would write the law of reflection on the board and illustrate with a diagram. Next I would show them a real example, using a light ray source, mirror, and protractor. Then we would discuss any questions the students might have.

B. I would first pose a question about reflection for the students to explore. The students could investigate using light ray sources, mirrors, and protractors, and then discuss their findings. I would close the lesson by giving them a summary of the law of reflection.

C. I would ask students to find out what they can say about light behaviour around mirrors by exploring on their own with an assortment of available items, including light ray sources, mirrors, and protractors. Then the students would report back on what they did and what they found out.

D. I would write the law of reflection on the board and illustrate with a diagram. Then I would have the students verify the law using light ray sources, mirrors, and protractors. We would then discuss their findings.
The questionnaires had three sections each containing four scenarios for Physical Science, and Biology. Teachers were instructed to respond only to the scenarios that reflected the subject they were teaching. In total, the questionnaire was administered to 514 science and in-service teachers in 145 schools in 6 districts.

Classroom observations
In order to collect data on teachers’ pedagogical practices in classrooms, lesson observations using a structured observation schedule were conducted. An observation schedule designed to measure how teachers use the SMASSE ASEI principles was used. A team of three researchers was trained in the use of the observation schedule until an inter-rater reliability of was reached. In total 26 classroom observations were conducted. The lessons were distributed as follows in the various kinds of secondary schools; nine, eleven and six for grant aided, conventional and CDSS, respectively. Twenty two of the teachers observed were male and four were female. It was planned that two lesson observations would be conducted per school for all the 18 sampled schools but due to logistical problems (for instance mid-term holidays), this was not possible. Seventeen of the teachers were qualified with MED/BED or DipEd. However, five had general degrees, two had general diploma and one had MSCE. Ten of the teachers had attended SMASSE trainings three times while five had attended four times, three attended twice, five attended once and two had not attended any.

Data analysis
The quantitative data from questionnaires and classroom observation schedules were analysed using SPSS software. The analysis was in form of descriptive statistics such as frequency counts, percentages, and cross-tabulations.

Findings of the study
Characteristics of the teachers
The findings from the analyses have shown that the majority (69%) of the teachers who completed the questionnaires came from the Community Day Secondary Schools (CDSS), with only 25 percent and 6 percent respectively coming from the conventional and grant aided secondary schools. Only 13 percent were female. 86 percent of the teachers had received SMASSE trainings while only 14 percent had not (see Figure 2)

![Percentage of teachers by number of INSET trainings](image)

**Figure 2.** Teachers and SMASSE trainings

---

1 A combination of Physics and Chemistry
The highest academic qualifications of the teachers are summarised in Table 3. In general, about 60 percent of the teachers could be considered to be qualified to teach at secondary school level with a diploma, degree or Masters in Education. The rest of the teachers were not qualified and 28 percent only had a Malawi School Certificate of Education (MSCE) which is a school leaving certificate.

**Table 3. Academic qualifications of the teachers**

<table>
<thead>
<tr>
<th>Academic qualification</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bed or MED</td>
<td>25</td>
</tr>
<tr>
<td>General degree</td>
<td>6</td>
</tr>
<tr>
<td>Diploma in Education</td>
<td>32</td>
</tr>
<tr>
<td>General diploma</td>
<td>4</td>
</tr>
<tr>
<td>MSCE</td>
<td>28</td>
</tr>
<tr>
<td>Other</td>
<td>2</td>
</tr>
</tbody>
</table>

Slightly over half (52 %) of the teachers had over 5 years of teaching experience while only 15 percent had less than 3 years teaching experience.

**Pedagogical perceptions of the teachers**

Figure 3 shows that on average the majority of the teachers perceptions of pedagogy are oriented more towards the more student centred guided inquiry (41%) and open discovery pedagogies (26%) with about a third (29%) orienting towards direct interactive and only 4 percent for the traditional direct didactic methods. This trend is reflected on the concepts of light reflection, stimuli and response and breathing mechanisms. On acid base indicators, the majority (40%) of the teachers opted for open discovery although an equally large percentage (35%) also opted for guided inquiry. However 58 percent of the teachers opted for guided inquiry on the concept of photosynthesis. Overall therefore, the figure shows that the teachers have moved from the traditional didactic methods to the more student centred pedagogy in their perceptions.
Table 4 shows variations in choice of pedagogy by gender. The general picture is that there are no major differences between male and female teachers in their pedagogical perceptions since the largest percentage of both male and female teachers had a tendency to opt for guided inquiry (42% males against 40% females) and the second was open discovery (25% males and 26% females) on average. However, a bigger difference is found on choice of direct interactive where 20% of male teachers and 24% of female teachers opted for it.

Table 4. Pedagogical perception by gender of the teachers

<table>
<thead>
<tr>
<th>Question</th>
<th>Direct Didactic</th>
<th>Direct Interactive</th>
<th>Guided inquiry</th>
<th>Open discovery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>F</td>
<td>M</td>
<td>F</td>
</tr>
<tr>
<td>Physical concepts science</td>
<td>5</td>
<td>2</td>
<td>23</td>
<td>32</td>
</tr>
<tr>
<td>Biology concepts</td>
<td>3</td>
<td>2</td>
<td>31</td>
<td>38</td>
</tr>
<tr>
<td>Average</td>
<td>5</td>
<td>4</td>
<td>20</td>
<td>24</td>
</tr>
</tbody>
</table>

**Practices of in-service teachers**

Several aspects of the lessons, in tandem with the SMASSE key principles, were assessed including use of student activities, student centredness, experimentation and improvisation using a classroom observation schedule. The findings are discussed under each of the principles. As mentioned earlier, 26 classroom observations were conducted with 22 being for male teachers.

**Incorporation of student activities**

An analysis of the lesson observations (see Figure 4) shows that half of the teachers incorporated activities in their lessons although only 41 percent ensured that the students are...
engaged in the activities, 39 percent ensured that the activities arouse student interest and 41 percent ensured that the activities enhance students’ understanding. Again less than half (40%) of the teachers helped the students to draw a clear relationship between the activities and the theories under study.

![Student activities

Figure 4. Incorporation of student activities in lessons

Figure 5 presents findings that reveal incorporation of student activities by gender. The figure shows that female teachers are consistently performing better, with close to two-thirds, in terms of incorporation of student activities and ensuring that the activities engage the learners to enhance understanding and drawing relationships between the activities and the concepts under study.

![Incorporation of student activities by gender

Figure 5. Incorporation of student activities by gender

Student-centredness

Figure 6 summarises the findings on student-centred characteristics of the lessons. On average, only about a third of the teachers used student-centred pedagogy in their lessons. In 46 percent of the lessons, students did something to show to the class; in 34 percent students gave their prior experiences; in 30 percent students presented own findings and the teacher summarised the lesson; in 20 percent students verified predictions on their own; in 19 percent students made own predictions and discussed differences in own predictions. This shows a
gradual decline in use of student-centred pedagogy from one step to the other meaning that there was minimal genuine student centred-ness in the lessons. The findings on student evaluation of the lessons were the worst with only 4 percent of the teachers allowing this to happen. It would appear from these findings that student-centeredness pedagogy as expected by the SMASSE project, which is itself central to inquiry (whether guided or open) has not yet been internalised by the teachers.

Figure 6. Student-centred lessons

An analysis of application of principles of student centred lessons by gender illustrated in Figure 7 shows that while female teachers have a greater tendency to incorporate activities in their lessons as discussed earlier, they still maintain a lot of control as only about a quarter (25%) allowed students to make their own ideas explicit and predict and explain their own theories. Female teachers never gave a chance to their students to evaluate the lesson and often summarised the lessons themselves without involving the students. Although half of the male teachers allowed the students to do something to show to the class, it is only in 36 percent of their lessons that students were given a chance to make their ideas explicit and in 32 percent students were allowed to present own observations.

Figure 7. Student centred activities by gender
Experimentation

Figure 8 shows that experiments were conducted in 39 percent of the lessons, in 23 percent teachers given the students a chance to relate the experiment clearly to theory and in 12 percent students were given a chance to deduce theories from the experiments. Again this contradicts teachers’ claims on use of guided inquiry and open discovery pedagogy as alluded to in their perceptions.

![Experiment Graph](image)

**Figure 8. Use of experiments**

Figure 9 however shows that while female teachers have a higher tendency to use experiments in their lessons (50% female compared to 36% male), none ensured that her students deduce theories or concepts from the experiments (compared to 14% male) although about a quarter were able to relate the experiment to some theory. This further shows greater female teacher control even in situations whether they appear to want to give students more autonomy through use of experiments.

![Use of experiments by gender](image)

**Figure 9. Experimentation by gender**
Improvisation

Figure 10 shows that use of improvised materials and utilization of locally available resources were observed in 19 percent and 41 percent of the lessons, respectively. Equally evident was simplification of activities for efficient resource use which was observed in 39 percent of the lessons.

Figure 10. Improvisation in lessons

Figure 11 shows that male teachers improvise more than female teachers (23% male and 0% female); utilise materials available in students’ immediate environment more than female teachers (50% male, 0% female) and simplified activities for efficient resource use more than female teachers (40% male, 25% female). It is possible that due to household chores after work, female teachers do not have enough time to develop improvised materials unlike their male counterparts. However this raises doubts on the kind of activities that female teachers prepare for their students.

Figure 11. Use of improvisation by gender
Conclusions

The findings of the study have shown that while teachers perceive to have gradually moved from the traditional didactic teaching methods where they exercise a lot of control with students given very little autonomy, to the more student centred pedagogy of guided inquiry and open discovery in theory, in their perceptions, in practice this does not seem to be the case. Student engagement with science content (Minner et al., 2009) through incorporation of student activities was evident only in half the lessons and only in slightly above a third were experiments conducted. With use of improvisation being limited to only about a fifth of the lessons in a country where equipment and chemicals are not commonplace like Malawi, one doubts the level of inquiry that the students are involved in during science lessons which is contrary to the perceptions of the teachers. Female teachers however display a greater tendency to include activities in their lessons in tandem with inquiry pedagogy. However these teachers give little autonomy to the students to use the activities including experiments conducted to generate own meanings and understandings.

Student responsibility for learning and student thinking as elements of inquiry pedagogy (Minner et al., 2009) seem to have suffered much more in the practice of the teachers. Overall, less than a third of the lessons were student centred. This means that in the majority of the lessons students were not given chancels for taking own responsibility such as making their prior ideas explicit and using them to make sense of the new learning, making predictions and generating own knowledge which they communicate to others including the teacher. In lessons where the teachers seem to exercise a lot of control, there is minimal student motivation, which is another aspect of inquiry teaching (Minner et al., 2009).

The findings of this study suggest that what teachers perceive to be their pedagogy is not what they do in practice. The teacher practices appear to be still on the direct didactic and direct interactive end of the pedagogy continuum with high teacher control and low student autonomy. Science is taught as knowledge that is static. Although large classes, lack of equipment and laboratories were given as reasons for their practices during FGDs, the fact remains that the teachers practices are resistant to change. Perhaps there is need for the teachers to receive an additional push to ensure that they practice what they know or are taught through in-service training. Teacher supervision of teaching, which is almost non-existent currently, should be enhanced at this level. Head teachers of the schools should not be given a teaching load but instead monitor the teaching practices of the teachers regularly. Secondary Education Methods Advisors should also do their supervisory role which they are not performing currently.

One possible reason for the mismatch between perceptions and practice could be teacher beliefs and attitudes. Previous research has shown that beliefs and attitudes sometimes prevent teachers from fully implementing what they have learnt. To this effect it might be useful to emphasise the SMASSE principles during pre-service teacher training so that the trainees capture these earlier before joining the teaching profession. Teacher trainers should thus be more vigilant in ensuring that their trainees acquire this knowledge and skills during pre-service teacher training.

References


292


Effects of High School Students’ Chemical Concept Understanding Level on Achievement in Kreb’s Cycle

Ikhifa Grace Onyenenu1 & Chukunoye Enunuwe Ochonogor2
Institute for Science and Technology Education (ISTE) University of South Africa, Pretoria
1Ikhifagraceo@yahoo.co.uk; 2Ochonec@unisa.ac.za

This study investigated the effects of student’s chemical concepts understanding level on their achievement in bio-chemical topic of Kreb’s cycle among high schools in Delta State of Nigeria. The design of the study was quasi experimental which involved the Non-Randomized Control-Group Pre-Test and Post-Test. The population of this study consist of all senior secondary (II) students in the twenty (25) local government areas of Delta State. Six secondary schools randomly selected from the three senatorial districts were used for the study, out of which three (3) schools were for treatment and the other three (3) for control groups. Intact classes were used for both groups. The total sample was made up of five hundred and ninety two (592) students. One (1) research question was raised and answered and one (1) null hypothesis was formulated and tested at p ≤ .05. The research instruments whose internal consistency of reliability co-efficient was calculated to be 0.76 applying Kuder Richardson formula 20 (K-R20) were the Test of Students Understanding of Chemical Concepts (TOSUCC) and Biology Achievement Test (BAT) which measured achievement in the bio-chemical topic of Kreb’s cycle. This shows evidence of internal consistency of the instrument. The data collected were analysed with Analysis of Covariance (ANCOVA) statistic to test the hypothesis and Mean Rating to answer the research question. Results showed that students’ chemical concept understanding level is low and there is significant difference between students who have high level of understanding of chemical concepts and those with level of understanding of chemical concepts in their achievements in Kreb’s cycle. The implications of these findings were highlighted and recommendations made.

Introduction

Biology, otherwise known as Life Science is a science subject which directs learners towards a number of important careers such as medicine, radiography, nursing, biology or life science education, botany, and so on. Having goals and aspirations to enter these professions is not enough; rather learners need understanding of chemical concepts to achieve their goals in the subject (Mhlamvu, 2008). Biology is offered by majority of students at the high school level in Nigeria. Aremu (2004) said in spite of the importance and popularity of biology among Nigerian students, performance has been poor. This according to Ahmed (2008) has been of great concern to many educators, parents, psychologists, researchers and curriculum planners.

Biochemistry is a branch of chemistry and biology defined by Robert (2006) as “the science concerned with the chemical basis of life”. Kreb’s cycle is one of the important bio-chemical topics in the senior secondary school biology curriculum. Muwanga (2005) said that the challenge of teaching biochemistry is that learning involves comprehension of objects and processes that cannot be seen or experienced. Many integrated science teachers skip the teaching of the chemical concepts that would lay foundation to the understanding of biochemical topics.

Kernerman (2010) defined a concept as a general notion or ideas of conception. Concepts enable us to organize and interpret data. Concepts also enable us to respond effectively to the
complexity of the world. Concepts are identified with words and symbols. To comprehend a subject is to know the meanings attached to the words which represent its concepts. Once concepts have been learned, such can be applied to or generalized in new instances that share the same essential attributes.

The understanding of Kreb’s cycle requires the knowledge of some chemical concepts in the junior secondary school integrated science curriculum. Some of the chemical concepts are matter, physical and chemical changes, compounds and mixtures, elements, chemical symbols, atomic structure, valency, chemical formula, and simple equations. Beskeni, Yusuf, Awang, and Ranjha (2011) investigated how effective prior knowledge can help in the understanding of difficult chemistry concepts at secondary school level and found that students’ prior knowledge must be considered in any classroom situation for effective teaching and learning of biology and chemistry. The key to understanding the subject is to understand its concepts. Kreb’s cycle also known as Citric Acid Cycle is a critical process of enzyme-driven chemical reactions for all cells that use oxygen for cellular respiration.

**Conceptual Framework**

The citric acid cycle is also known as tri-carboxylate cycle is a metabolic pathway located inside the matrix of the mitochondria that converts the end products of metabolism into energy. It was originally known as Kreb’s cycle, named after Hans Krebs who worked out the details in the early 1930s. The citric acid cycle requires oxygen and two cycles to metabolize a molecule of glucose. The acetyl CoA (2C) enters the Kreb’s cycle and combines with oxaloacetic acid (4C) to make citric acid (6C). The yield of the reaction is summarized below.

<table>
<thead>
<tr>
<th>Table 1. Summary of the Kreb’s cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net yield from Kreb’s cycle (2 turns)</td>
</tr>
<tr>
<td>6NADH2</td>
</tr>
<tr>
<td>2FADH2</td>
</tr>
<tr>
<td>4CO2</td>
</tr>
<tr>
<td>2ATP</td>
</tr>
</tbody>
</table>

The acetic acid enters into a Kreb’s cycle where it is joined to a 4 -carbon acid (oxaloacetic acid or oxaloacetate) present in the mitochondria to form citric acid (citrate) (a 6 - carbon acid). One molecule of carbon (IV) oxide and two atoms of hydrogen are removed by oxidative decarboxylation and NAD respectively to form a 5 - carbon ketoglutaric acid. Another molecule of carbon (IV) oxide and two atoms of hydrogen are further removed from ketoglutaric acid to form a 4- carbon succinic acid. Two atoms of hydrogen are removed from succinic acid to form a 4 - carbon malic acid. Furthermore, two hydrogen atoms are removed from malic acid to form a 4 - carbon oxaloacetic acid. The oxaloacetic acid then combines with acetic acid again and the cycle is repeated. The hydrogen atoms removed in successive stages combine with molecular oxygen to form water.

The overall reaction of glycolysis and kreb’s cycle is represented by an equation

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O + 38\text{ ATP}$$
ATP (Adenosine triphosphate)

1) ATP (Adenosine triphosphate) is a water soluble energy carrying small molecules that is transported round the cells.

2) It is manufactured from nucleotide base adenine phosphate groups making ATP a phosphorylated nucleotide.

3) ATP carries energy to any part of the cell that needs energy.

4) One reaching the cell, ATP is broken down by enzymes called ATPase to ADP (Adenosine diphosphate) to release chemical energy for metabolic activities.

In ATP synthesis, kreb’s cycle is the most important source of energy in metabolism. Understanding of kreb’s cycle is crucial or essential for any student hoping to gain understanding of cell biology. The cycle itself is complex and must be taught in an effective memorable manner to fully understand it. The importance of kreb’s cycle cannot be over emphasized. Willey, Sherwood and Woolverton (2008) gave the role of ATP as follows:

- ATP is used to store energy captured during exergonic reactions so it can be used to store energy.
- It is important for oxidation reductions in energy conservation.

Idodo-Umeh (2009) gave the importance ATP in the following ways.

- Living organisms (especially animals) require energy to do work.
- Electric organs of fish convert ATP into electrical energy which forms electric current for capturing prey or weapon for defense.
- Transmission of nerve impulses requires ATP.
- Production of heat needed to keep the body warm requires ATP.
- Active transport requires ATP to take place.
- Energy ATP is needed by some animals to produce light.
- ATP is required for the synthesis of protein, starch, cellulose, fats and oil.
- Cell division requires ATP (energy).

Statement of the Problem

The problem of this study is to find out the effects of students chemical concept understanding level on their achievement in the biochemical topic of Kreb’s cycle. To address the problem the following research questions are raised.

1. What is the effect of students’ chemical concept understanding level on their achievement in the topic of Krebs’ Cycle?

The following null hypothesis was tested at P ≤ .05 for the study:

There is no significant difference in mean scores between students who have high understanding level of chemical concepts and those of low chemical concept understanding in their achievement in Kreb’s cycle test.
Significance of the Study

The findings of this study would be beneficial to teachers, researchers, curriculum planners as well as the Behavior Changing Agents to have greater awareness and understanding of the various interactions and factors involving variables that predict the academic performance and achievement of students.

The student will benefit greatly from this study as because they will receive higher academic success as their teachers implement the recommendations of this study and will eventually contribute to the scientific, industrial and technological advancement of the nation. Indeed, the findings of this study could serve as basis for designing intervention programs to improve students’ achievement levels biochemical topics in particular and biology or Life Science in general.

Scope and delimitation of the Study

The study was carried out on the senior secondary two (SS 2) students in Delta State public schools. The study was limited strictly to senior secondary two biology content. Chemical concepts covered in the study were matter, physical and chemical changes, elements, compounds, mixtures, chemical symbols, formulae and equations, atomic structure while the biochemical topic was Krebs’s Cycle.

Research Methodology and Procedure

Research Design

The study made use of a quasi-experimental Non–Randomized pre-test, post-test control group design. Quasi experimental design involves the use of intact groups or classes and members are not randomly assigned to the groups and classes (Ary, Jacobs and Sorensen (2010)). The design is represented as follows:

<table>
<thead>
<tr>
<th></th>
<th>Pre-test</th>
<th>Treatment</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental Group</td>
<td>O₁</td>
<td>X</td>
<td>O₁</td>
</tr>
<tr>
<td>Control Group</td>
<td>O₁</td>
<td>-</td>
<td>O₁</td>
</tr>
</tbody>
</table>

The Experimental group of this study consisted of three hundred and three (303) intact class students from three (3) senior secondary schools randomly drawn from the 3 (three) senatorial zones of Delta State. The group was pre tested on chemical concepts, taught chemical concepts for four weeks and post-tested in chemical concepts. The students were pre-tested on Kreb’s Cycle, and later taught Kreb’s Cycle before being post-tested by administering the Biology Achievement Test (BAT).

On the other hand, the Control group consisted of two hundred and eighty nine (289) intact class students from three (3) senior secondary schools randomly drawn from the three senatorial zones of Delta State This group was pre-tested and post tested on chemical concepts, but not taught chemical concepts beyond what they knew from the lower classes. They were then pre-tested on Kreb’s cycle, and later taught Kreb’s Cycle. They were also post-tested using the Biology Achievement Test (BAT).

Population of the Study

The population of this study consist of all senior secondary II (SS II) students in the 25 (twenty five) local government areas of Delta State. The population is made up of males and
females students with a totaling 72,876 students with an average of 4,896 students in each senatorial zone.

Sample and Sample Technique

The research sample consist of 592 (five and ninety two hundred) SS II students from six public secondary schools, two drawn from each of the three (3) senatorial zones in the State. The schools are distributed as follow: three (3) schools (one from each zone) for Experimental group and three (3) schools (one from each zone) for control group were all used as intact classes.

Research Instrument

Two instruments were constructed and used in this study. They are:

1. Test of students understanding of chemical concepts (TOSUCC). The chemical concepts taught and tested in the study are physical and matter, chemical changes, elements, compounds and mixtures, chemical symbols, formula, equation and atomic structure. The instrument contained five (5) essay and twenty (20) short structured test items after validation and reliability test processes.

2. Biology Achievement Test (BAT) on kreb’s cycle. This test also contained five (5) essay and twenty (20) short structured test items after validation and reliability test processes.

The test items were administered to SS2 (Senior Secondary Two) students not used in the main study to pilot-test the instrument and generate data for reliability test. Kuder Richardson formula 20 (K-R20) was used to calculate the reliability coefficient of the tests to be 0.76.

Analysis of Data

T-test statistic generating means and standard deviations was used to formulate answer to research question and Analysis of covariance used to test the effect of the dependent variable on independent variables. ANCOVA was used to analyze collected to test the study hypothesis.

Results and discussions

Table 1. Distribution of Respondents by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>289</td>
<td>48.8</td>
</tr>
<tr>
<td>Experimental</td>
<td>303</td>
<td>51.2</td>
</tr>
<tr>
<td>Total</td>
<td>592</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 1 shows the frequency of control group as 239 (48.80%) and experimental group 303 (51.2%). In other words, 592 respondents took part in the study.
Table 2. t-test of Independent Samples in Pre-test Scores on Chemical Concept

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Df</th>
<th>t-cal</th>
<th>Sig (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRE-TEST</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>289</td>
<td>20.94</td>
<td>3.09</td>
<td>590</td>
<td>-2.25</td>
<td>P = 0.822</td>
</tr>
<tr>
<td>Experimental</td>
<td>303</td>
<td>20.54</td>
<td>3.08</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \alpha = 0.05 \)

Table 2 shows a t-calculated value of -0.225 and a P value of 0.822 at an alpha (\( \alpha \)) level of 0.05. Since the P value is greater than the alpha level, it showed that there was no significant difference between the two groups at the Pre-test stage. A 26.0-point mean score of the respondents was used to indicate the level of chemical concept understanding. Thus, a mean score of less than 26.0 and more than 26.0 is considered as indicative of low and high level of understanding of chemical concepts respectively. The mean scores for the control group 20.94 and experimental group 20.54 from the above table are low and very close to themselves. The closeness of the standard deviation values of 3.09 and 3.08 for control and experimental groups respectively further proves that no significant difference in the level of chemical concept understanding existed between the two group members. Therefore, the chemical concept understanding level of the students was low at the pre-test stage.

Table 3. t-test of Independent Samples in Post-test Scores on Chemical Concept

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Df</th>
<th>t-cal</th>
<th>Sig (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>POST-TEST (TOSUCC)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>289</td>
<td>25.94</td>
<td>3.60</td>
<td>590</td>
<td>-6.838</td>
<td>0.000</td>
</tr>
<tr>
<td>Experimental</td>
<td>303</td>
<td>27.93</td>
<td>3.49</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \alpha = 0.05 \)
Table 3 shows a calculated t-test value of -0.6.838 and a P value of 0.000 at an alpha level of 0.05 with the P value less than the alpha level. It shows that the null hypothesis which states that "There is no significant difference in mean scores between students who have high understanding level of chemical concepts and those of low chemical concept understanding in their achievement in Kreb’s cycle test" could not be accepted. Consequently, there is a significant difference in the level of chemical concepts understanding of those exposed to the instructions on chemical concepts and those who were not.

The above table further shows that the mean of the Experimental Group as 27.93 which is significantly greater than the mean of Control Group which is 25.94. Also, a significantly lower standard deviation value of 3.49 for the experimental group in the post-test compared to the 3.60 value for the control group shows that those exposed to chemical concepts instructions had better level of understanding of chemical concepts than who were not. Therefore, many more of the experimental group members scored higher marks that made them to achieve higher in the Biology Achievement Test on Kreb’s Cycle.

Table 4. Mean and Standard Deviation of Participants’ Post-test Achievements

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>289</td>
<td>16.1246</td>
<td>3.28270</td>
</tr>
<tr>
<td>Experimental</td>
<td>303</td>
<td>21.9406</td>
<td>2.42294</td>
</tr>
<tr>
<td>Total</td>
<td>592</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4 shows a post-test mean of the control group as 16.12 and a post-test mean for experimental group as 21.94. The mean for the experimental group is significantly greater than the mean for the control group with a significantly lower standard deviation of 2.42294 the control group having 3.28270. The implication of this is that the experimental group members achieved in terms of quantity and quality of scores more than the control group in Kreb’s Cycle topic. It also shows that an in-depth understanding of related chemical concepts aid comprehension and higher achievement in Kreb’s Cycle and other biochemical concepts.
Table 5. ANCOVA of students’ achievement in Kreb’s cycle

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>8048.093a</td>
<td>2</td>
<td>4024.047</td>
<td>1.294E3</td>
<td>.000</td>
</tr>
<tr>
<td>Intercept</td>
<td>1549.613</td>
<td>1</td>
<td>1549.613</td>
<td>498.258</td>
<td>.000</td>
</tr>
<tr>
<td>Pre Krebs’s Cycle</td>
<td>3044.621</td>
<td>1</td>
<td>3044.621</td>
<td>978.959</td>
<td>.000</td>
</tr>
<tr>
<td>Group</td>
<td>4298.493</td>
<td>1</td>
<td>4298.493</td>
<td>1.382E3</td>
<td>.000</td>
</tr>
<tr>
<td>Error</td>
<td>1831.826</td>
<td>589</td>
<td>3.110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>225878.000</td>
<td>592</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>9879.919</td>
<td>591</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. R Squared = .815 (Adjusted R Squared = .814)

Table 5 shows F value of 1.382E3 (i.e. $1.382 \times 10^3$) and a P value of .000 at an alpha level of 0.05. The P value is less than the alpha level, so the null hypothesis which states that ‘There is no significant difference in mean scores between students who have high understanding level of chemical concepts and those of low chemical concept understanding in their achievement in Kreb’s cycle test’ could not be accepted. Therefore, there is a significant difference in mean achievement scores between students who have high understanding level of chemical concepts and those who do not have, in Kreb’s cycle.

Table 6. Pair wise Comparisons of Students Achievement in Kreb’s Cycle

<table>
<thead>
<tr>
<th>(I) Group</th>
<th>(J) Group</th>
<th>Mean Difference (I-J)</th>
<th>Std. Error</th>
<th>Sig. a</th>
<th>95% Confidence Interval for Difference a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>Experimenter</td>
<td>-5.412*</td>
<td>.146</td>
<td>.000</td>
<td>-5.698 -5.126</td>
</tr>
<tr>
<td>Experimenter</td>
<td>Control</td>
<td>5.412*</td>
<td>.146</td>
<td>.000</td>
<td>5.126 5.698</td>
</tr>
</tbody>
</table>

Based on estimated marginal means

* The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Table 6 shows a mean difference of experimental against control as 5.41 and a P value 0.000. The P value is less than 0.05 ($P<0.05$) further proving that students who had understanding of chemical concepts achieved more in Kreb’s cycle than those who did not.
Summary of the Research Findings

The analyses of the results collected show the following findings.

1. The level of students’ chemical concept understanding is generally low under usual school setting. There is no significant difference between the experimental and control groups in their level of chemical concept understanding before being exposed to instructions on chemical concepts.

2. There is a significant difference in the level of chemical concepts understanding of those exposed to the instructions on chemical concepts and those who were not.

3. There is a significant difference between students who have understanding of chemical concepts and those who do not, in their achievement levels in Kreb’s cycle. This is because there is a significant difference between student’s chemical concept understanding level and their achievement in Kreb’s cycle. This implies that students who had understanding of chemical concepts perform better and therefore, achieved more in Kreb’s cycle than those who did not. This is a measure to show that Kreb’s Cycle is a body of various and related chemical concepts such as matter, physical and chemical changes, compounds and mixtures, elements, chemical symbols, atomic structure, valency, chemical formula, and simple equations as well as biochemical concepts.

This result is in line with the result of Beskeni, Yousuf, Awang, Ranjhal (2011), who said that the knowledge of biochemistry concepts requires the knowledge of chemical concepts. They investigated how effective prior knowledge can help in the understanding of difficult chemistry concepts at secondary school level and found out that prior knowledge has tremendous implication in the teaching and learning of chemistry. It is also in line with the work of Villafane, Sachel M, Bailey, Cheryl P., Loertscher, Jennifer, Minderhout, Vicky, Lewis, Jennifer E. (2011) who stated that learning depends on the application of previously learned concepts from general chemistry and biology to new biological context. They said also that for meaningful learning to take place there must be relevant concepts available within the cognitive structure which can be linked with the new material. Khan (2011) in his article said that existing level of understanding of concepts to the subject of chemistry among class IX students was not high.

Conclusion

Findings of the study and others cited in the literature reveal that

1. Chemical concept understanding of students is generally low. This may be one of the reasons why performance is poor in Senior Secondary School Examination (SSCE).

2. The treatment was very effective in enhancing achievement of the participants in bio – chemical topics. This shows that chemical concept understanding aids the understanding of biochemical topic of kreb’s cycle. If students understand chemical concepts, they will have better achievement in biochemical topics.

3. Good prior knowledge of chemical concepts is important for the teaching and learning of biochemical topics in biology or life science and that chemical concept understanding is a pre-requisite for the comprehension of biochemical topics. Finally, learning biochemical topics depends on the application of previously learnt chemical concepts.

Implication

1. Biology students in many schools in Delta State have low level of chemical concepts understanding. Students who had chemical concepts understanding performed better than
those without in biochemical topics. The low level of chemical concept understanding of biochemical topics by students has implication in their learning of the topics. There is the probability that secondary school biology and chemistry teachers do not teach for concepts understanding.

2. The treatment strategy reviewed in this study will provide educational psychologists, guidance counselors, teachers and principals of secondary schools with guidelines for future educational diagnosis aimed at improving educational system in Nigeria.

3. The study will enhance policy making of the federal government and non-governmental organizations on such and similar educational issues in Nigeria.

4. The study would serve as an eye opener to researchers on the educational system, as it will open doors for various similar researches on the issue.

Recommendations

1. Secondary school biology and chemistry teachers should teach for concept understanding of topics that are related to new topics and tasks before teaching them.

2. Curriculum planners and school administrators should always participate in workshops on concepts understanding especially for difficult topics like biochemical topics to update their knowledge in the effective process of enhancing students’ achievement in biology or life science.

3. Federal Government should:
   1. Fund, train and retrain biology and chemistry teachers on – job through workshops and seminars.
   2. Send inspectors of education especially those of science department for awareness training, and workshops on teaching for conceptual understanding. This would enable them to be competent to recommend books and other materials that are required for teaching for conceptual understanding.
   3. The various examination bodies like WAEC and NECO should include appropriate models of teaching in their syllabuses and make sure the course can be covered within the stipulated time. Enough time should be given for the topics to be covered
   4. Ensure high performance, competent, adequate and qualified teachers with defined areas of specialization and with teaching experience handle the students from Junior Secondary School 1(JSS1). Such teachers should be involved in examining, supervising and marking of the internal and external examinations; by this, will enhance their professional competencies.
   5. Schools and Government should provide all the necessary requirements and good environment to enable teachers adapt teaching for concepts understanding in the discharge of their duties.

References


Idodo-Ume (2009), College Biology. Idodo-Ume publishers Ltd. Benin City.
Mhlamvu V. N. (2008), Conceptual Understanding of Photosynthesis, Unpublished Dissertation Submitted to the Faculty of Education in Partial Fulfillment of the Requirement for the Award of the Degree of Master of Education in the Department of Mathematics, Science and Technology of Education, University of Zululand
Technology and Indigenous Knowledge
Long Papers
The Effect of Computer Simulations on the Speed of Writing Tests

Sam Kaheru 1 & Jeanne Kriek 2

1 Department of Professional Studies, University of Venda, South Africa.
2 Institute of Science and Technology Education, University of South Africa, South Africa.

1 samkaheru@univen.ac.za, 2 kriekj@unisa.ac.za

The focus of the study was on how computer simulations affect the speed of grade 11 learners (male/female) writing a test on the topic Geometrical Optics when a teacher centred approach is used. The assumption was that the moment knowledge or a skill ends up in the long term memory, it is performed with ease and therefore there is limited to no effort needed to perform a task, meaning the speed of performance is increased. The study was done in a rural area in South Africa, in the Limpopo Province in the district of Vhembe. The theoretical framework was based on the information processing model. Within the non-equivalent quasi experimental design a switching replications design study was used whereby 105 learners in four schools took part. This study found that the speed of writing a test decreased through the use of simulations and was contradictory to what was expected. The recommendation for further research included, having the performance in the test and also investigating whether learners using a preferred language would perform better.

Background to the study

South Africa and specifically the Limpopo Province is confronted with a lack of laboratories (NEIMS, 2009). Therefore, there is a need to explore a way to address the lack of resources and a way could possibly be an alternative technology, for example computer simulations to develop knowledge and skills in a chosen topic geometrical optics. Geometrical optics is a topic that deals with lenses needed in telescopes, a section prescribed when this study was done in the South African National curriculum statement (NCS) in physical sciences for Grade 11 (Department of Education, 2008). This topic was also chosen because South Africa was successful in its bid for an award for an international telescope project referred to as Square Kilometre Array (SKA) project to construct and host the largest radio telescope in the world for space exploration, and was therefore relevant for research.

Computer simulations are an increasing mode of teaching and learning as they provide hands on opportunities (Zacharia, 2007; Zacharia et al, 2008). It can also be used in a teacher centred environment as demonstration tool which could address the under resourced schools (Kaheru & Kriek, 2014). It offers the advantage of flexibility, promoting students’ active engagement in conceptual change, higher-order thinking and reinforced practice (Smetana & Bell, 2012; Dega et al, 2013). In addition when learning abstract concepts like electromagnetic fields, students have the opportunity to confront their own conceptions and receive immediate feedback (Dega et al, 2013).

In order for information to go to the long term memory, it is important that it passes through the working memory; the working memory is limited with regard to how much information units it can process. It is important to determine the appropriate load and is called cognitive load. The cognitive load (Paas, Renkl, & Sweller, 2003; Mayer, 2002) is what will be researched in terms of what and how computer simulations can change the cognitive load especially the load that leads to taking the information to the long term memory.
The long term memory has unlimited capacity. One of the outcomes for long term memory is “automation” (Paas & Sweller, 2012, p. 25; van Merrienboer et al., 2003). The moment knowledge or a skill ends up in the long term memory, it is performed with ease. There is limited to no effort needed to perform a task, meaning the speed of performance is increased.

With this backdrop it was expected that possibly the use of computer simulations would be able to increase the speed of answering questions in the test since possibly mastery had increased (Paas, 1992; Finkelstein, et al., 2005; Paas & Sweller, 2012).

This study focussed on how computer simulations affect the speed of answering questions in the test used as instrument.

**The research question for the study was:**

What is the effect on the speed of writing a test when computer simulations are used by Grade 11 learners (male/female) when the topic Geometrical Optics is taught using a teacher centred approach?

**Theoretical framework**

The theoretical framework used for this study was the Cognitive Load Theory (CLT) and the Cognitive Theory of Multimedia Learning (CMML). Cognitive load theory states that when knowledge or a skill are being learnt the brain experiences a cognitive load. There are a possible three components of the load, namely intrinsic, extraneous and germane load. The intrinsic load refers to the inherent difficulty of the subject or unit. An example would be looking at the subject Physics which is regarded as being difficult. The second component of cognitive load is the extraneous load which is due to the links or connections within what is being learned. It can be said that the relationships of the new knowledge and on what the learner knows could be linked to this. What instruction tries to do is to reduce this extraneous load. If the extraneous load is reduced then the third component, germane load is reduced. Germane load is the cognitive load which leads to understanding and what is learned ending up in the long term memory. The germane load leads to discrete or disparate knowledge pieces being put together in “chunks” or at times called “schemas” and it would be easy to memorise these since they have connections in the mind. The cognitive load theory posits that once what has been learned ends up in the long term memory a learner cannot forget what has been learnt. It also goes further to indicate that once it is in the long term memory it can easily be retrieved. One does not have to think about it, it is more or less automatic. It is this being automatic that led to the study that if they have committed the information to long term memory then it should be quite easy to retrieve it.

The theoretical framework combines the two theories and the teacher centred approach that was used for the study. It is indicated in Figure 1.
The cognitive load and the contribution of the cognitive theory of multimedia learning are indicated. The cognitive theory of multimedia learning contributes to the model in the way that it states the channels in which the learning is processed have a limited amount of information which can be processed. When the information that have to be acquired comes in only a small amount will be given attention (Chou, 1998; Mayer & Chandler, 2001; Muller et al., 2008; Zheng et al., 2008). The channels that were alluded to in the study were the visual and the audio since the teacher was also talking to complement the computer simulation. The whole scenario is captured in a teacher centred approach at times called traditional approach (Hake, 1998; Schwerdt & Wuppermann, 2011; Taşoğuğlu & Bakaç, 2010), where what the teacher does is focused on him, he is the main player and he is trying to make sense to the learners of what is happening. In the case where the computer simulations are not used the teacher tries to use other examples that are familiar in the life of the learners. This study is part of a bigger study and focus on the speed of writing the test. The hypothesis is, if the automatic mode is achieved then it should be very easy to retrieve the information and use it.

Research Methodology

Research design: A non-equivalent quasi experimental design was chosen whereby learners in four schools took part in the study (Trochim, 2006). It was chosen since the schools which participated would have to work with intact schools not part of the classes, to avoid disrupting the school systems. Learners were assigned randomly to classes at the beginning of the school year. The learners participated intact and were not divided differently from what they normally were. For the purposes of the research, it was the same educators in each of the schools who taught the same classes. The number of learners was not made equal but the one provided was used. The timetable remained the same and could not be altered to address the researcher’s needs, because the learners had to follow the sequence of topics at the same time allocated as it was prescribed in the “Pace setters”. Pace setters is an initiative from the Department of Education of the Limpopo province which prescribe to the teachers what specific content needs to be taught in a specific timeframe and they were not allowed to

---

**Figure 1.** Showing the Cognitive load theoretical framework of the study (Kaeru, Mpeta, & Kriek, 2011)
deviate. The only difference was the use of computer simulations different from their normal classroom situation.

Within the non-equivalent group design a switching replications design (Trochim, 2006; Alexander & Winne, 2006) was used. By design each of the treatment groups had a control built in. The switching replications design was chosen for this study since it increased internal validity with regard to subjects that may have contact with one another, it reduced rivalry (Kothari, 2004). Each group had turns at becoming a treatment and control in the course of the study. The disadvantage of this could be that there could be a continual improvement even after the treatment would have been withdrawn (Trochim, 2006). For the purposes of this research, it was to be presumed that if a treatment was strong enough to even continue after the treatment has been withdrawn then it would mean it had a very strong effect.

\[
\begin{array}{cccccc}
N & O & X & O & O \\
N & O & O & X & O \\
\end{array}
\]

**Figure 2.** Switching replications design for research

The N (see Figure 1) indicates that it was a non-random sampling and assigning of the groups. Schools were requested to participate and where they accepted it was decided which school would start as an experimental group and which would be in the treatment group.

The O (see Figure 2) indicates the observations made where a research instrument was used. The instrument was a split timer. To further explain the top line, it can be summarised as OXOO where the three O’s in the order they appear are pre-test, post-test1 and post-test2. These tests were the same.

The X (see Figure 2) indicates the use of computer simulations with a teacher centred approach during that period. Where there is just a space between the Os it indicates that no computer simulations were used instead, it was simply a teacher centred approach.

The two lines in which the design is shown are also significant. As shown in Figure 2, if one considers the first arrangements in the top line of OXO and the bottom arrangement of OO it means in the first instance there is a group using computer simulations on top (treatment) and one which is not using computer simulations, the one below. Then the treatment is switched and now the top group is the control of the bottom group.

**Participants:** The study involved 104 learners in four schools in a district in the Limpopo province in South Africa. There were 50 male and 54 female learners. All were in grade 11 with the age ranging from 16 to 20 years old.

**Data collection:** The three tests; pre-test, post-test 1 and post-test 2 were identical in all respects. Before the unit, a pre-test was written. The unit was covered in one week where after the first week post-test 1 was written. After the second week, post-test 2 was written. The language of the test was English which was a second, third or fourth language of the learners, it was not their home language. The three tests were identical in all respects. Learners were not told that they were going to write the same test. The tests were made out of test items, covering the knowledge and skills. The tests had multiple choice items. Kazeni(2005) showed how the importance of written test items and their validity in testing skills items and knowledge.
Data analysis: Descriptive statistics based on the gender were used considering the groups taught with computer simulations and those who did not use computer simulations. Independent samples t-test and the paired samples t-test were also used to determine if there was a difference in the samples and also if there were changes as a result of the teaching using the computer simulations (CS) or lack of.

Instrument: The duration of writing the test for each individual learner was recorded using a split timer device. The split timer was started for all the learners and each of the learners as s/he finished it would be split or paused and a number of the sequence of submission would be put on the answer sheet. The sheets were collected as soon as a learner would indicate that they had finished. This would to a very good approximation indicate that the learner had taken the time as indicated by the split timer. In all the three tests the same timing was done.

![Split Timer to record time](Online Stopwatch, 2014)

Figure 3. Split Timer to record time  (Online Stopwatch, 2014)

Validity of the instrument

The timer would be started when the learners started writing the test. It was important that all the learners who were in the class would be started off at the same time. As soon as the learner indicated that he had finished then the split timer would be stopped for that particular learner. The order in which he had finished would be written on the script. It was important that the research assistants were given the same training and the same understanding of how to take readings and use the split timer. This was done to ensure that the times written would be the same. There were practice times to check if it was done in the same way to make sure there was consistency.

Sources of not being valid would be where the learners arrive after the test had started. In order not to affect the actual times, the learner would have to have his own starting time so that the actual finishing time would be taken for the individual participant. This happened only in School A when three learners arrived after the test had started for the second post test where each of the learners’ times was taken separately.

Reliability of the instrument

The reliability of the split timer would be the user starting it on time and also stopping it when it would be necessary. There could be sources of error with regard to the timer not being started on time and not being stopped on time. Other sources of error included (a) the users not taking the reading correctly (b) the zero error of the split timer was + - 0.0005 seconds. The systematic error be in starting late would be larger however as indicated all these were very small. Since the times being talked of all were of the order of 17 minutes, the split timer readings were very reliable.
Teaching in a teacher centred environment with the use of computer simulations

The learners were taught a topic on geometrical optics wherein one of the two groups was taught by the a teacher using the teacher centred approach while the other one was taught by a teacher using computer simulations in a teacher centred environment. The unit on geometrical optics could be conveniently divided into two sections, the first one was on the characteristics of lenses and how images are formed and the associated changes in variables. The associated changes in variables include image distance, object distance, size of the image and objects and magnification. The second section may be categorised as the practical applications to instrumentation or use. How the eye is able to “see”, issues of long sightedness and short sightedness; how telescopes and microscopes work and the same related issues as in the first section of the image and object distances and the different heights. The computer simulations were used in each of the different sections but for the different groups.

Results and discussion

The following data and subsequent analysis leads to conclusions on how the speed of writing tests was affected by the use of CS. The Without CS indicates that the students were not using simulations whereas the With CS indicates that the learners were using computer simulations at that time. It is important to emphasise though the categorisation for Without CS is used for the Pre-test it indicates the group that did not use the computer simulations prior to the post-test 1, the same reasoning applies to the With CS for the pre-test.

Independent Samples t-test based on use of CS

Table 1. Descriptive statistics for writing times for pre-test, post-test 1 and post-test 2

<table>
<thead>
<tr>
<th>Condition</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Writing Pre-test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without CS</td>
<td>45</td>
<td>24.23</td>
<td>5.18</td>
<td>0:00:46.33</td>
</tr>
<tr>
<td>With CS</td>
<td>58</td>
<td>23.14</td>
<td>4.91</td>
<td>0:00:38.69</td>
</tr>
<tr>
<td>Writing Post-test 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without CS</td>
<td>46</td>
<td>16.16</td>
<td>2.57</td>
<td>0:00:22.74</td>
</tr>
<tr>
<td>With CS</td>
<td>55</td>
<td>17.58</td>
<td>4.28</td>
<td>0:00:34.63</td>
</tr>
<tr>
<td>Writing Post-test 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TimeWith CS</td>
<td>35</td>
<td>14.30</td>
<td>1.63</td>
<td>0:00:16.52</td>
</tr>
<tr>
<td>Without CS</td>
<td>55</td>
<td>12.93</td>
<td>4.13</td>
<td>0:00:33.44</td>
</tr>
</tbody>
</table>
Table 2. Descriptive statistics for writing times based on gender

<table>
<thead>
<tr>
<th>Gender</th>
<th>Condition</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Writing time Pre-test Without CS</td>
<td>22</td>
<td>24.67</td>
<td>5.30</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>With CS</td>
<td>27</td>
<td>22.34</td>
<td>3.98</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>Writing Time Post-Without CS</td>
<td>23</td>
<td>16.63</td>
<td>2.73</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>With CS</td>
<td>26</td>
<td>16.69</td>
<td>4.00</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>Writing Time Post-With CS</td>
<td>17</td>
<td>14.21</td>
<td>1.28</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>Without CS</td>
<td>26</td>
<td>12.65</td>
<td>4.07</td>
<td>0.80</td>
</tr>
<tr>
<td>Female</td>
<td>Writing time Pre-test Without CS</td>
<td>23</td>
<td>23.80</td>
<td>5.14</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>With CS</td>
<td>31</td>
<td>23.84</td>
<td>5.57</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Writing Time Post-Without CS</td>
<td>23</td>
<td>15.69</td>
<td>2.36</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>With CS</td>
<td>29</td>
<td>18.38</td>
<td>4.43</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>Writing Time Post-With CS</td>
<td>18</td>
<td>14.38</td>
<td>1.94</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>Without CS</td>
<td>29</td>
<td>13.17</td>
<td>4.25</td>
<td>0.79</td>
</tr>
</tbody>
</table>

The descriptive statistics given in Table 1 and Table 2 consider the groups those taught using computer simulations and those that did not use the computer simulations at the various stages and then the descriptive statistics based on the gender was provided.

The analysis that follows used the independent samples t-test and the paired samples t-test. The tests considered if there was a difference in the samples and also if there were changes as a result of the teaching using the computer simulations (CS) or lack of.

Writing Time Pre-test Without CS and With CS

Given that the homogeneity of variance by the Levene’s test, F(1, 101) = 0.001, p = .98 (p > .05) was upheld, a test assuming equality of variances was calculated (see Table 1 and Table 3). The result of this test indicated that there was no significant difference in the scores t(101) = -1.09, p = .28 (p > .05) and d = 0.28 (small effect). These results suggest that the writing time for the pre-test of those in the Without CS (M = 24.23, SD = 5.18) and With CS (M = 23.14, SD = 4.91) conditions with regard to writing times were the same (see Table 1).
Table 3. Levene's Test and Independent t-test for writing times

<table>
<thead>
<tr>
<th></th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>Sig.</td>
</tr>
<tr>
<td>Writing Time Pre-Test</td>
<td>.001</td>
<td>.98</td>
</tr>
<tr>
<td></td>
<td>Equal variances assumed</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equal variances not assumed</td>
<td></td>
</tr>
<tr>
<td>Writing Time Post-Test1</td>
<td>12.88</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>Equal variances assumed</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equal variances not assumed</td>
<td></td>
</tr>
<tr>
<td>Writing Time Post-Test2</td>
<td>23.41</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Equal variances assumed</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equal variances not assumed</td>
<td></td>
</tr>
</tbody>
</table>

Writing Time Post-test 1 Without CS and With CS

Given that the homogeneity of variance by the Levene’s test, $F(1, 99) = 12.88$, $p = .001$ ($p < .005$) was not upheld, a test not assuming equality of variances was calculated (see Table 1 and Table 3). The result of this test indicated that there was a significant difference in the scores $t(90) = -2.06$, $p = .04$ ($p < .05$) and $d = 0.39$ (small effect size). These results suggest that the writing times of those in the Without CS ($M = 16.16$, $SD = 2.57$) and With CS ($M = 17.58$, $SD = 4.28$) conditions with regard to writing times were not the same (see Table 1).

Writing Time Post-test 2 Without CS and With CS

Given that the homogeneity of variance by the Levene’s test, $F(1, 88) = 1.87$, $p = .000$ ($p < .001$) was not upheld, a test not assuming equality of variances was calculated (see Table 1 and Table 3). The result of this test indicated that there was a significant difference in the scores $t(76) = -2.20$, $p = .03$ ($p < .05$) and $d = 0.40$ (small effect). These results suggest that the writing times of those in the With CS ($M = 14.30$, $SD = 1.63$) and Without CS ($M = 12.93$, $SD = 4.13$) conditions with regard to writing times were not the same (see Table 1).

Independent Samples t-test based on gender

An analysis was made based on the gender of the learners in terms of the Writing time.

Male Writing time Pre-Test

Given that the homogeneity of variance by the Levene’s test, $F(1, 47) = 1.51$, $p = .23$ ($p > .05$) was upheld for the male learners, a test assuming equality of variances was calculated (see Table 4). The result of this test indicated that there was no significant difference in the writing time $t(47) = 1.76$ and $p = .09$ $p > 0.05$ (see Table 4) and $d = 0.50$ (medium effect).
These results suggest that the male learners in the group Without CS ($M = 24.67, SD = 5.30$) and in the group With CS ($M = 22.34, SD = 3.98$) conditions were not significantly different (see Table 2). There was a higher mean for the male learners in the group Without CS. This indicates that the male learners in the Without CS wrote longer than those With CS though there was no significant difference.

**Male Writing time Post-Test 1**

Given that the homogeneity of variance by the Levene’s test, $F(1, 47) = 3.68, p = .06$ ($p > .05$) was upheld for the male learners, a test assuming equality of variances was calculated (see Table 4). The result of this test indicated that there was no significant difference in the writing time $1,t(47) = 0.06$ and $p = .95$ ($p > 0.05$)(see Table 4) and $d = 0.02$ (very small effect). These results suggest that the male learners in the group Without CS ($M = 16.63, SD = 2.73$) and in the group With CS ($M = 16.69, SD = 4.00$) conditions were not significantly different (see Table 2). There was a higher mean for male learners in the With CS groups.

**Table 8. Levene’s Test and Independent t-test for writing times based on gender**

<table>
<thead>
<tr>
<th>Gender</th>
<th>Levene's Test for Equality of Variances</th>
<th>t-test for Equality of Means</th>
<th>95% Confidence Interval of the Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>Sig.</td>
<td>t</td>
</tr>
<tr>
<td>Male</td>
<td>Writing time Pre-Test</td>
<td>Equal variances assumed 1.51</td>
<td>.23</td>
</tr>
<tr>
<td></td>
<td>Writing time Post-Test</td>
<td>Equal variances assumed 3.68</td>
<td>.06</td>
</tr>
<tr>
<td>Female</td>
<td>Writing time Pre-Test</td>
<td>Equal variances assumed .70</td>
<td>.41</td>
</tr>
<tr>
<td></td>
<td>Writing time Post-Test</td>
<td>Equal variances assumed 12.38</td>
<td>.001</td>
</tr>
</tbody>
</table>

314
This indicates that the male learners in the With CS wrote longer than the male learners Without CS though there was no significant difference.

**Male Writing time Post-Test 2**

Given that the homogeneity of variance by the Levene’s test, $F(1, 41) = 17.54, p = .000 (p < .001)$ was not upheld for the male learners, a test not assuming equality of variances was calculated (see Table 4). The result of this test indicated that there was no significant difference in the writing time Post-Test 2, $t(32) = 1.82$ and $p = .08 > 0.05$ (see Table 4) and $d = 0.48$ (small effect). These results suggest that the male learners in the group With CS ($M = 14.21, SD = 1.28$) and in the group Without CS ($M = 12.65, SD = 4.07$) conditions were not significantly different (see Table 2). There was a higher mean for the With CS, who had just been taught using CS. This indicates that the male learners in the With CS wrote longer than their counterparts Without CS but with no significant difference.

**Female Writing time Pre-Test**

Given that the homogeneity of variance by the Levene’s test, $F(1, 52) = 0.70, p = .41 (p > .05)$ was upheld for the female learners, a test assuming equality of variances was calculated (see Table 4). The result of this test indicated that there was no significant difference in the writing time 1, $t(47) = 1.76$ and $p = .08 > 0.05$ (see Table 4) and $d = 0.01$ (very small effect). These results suggest that the female learners in the group Without CS ($M = 23.80, SD = 5.14$) and in the group With CS ($M = 23.84, SD = 5.57$) conditions were significantly different (see Table 2). There was a higher mean for the With CS. This indicates that the female learners in the With CS wrote for a slightly longer time than With CS though there was no significant difference.

**Female Writing time Post-Test 1**

Given that the homogeneity of variance by the Levene’s test, $F(1, 50) = 12.38, p = .001 (p < .005)$ was not upheld for the female learners, a test not assuming equality of variances was calculated (see Table 4). The result of this test indicated that there was a significant difference in the writing time 2, $t(44) = -2.81$ and $p = .01 < 0.05$ (see Table 4) and $d = 0.76$ (medium effect). These results suggest that the female learners in the group Without CS ($M = 15.69, SD = 2.36$) and in the group With CS ($M = 18.38, SD = 4.43$) conditions were significantly different (see Table 2). There was a higher mean for the female learners in the group With CS. This indicates that the female learners in the group With CS wrote longer than their counterparts Without CS.

**Female Writing time Post-Test 2**

Given that the homogeneity of variance by the Levene’s test, $F(1, 45) = 7.87, p = .007 (p < .01)$ was not upheld for the female learners, a test not assuming equality of variances was calculated (see Table 4). The result of this test indicated that there was no significant difference in the writing time 1, $t(42) = 1.32$ and $p = .19 > 0.05$ (see Table 4) and $d = 0.34$ (small effect). These results suggest that the female learners in the group With CS ($M = 14.38, SD = 1.94$) and in the group With CS ($M = 13.17, SD = 4.25$) conditions were not significantly different (see Table 2). There was a higher mean for the With CS, who had just been taught using CS. This indicates that the female learners in the With CS took longer to write the test than Without CS though there was no significant difference.
Summary

Data from the earlier tables are summarised below. Looking at the summary in Table 5, it is important to note that those learners using CS took longer except for the pre-test cases. The male learners, who were using CS, despite being a small effect change, took longer to write post-test 1 than those who were not using CS.

The female learners using the CS for the first time also took longer to write and register a medium effect size of 0.76, very close to high effect size and it was significant. The use of the CS indeed increased the time of writing.

Table 5. Groups that took longer to write the test

<table>
<thead>
<tr>
<th>Description</th>
<th>Wrote for longer time</th>
<th>Significant</th>
<th>Cohen’s d</th>
<th>Using computer simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without CS and With CS Pre-Test</td>
<td>Without CS</td>
<td>No</td>
<td>0.28</td>
<td>No, pre-test</td>
</tr>
<tr>
<td>Without CS and With CS Post-Test1</td>
<td>With CS</td>
<td>Yes</td>
<td>0.39</td>
<td>Yes</td>
</tr>
<tr>
<td>Without CS and With CS Post-Test2</td>
<td>Without CS</td>
<td>Yes</td>
<td>0.40</td>
<td>Yes</td>
</tr>
<tr>
<td>Male writing Pre-Test</td>
<td>Without CS</td>
<td>No</td>
<td>0.50</td>
<td>No, pre-test</td>
</tr>
<tr>
<td>Male writing Post-Test 1</td>
<td>With CS</td>
<td>No</td>
<td>0.02</td>
<td>Yes but with very small effect size</td>
</tr>
<tr>
<td>Male writing Post-Test2</td>
<td>With CS</td>
<td>No</td>
<td>0.48</td>
<td>Yes</td>
</tr>
<tr>
<td>Female writing Pre-Test</td>
<td>With CS</td>
<td>No</td>
<td>0.01</td>
<td>No, pre-test</td>
</tr>
<tr>
<td>Female writing Post-Test 1</td>
<td>With CS</td>
<td>Yes</td>
<td>0.76</td>
<td>Yes</td>
</tr>
<tr>
<td>Female writing Post-Test2</td>
<td>With CS</td>
<td>No</td>
<td>0.34</td>
<td>Yes</td>
</tr>
</tbody>
</table>

An indication of Table 5 with regard to the groups that took longer to write the test shows that the groups that were using the CS took longer to write. It is interesting that the only time where they did not take longer was in the pre-test, true for the general groups, female alone and the male alone. It is important to highlight too that though there were changes, some of the changes (in this greater for the CS group) were not significant, which occur in post-test 2 for female and male participants, and for the male in post test 1. The biggest significant effect size occurs for the female in post-test 1, with an effect size of 0.76

Conclusion

The findings show that the speed of writing a test decreased through the use of simulations and are consistent over the different groupings. Both the male and female learners who used CS took longer to finish writing. This was contradictory to what was expected. Research that indicates that the moment information ends up in the long term memory, it is performed with ease and therefore there is limited to no effort needed to perform a task, meaning the speed of performance is increased (Paas & Sweller, 2012; van Merrienboer et al, 2003). This was not found in this study. In the extensive study it was found that learners who took longer (groups using CS) performed better, but this is not dealt with in this paper. A possible reason for taking longer to write the tests could be that the learners were reading carefully and trying to
understand what is asked in order to answer the questions correct and hence the better performance. Further future work could be, linking the performance of the test with the speed in the context of using CS. The fact that the current research was as a result of teacher centred it could be investigated with regard to learner centred. A final suggestion for further research would be the preferred language of the learner, if the conditions were given in the preferred language would the results be the same?

**Bibliography**


An Experiment with Peer Instruction in Computer Science to Enhance Class Attendance

C. M. Keet
University of Cape Town, South Africa
mkeet@cs.uct.ac.za

Class attendance of computer science courses in higher education is typically not overwhelming. Anecdotal reports and the authors’ experiences with a low-resource mode of peer instruction indicated increased class attendance after a lecture with such concept tests. This has been evaluated systematically with a 3rd-year computer science module using a medium-resource, software-based, Audience Response System (‘clickers’). Results show there is neither a positive nor a negative relation between lectures with peer instruction (PI) and class attendance. The student participation rate in software-based voting decreased and some decline in lecture attendance was observed. Thus, PI itself could not be shown to be a useful strategy to enhance class attendance. Notwithstanding, the students’ evaluation of the use of PI was a moderately positive.

Introduction
Decreasing class attendance has been noted as a problem for most computer science modules in the Computer Science Department at UCT, and it has been observed (anecdotally) at other institutions of higher education as well. Various hypotheses and opinions exist to explain why this is the case. Among others, it may be because of the standard practice of the ‘sage on a stage’ adopted due to the theoretical, abstract, nature of the material, which can make a lecture dull. Lectures and lecturers do add value in various ways, however, and students miss out not attending them1. The assumption is that a different set-up of the lecture, being a more interactive way of knowledge transfer and learning, will increase attendance. The challenge, then, is how to make the lectures at least less dry and passive ‘sponge’-like so as to make students want to come to lectures.

Several possibilities to change the lecture format exist, such as the “flipped classroom” (Tucker, 2012), converting at least part of the lecture into a tutorial to give students a feeling of hands-on experience compared to a set of slides, peer instruction (Crouch and Mazur, 2001), and other research-based options (Borrego et al., 2013). Peer instruction (PI) not only encourages students to be active participants in class, but—moreover—it has been shown that students will learn the concepts better (Crouch and Mazur, 2001). PI has been used in computer science education, with positive results both regarding student learning outcomes and grades and their predominantly positive opinion of lectures with PI, and it is gaining momentum in education research for computer science and engineering (Bailey Lee et al., 2013; Borrego et al., 2013; Koppen et al., 2013; Simon et al., 2013; Zingaro & Porter, 2014), with as main online resource http://www.peerinstruction4cs.org. The authors’ personal experiences with peer instruction in a low resource setting have been positive, most notably the students’ joy of doing quizzes and it generated an increase in lecture attendance each lecture after such quizzes were held. However, those numbers of participation and attendance

1 Reasons for what they ‘miss out on’ is a separate line of investigation, not pursued here.
were only estimated, for they were carried out in a low-resource setting\(^1\). There are only few other works mentioning PI and lecture attendance (Duncan, 2006; Kaleta & Joosten, 2007), with unclear results. The hypothesis, then, is that peer instruction in computer science increases lecture attendance.

To examine the effect of peer instruction on lecture attendance, a subset of the lectures of a large 3rd-year computer science course on networks each had 2 concept tests (‘quiz questions’) with PI, where class attendance was counted at the commencement of each lecture and during the first concept test. To obtain a measure of results, medium-resource software-based ‘clickers’ (Audience Response Systems, ARS) were used, befitting a course on computer networks. The results show that peer instruction did not increase class attendance among the computer networks students after a lecture with PI, and percentage of voting in the questions declined with each instalment despite that peer discussion had a beneficial effect on voting results. Notwithstanding the measured decline, course evaluation revealed a moderately positive opinion of PI.

**Literature review: PI in CS**

Peer instruction, in short and procedurally, is about students teaching each other (their peers), where first a multiple choice question (concept test) is posed by the lecturer in class, students vote on an answer, they then discuss the question and answers with their neighbours in the lecture, then they vote again on the same question, and finally there is a class-wide discussion of the question. This makes a lecture at least more active compared to knowledge transfer by means of a monologue by the lecturer (also referred to colloquially ‘sage on a stage’). Such a general shift toward the implementation of the theory of active learning may have been encouraged by the ARS technologies, but experiments to date, which are mostly carried out in STEM fields, focus more on the implementation and effects of ASR for PI on learning outcomes and student perception rather than pedagogical theories or what the right conditions are for using ARSs (Good, 2013). A recent literature review by Good (2013) on the use of ARS for PI notes its overall positive effects, albeit with various ifs and buts. Regardless, peer instruction in Computer Science (CS) is gaining momentum after its successful introduction in physics (Crouch & Mazur, 2001) and elsewhere, such as genetics, astronomy, and veterinary dermatology (Duncan, 2006; Good, 2013; Smith et al., 2009).

PI in CS is still at the stage of having to argue its usefulness. This is in particular for higher-level CS courses that typically deal with very abstract ideas and concepts, with complex judgement of design trade-offs, and advanced mathematics that does not seem to lend itself well for bite-sized quizzes. Bailey Lee et al. (2013) validated empirically repeatable student gains and overwhelming positive feedback with the courses they experimented with, being the engineering-oriented computer architecture and mathematics-oriented theory of computation (though the latter is taught only to a very limited extent in South Africa (Keet, 2013)). Although the quiz questions (available from peerinstruction4cs.org) are not as deep as the Gradiance multiple-choice questions based on “root questions” (Ullman, 2005), they demonstrate that concept questions for such advanced theoretical courses are feasible. To the best of our knowledge, no such resources exist for the topic we focus on (computer networks). While the setting of experiments on PI in CS typically occurs at large research-led

---

\(^1\) That is, without ‘clickers’ to measure participation: it proceeds by using the fingers of one’s hand against one’s chest to indicate a vote so that the lecturer can get an impression of the votes. In practice, it also occurred that students said their vote aloud.
universities, it also has been shown to be effective at small so-called ‘liberal arts colleges’ (Porter et al., 2013).

Students generally have a positive opinion of students about PI (e.g., Duncan (2006), Good (2013) and references therein). Simon et al. (2013) investigated in a targeted experiment on students’ perceptions of PI for CS as well. They concluded that students find lectures with peer instruction overwhelmingly “interactive” (argue/explain concepts, learn from or teach partner) as compared to plain “active” (listening and/or note-taking), with the former decidedly deemed positive by the students for a broad range of reasons, such as valuing the interaction in the classroom with peers, higher perceived approachability of the lecturer, and a community spirit.

Besides the basic or ‘classic’ process of PI as outlined at the start of this section, several variations exist that have been shown to improve the overall positive effects of PI, such as PI for marks (Zingaro & Porter, 2014) and the measured added benefits of lecturer-led class-wide discussion (Zingaro & Porter, 2014). Further, group discussion works best when between 35-70% of the students give a correct answer during the first vote (Crouch & Mazur, 2001), although voting for a wrong answer with subsequent discussion is beneficial to learning anyway (Smith et al., 2009). Other modifications have to do with the actual voting. Besides the tried-and-tested hardware-based ARSs, medium resource software-based ARSs are being used (e.g., Koppen et al., 2013), which are typically conducted with the students’ laptop or smartphone and a wireless connection to an online resource. This may be considered exceedingly applicable in CS courses, because CS students tend to have mobile devices more often than in other disciplines. Some decline in quiz participation with the online voting systems have been observed over the duration of a course in the semester, but 75% of the students still actively participate in answering the question even though not pressing the button to vote (Koppen et al., 2013).

Thus, there are several initial experiments and results with PI in CS that focus on feasibility, improved student learning, and student experience. To the best of our knowledge, no experiment has been conducted to assess whether PI in CS will increase lecture attendance (or at least not decrease it). Duncan (2006) does state it can be used to increase class attendance, but a reference to the study claiming an increase from 60-70% to 80-90% attendance after the introduction of clickers at the University of Colorado is missing. Kaleta & Joosten (2007) indirectly indicate that PI may increase class attendance, as students like the active learning and 64% of the students would sign up for another course that uses clickers; similar indirect indications can be found elsewhere (see, e.g., Duncan (2006) and references therein). Thus, this still leaves open whether including concept questions will increase class attendance, or at least not decrease it.

Materials and Methods

We first describe the set-up of the experiment, and subsequently describe the setting of the course where the experiment was conducted.

Set-up, running, and evaluation of the peer instruction

This section outlines the overall set up and the motivation of selection of the PI technology chosen.

The methodology can be summarised as follows.

1. Decide on medium-resource (wifi and software) PI set up, comprising: a) Examine options and choose which online tool to use; b) Devise a back-up solution if either it
does not work in the venue for some reason or too many students do not have a laptop or smartphone to participate.

2. Select the lectures when to use PI: given that there will be 19 lectures (see below): lectures 3, 7, 11, and 15.

3. Create questions for the relevant sections of the course material, given the indication when to use PI.

4. Record the number of students in class for each of the 19 lectures by silent manual headcount by the lecturer.

5. Make any other notes that may be relevant; e.g., the lecture is on a day between public holidays, how the quiz participation is perceived, etc.

6. Conduct PI and record participation in the quiz and the answers, being both the first answer and, where applicable, the repeat answer after the peer discussion. The process is modified slightly to incorporate the observations by Crouch & Mazur (2001) that peer discussion is most effective when 35-75% had the question correct in the first vote, and that class-wide discussions do contribute to understanding (Zingaro & Porter, 2014) and to stimulate discussion for the <30% correct answer instances; see Figure 1.

7. Evaluate the data in standard spreadsheet software, including the number of students attending, number of votes cast, average number of votes cast, and the percentage of participation in the quizzes (calculated as [avg no. of votes for instalment]/[no. of students attending that lecture]*100).

8. Use the general course and lecturer evaluation forms to obtain feedback about the PI specifically:
   - Lecturer evaluation: One closed question is added, being “Quizzes: To what extent did they make the lectures more interesting or useful? (1: not at all … 5: very interesting and/or useful)”, and the question-associated “comments” section.
   - Course evaluation: The standard question on quizzes is used, being “Quizzes: were you satisfied with the content and frequency of quiz questions in the lectures?”, which has as answer options ‘poor’, ‘below average’, ‘average’, ‘good’, ‘excellent’.

![Figure 1. PI in the classroom. A: original version; B: slightly modified PI process (with input from experiences and suggestions by Crouch & Mazur (2001) and Zingaro & Porter (2014)).](image-url)

Concerning the materials, the wireless connection and online voting with a ‘software-based clicker’ will be used. In order to determine suitable ‘clicker software’, several sources were
consulted and tools explored. This concerns principally the clicker software evaluation by the Princeton University Educational Technologies (McGraw Centre for Teaching & Learning, 2012), covering 19 different software-based ARSs. Several software ARSs did not appear to meet the basic requirements upfront (Socrative\(^1\) and eClicker\(^2\)), and four were selected for evaluation, being Google Forms\(^3\), Mentimeter\(^4\), Pinnion\(^5\), and Qurio\(^6\) that all have a free version. Google Forms and Pinnion are too cumbersome for releasing the individual questions to the students in a classroom setting and Qurio had an annoying website. This left Mentimeter as best option for this experiment. It has a limited free version, the backend administrator side is easy, and each question can be released on the fly with a simple URL, and has an easy reset button for the revote. The free version does not permit export of the results, which can be remedied by taking a screenshot of the results page. Limitations for which there are no workarounds are that it does not allow for pictures in the question, and the answer options have a maximum of 100 characters. For those questions where the answers were longer than 100 characters or had a diagram, the ‘offline’ version of the question was projected on screen during voting.

The time-consuming aspect for the set-up is the generation of good concept test questions, most of which have to be developed specifically for this experiment. The questions will be developed ‘offline’ in a separate file, which functions as backup in case of a network outage, and subsequently copied into Mentimeter for use in the lecture. A selection of the full set of prepared questions is included in the Appendix, and one is depicted in Figure 2 and explained afterward.

![Figure 2. One of the more advanced concept questions later in the course](http://www.socrative.com/)

\(^1\) http://www.socrative.com/
\(^2\) http://eclicker.com/
\(^3\) http://docs.google.com
\(^4\) http://www.mentimeter.com
\(^5\) http://www.pinnion.com/
\(^6\) http://www.qurio.co/
This question for lecture 15 (Figure 2) tests whether students can apply the Address Resolution Protocol (ARP) and, more importantly, understand the conceptualization of the division between, and separation of, data link layer and network layer services, which has as further underlying principle the idea of layering (which is part of the “great principles in CS”). Option A can be true, because of host A’s broadcast, which is a basic ARP operation, and so is what is described in option B. Option C versus D is really about the separation of link and higher-level network layer services, for which a basic understanding of ARP is required. Link layer MAC addresses (like 49-BD-D2-C7-56-2A) only operate within that subnet cf. IP addresses (like 111.111.111.111) that operate at the network layer and, unlike MAC addresses, can pass through router R. Thus, the IP address of hosts A and B can pass through R, but the MAC addresses remain on each subnet, i.e., D is correct. The consequence of this separation is that when router R receives a packet addressed to 449-BD-D2-C7-56-2A, it will discard it before even looking at the IP address, because its address on that side is E6-E9-00-17-BB-4B (evaluating that that packet is not for itself), thus C is the correct answer: a packet with those destination addresses can never arrive at host B.

Setting of the course

The experiment will be conducted in the “Networks course” in 2014, which is the third block of the larger course CSC3002F taught in the first semester. CSC3002F is split into a block on operating systems, one on functional programming, and, last, computer networks. Practically, this means that computer networks is taught from mid-April to the end of May over 19 lectures, one each working day from 9:00-9:45. With the current academic timetable, there are many public holidays in that period. Class attendance is highly recommended, but not compulsory, and there are no attendance registers. Labs are largely self-directed, and students are expected to spend two afternoons each week on the exercises and the assignment. The course is compulsory for all computer science students at level 3, and it is elective for computer engineering students, comprising typically about 100 registered students.

Computer networks is a regular course and is internationally not known for being comparatively easy or hard. The course content consists of chapters 1-6 of the textbook (Kurose and Ross, 2013), which amounts to roughly 3 lectures per chapter. The book’s instructor materials has 90-140 slides per chapter, which would amount to 1 minute per slide if used in full. The number of slides were reduced, creating time for interaction with the class whilst ensuring maintaining compliance with the ACM curriculum guidelines for computer science (Joint Task Force ACM & IEEE, 2013).

Results

We describe first the results on PI and class attendance, then the course and lecturer evaluation, and close with some observations on the PI learning curve for students.

Peer Instruction and class attendance. 104 students are registered for the course, and at the first networks lecture, about 60 students attended. Student attendance per lecture is depicted in Figure 3; they are rounded up from the count at the start because several students typically arrive in the first 5-10 minutes of the lecture. The relative dip in attendance of lecture 6 may be due to it being on Friday, May 2, where May 1 was a public holiday, suggesting an extended weekend, and lecture 12 (May 14) was at the same date as the assignment deadline. A “topical relevance” email was sent after lecture 9 about net neutrality, which had its main relevance for lecture 11, and a slight increase in attendance was noted, but this may also be because it was the lecture before the assignment deadline.
Figure 3. Student attendance (Y-axis) at each lecture (X-axis), and averages of amount of votes for the concept questions.

The schedule set out for the quiz questions—lectures 3, 7, 11, and 15—was not fully adhered to due to being out of step with sensible PI times with respect to the material covered. Peer instruction was used during lectures 3, 7, 10, and 15. Because the material was covered at a slightly different pace than initially planned, overall, 22 questions were prepared and 10 have been used. Table 1 shows the voting response rates by question. The average amount of votes per question for an entire PI session is included in Figure 3. It can be calculated from this data that the percentage of students participating in the concept tests shows a gradual decline: from 67% for the test question in the first lecture, to 57% in lecture 3, 50% in lecture 7, 43% in lecture 10, down to 40% in the final instalment during lecture 15. No increase in attendance after the more interactive lecture with peer instruction can be observed. There is also no substantial decrease after the lecture with peer instruction, although there is a slow gradual decline in attendance from about 45% to about 25-30%.

Course evaluations. There were 43 answers for the question about quizzes. Aggregating the 5 options to three, then 11 considered it below average, most students considered it average (n = 22, i.e., 51%), and 10 above average, which is overall slightly positive. The lecturer evaluation form’s question about the quizzes had 26 answers (of which 7 chose N/A) with a mean of 3.38 out of 5. A total of 6 students considered it below average, 7 average, and 13 above average (i.e., chose 4 or 5 on the 5-point Likert scale), which is overall more positive than the course evaluation feedback. Furthermore, there were PI-specific comments on the PI-question and overall comments sections in line with observations by others (see Discussion):

- "I liked the quizzes, they made the lectures more interactive."
- "It was good to see a computer science lecturer actually make use of technology other than powerpoint."
- "The online voting tool that [xxx] used made the lectures more interesting and useful, as it encouraged class participation."
- "Using GoVote really added to the lectures."
- "enjoyed the online classes quizzes helped me stay awake!"

There were three comments in the open-ended comments section of the type “those lectures were quite boring”, but it cannot be determined whether those students had attended the PI lectures (their self-reported lecture attendance was 0-50% or 50-80%).
Table 9. Count of the votes for each question; v1 and v2 denote first vote and revote, respectively; Lx lecture number; Qx question number (see Appendix A for questions).

<table>
<thead>
<tr>
<th>Question – topic</th>
<th>Answer option</th>
<th>First vote</th>
<th>Revote</th>
<th>No. votes cast</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1Q1 – warming up question</td>
<td>A 1</td>
<td>B 29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C 1</td>
<td>D 4</td>
<td>E 5</td>
<td></td>
</tr>
<tr>
<td>L3Q1 – protocols</td>
<td>A 2</td>
<td>B 0</td>
<td>C 2</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>D 23</td>
<td>E 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3Q2 – switching and ISPs</td>
<td>A 14</td>
<td>B 5</td>
<td></td>
<td>First vote: 30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C 5</td>
<td></td>
<td>Revote: 24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L3Q4 – TCP/IP stack</td>
<td>A 2</td>
<td>B 3</td>
<td>C 7</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L7Q1 – sockets</td>
<td>A 2</td>
<td>B 6</td>
<td>C 7</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L7Q2 – UDP</td>
<td>A 1</td>
<td>B 0</td>
<td>C 1</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D 21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L10Q1 – 3-way handshake</td>
<td>A 6</td>
<td>B 4</td>
<td>C 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>First vote: 14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Revote: 12</td>
</tr>
<tr>
<td>L10Q2 – TCP congestion control</td>
<td>A 1</td>
<td>B 2</td>
<td>C 5</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D 5</td>
<td></td>
</tr>
<tr>
<td>L15Q4 – basic ARP</td>
<td>A 1</td>
<td>B 10</td>
<td>C 1</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D 1</td>
<td></td>
</tr>
<tr>
<td>L15Q5 – advanced ARP</td>
<td>A 4</td>
<td>B 3</td>
<td>C 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>First vote: 14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Revote: 15</td>
</tr>
</tbody>
</table>
Peer instruction learning curve. There was a learning curve for peer instruction among students, and an initial hesitation to participate. The hesitation was due to the fact that multiple students were afraid their answers were recorded to the extent that it would be revealed who answered what, and the syllabus of the course as a whole announced that quizzes may be for marks. It had to be repeated several times it was not for marks and that Mentimeter does not log who voted what\(^1\). Regarding the learning curve of peer instruction itself, the results of L3Q2 compared to L7Q1 show this well (see Table 1). For the first real instalment, L3Q2, it still was about voting, then waiting and chatting, and then re-voting. Form the voting pattern it can be seen that the students who had it wrong the first time (option C, which was explained why it was not correct) simply did not vote the second time—30 votes versus 24 votes, where 5 had been given to option C—with the rest voting as they had voted before. In contrast, in lecture 7 they were very explicitly instructed they really had to convince their neighbours of their answer and not use the opportunity to catch up with what they did in the weekend or chat about other things like the previous time (L3Q2). This had the desired effect, as can be seen by comparing the First vote and Revote of L7Q1 in Table 1, and it generated joy and laughter in class when the results of the re-vote were shown (see Figure 4). While peer instruction in the sense of enhancing concept understanding was not the aim, its beneficial effect is nicely illustrated with L7Q1’s vote and revote results, which was also observed with L10Q1 and its revote. L15Q5 (recall Figure 2) had too few correct answers in the first vote—it was almost a random distribution (see Table 1)—and, as observed elsewhere (Crouch & Mazur, 2001), peer discussion is then fruitful to find the correct answer only to a very limited extent.

**Figure 4.** Screenshot of the results of Lecture 7, Question 1, re-vote after discussion.

Discussion

The low first-lecture attendance of about 60%, and from the second lecture 45% of the registered students, indicate that a majority is ‘lost’ upfront and likely will not even have been aware peer instruction was being used. It suggests that a core of students diligently attends lectures regardless the quality of the lectures, and it might not make a difference what

\(^{1}\) Mentimeter most probably does log usage (IP address, at least), but this information is not available to Mentimeter users.
is done during the lectures. Either way, PI specifically did not increase lecture attendance. The comments in the lecturer and course evaluations, however, do suggest PI is deemed an improvement. The type of feedback obtained from the evaluation, including different motivations for the positive opinions, concurs with Simon et al.’s (2013) data about the wide range of reasons for liking peer instruction and suggests it may be useful to continue with peer instruction, even if only for the benefit of the minority who attends lectures.

It cannot be excluded that PI was not explained sufficiently well, which is known to contribute to lack of enthusiasm to participate (see Good (2013) and references therein), and it could be that it has not been used often enough so that its effects would be too limited to note (Duncan, 2006), which can be evaluated with successive instalments of the course. Also, one could conjecture that the book may be clear enough so that students may be able to master its contents without attending the lectures; however, testing such a hypothesis would be unethical, and the exam results were not very good. Overall, it remains guesswork as to why many students do not attend lectures and there are many other research-based instructional strategies one can choose from, such as Think-aloud-paired problem solving and Think-pair-share (Borrego et al., 2013). To better choose interventions to increase lecture attendance, it would be useful to first find out why attending students do attend, and why those who do not, stay away, compared to more in-depth PI evaluation, such as validating the known Normalised Gain of PI (about 34%-45% (Bailey Lee et al., 2013)).

Several other useful observations can be made based on the data obtained. First, decline in participation as measured by Mentimeter in first instance might indicate that the students got bored with ‘quizzes’ and they were unfamiliar with them in this form. The more likely reasons probably have to do with the more mundane aspects of timeously logging on to the wireless network or the mobile device being low on battery, which were observed by Koppen et al. (2013), but they also found out that in the absence of a mobile device to vote, only 1% “didn’t think along”, some thought about the question and the vast majority worked together with their neighbour(s), having validated results obtained elsewhere (Smith et al., 2009). Thus, a lower measured voting rate does not imply lower participation rate.

Second, none of the tested software ARSs was ideal. Mentimeter’s artificial limit on question and answer lengths does not fit CS well, and an option to use diagrams will make the presentation better compared to the clumsy dual view of the voting page online and the ‘offline’ question in OpenOffice Writer. The need for more characters and diagrams is implicitly also present in the computer architecture and theory of computation PI questions—9 of the 11 questions in (Bailey Lee et al., 2013) go beyond Mentimeter’s question capabilities—and theory of computation and several other CS courses also need mathematics symbols. Failing extant software-based ARSs, we plan to design our own one for the next instalment.

Third, reflections can be made on the so-called “fidelity of implementation” of the research-based instructional strategy, which varies widely in general (Borrego et al., 2013). This experiment also had one of the many possible permutations that stay faithful to the classic PI process (recall Figure 1). Besides the emphasis on lecture attendance and the observation that discussion among students improves their understanding, it may be that some of the questions were not as good as they should have been. Unlike in physics, a concept inventory does not exist for CS, and, to the best of our knowledge, no concept tests existed yet for networks and thus had to be created for this course. The quality of Concept Tests, then, rely in part on knowing all common misconceptions and to have those woven into the answer sets. Some of the questions prepared could possibly be categorised under, and pushed into, pre-lecture
readings and self-tests (see Appendix for a selection of the prepared questions). Although adding pre-lecture quizzes to the overall PI process may be beneficial (Crouch & Mazur, 2001), it will require a substantial change in learning pattern of the students and such pre-lecture quizzes have the danger that when a student does perform well on them, it may give a false sense of security of knowing all testable content. Notwithstanding, the textbook is quite detailed in its explanations, and it may be tried out for the next instalment. Further, the amount and difficulty of the concept test questions and PI intervention will be extended for the next instalment of the course—if not for increased attendance, then at least a better grade for the course, which in turn may motivate students to come to the lectures.

Conclusions

Using peer instruction has not resulted in increased class attendance. It has not resulted into a stark decline due to the quizzes either. The student participation rate in voting decreased from 57% to 40% and an overall decline in attendance was observed from about 45% to 25-30%. Whether the participation is thanks to the peer instruction, or because a hard-core group of students diligently attend lectures anyway, is unclear. Student evaluations indicate a moderately positive opinion of the use of peer instruction. Together with noted improvements in course grades elsewhere, the results are in favour of continuation of peer instruction. Nevertheless, it would be useful to determine why students stay away so as to devise an experiment that may have a higher likelihood to be effective in increasing lecture attendance.

References


**Appendix A: Selection of the Quiz Questions**

Notation for the questions numbers: Lecture numberQuestion number, boldface: question used during that lecture, plaintext: not used; italics: correct answer.

**L3Q4**: Consider the IP stack. Which of the following is true?

A. Application layer: networked applications, like FaceBook
B. Transport layer: process-process data transfer
C. Network layer: packets sent through various ISPs from source to destination
D. Link layer: data transfer between neighbouring routers
E. Physical layer: bits on the transmission medium

Many new terms have been introduced at this stage, and this question is about putting the right ones together and to go over the notion services at each layer. Hence, this is essentially a recall question.

**L7Q2**: Which of the following statements is true about UDP?

A. UDP does not do demux
B. UDP uses a selective repeat as part of its protocol, to make sure all the packets arrive
C. The connection-oriented aspect for UDP is handled by the lower (network) layer
D. There is no handshaking between sender and receiver in UDP

The underlying issue is that UDP is connection-less, unlike TCP, which has several processes and consequences, no handshaking is one of them.

**L10Q1**: Why is a three-way handshake needed for TCP connections, rather than a two-way handshake? Choose the best option.

A. *With a two-way handshake, then if the client terminated after the SYN message, but the server receives and accepts the SYN message, connections are left open but there is no client anymore, which is an undesirable state and is prevented by 3-way handshake.*
B. The three-way handshake is needed because the server needs to know whether the client accepts the sequence number proposed by the server, which is not guaranteed to with only a two-way handshake.
C. The server needs to know from the client whether the client has received its SYNACK message correctly, so that it can open the port it has reserved for the client, whereas with a two-way handshake it would never open the port because there is no guarantee that the client is still alive for the connection.

This question probes deeper into handshaking, and understanding the reasons behind it.

**L10Q5**: BGPs. Consider the network below in which network W is a customer of ISP A, network Y is a customer of ISP B, and network X is a customer of both ISPs A and C. What routes will X advertise to A?
A. It will advertise C, since it is directly connected to it.
B. It will advertise C and B, since they are ISPs other than A.
C. It will advertise C, B, and Y, since they are reachable without going through A.
D. X will not advertise any routes to A, since X is a customer network, not a transit network.

This is about grasping the flow of routing information, and understanding who advertises what to whom, and why.

L15Q1. Which of the following is/are link-layer services? Choose the best option.

A. Reliable delivery between adjacent nodes, with error detection and correction
B. Semi-reliable delivery between adjacent nodes with flow control and error detection only
C. Unreliable delivery, but with flow control and choice between half-duplex and full-duplex to mitigate that

This is a recall question, with a few newly introduced concepts.

Contextual note: this is a recall question, with some added challenge on newly introduced concepts.
Emancipating Secondary School Teachers from their Technology Knowledge and Pedagogical Challenges: An Action Research Study

¹Tomé Awshar Mapotse & ²Mishack Thiza Gumbo

¹ ²University of South Africa (Unisa)

¹ ²Department of Science and Technology

¹mapotta@unisa.ac.za; ²gumbomt@unisa.ac.za

This paper reports the findings from an action research (AR) study which was conducted to help emancipate secondary school teachers from the challenges that they faced in the knowledge and teaching of Technology at Mankweng Circuit of Limpopo Province between 2010 and 2012. The paper responds to the following research question: “How can AR interventions be used to strengthen subject knowledge and pedagogy of South African secondary school Technology teachers?” The findings of the study are crucial in informing the professional development of Technology teachers. Reconnaissance, which we called Phase 1, was instrumental in identifying the challenges faced by teachers. Data collection in this phase was done through observation, interview and questionnaire in five selected secondary schools in Mankweng Circuit of Limpopo Province. Phase 1 was followed by intervention in Phase 2 to address these challenges. Field notes, logs of meetings and audio-visual pictures of class presentations were used to gather data in this phase. The main focus of this paper is on Phase 2. The Phase 1 findings revealed the Technology teachers’ challenges in teaching experience, lesson planning, assessment, level of internal and external support, teaching-learning resources, interpretation of curriculum policy and implementation and teacher-learner ratio. Findings for Phase 2 revealed some level of improvement between the before- and post-intervention stages.

Background

Kufaine and Nyirenda (2013) claim that science and technology should equip the individual with knowledge, skills, values and attitudes in order for the graduate to contribute towards the socio-economic development of a nation. Science and technology is thus viewed as a catalyst for the development of the individual and the nation (Nampota, Thompson & Wikeley, 2009). However, in the South African context Technology teachers are still lacking in their competencies to teach Technology as per the findings from Phase 1. In fact, The Minister of Department of Basic Education (DBE), Motšekga (2011) avers that there is a wide agreement amongst education stakeholders that subject knowledge amongst teachers is often well below what it should be.

The challenges that teachers faced are much related to curriculum developments in South Africa. The introduction of Technology Education packaged in a few-times-reviewed curriculum has raised interesting issues which include the conceptual understanding of Technology (Shafër, 1999). The introduction of Technology Education has posed different challenges compared to other subjects (Rauscher, 2010). These challenges are exacerbated by the fact that Technology teacher training has been combined with other subjects despite its unique nature and approach being much different from other subjects. Technology Education is also relatively new compared to other restructured subjects from the old curriculum. The majority of in-service teachers found it difficult to understand the curriculum (De Jager, 2011), which was ultimately modified to Curriculum and Assessment Policy Statement (CAPS), which is actually the implementation strategy of the National Curriculum Statement.
Thus, the NCS remains, while CAPS is an implementation guidelines (Motshekga, 2010; De Jager, 2011). However, Technology teachers seem to experience challenges as it is revealed in this study; 99% of these teachers are not qualified to teach Technology (DoE Gauteng Memo 202, 2004). Conducted studies in the field reveal attempts to strengthen Technology teachers’ subject knowledge and pedagogy. Stevens (2006), De Vries (2007), Middleton (2009), Techno Moodle (2010), Williams and Gumbo, (2011), Mapotse, (2013) and Yager (2013) attempt to raise some aspects on pedagogy and content knowledge. However, little research has been conducted to strengthen the teachers’ pedagogy and content knowledge through AR.

During the evaluation of student teachers' practice teaching in the mentioned circuit above it was observed that teachers lacked some knowledge about the teaching of Technology. The assigned mentor teachers to practicing students were not in possession of the Technology curriculum policy documents in the first place. The planning of the learning programmes and lessons was disorderly in the sense that it was not done according to the phase or grade. Different topics were treated and this was confusing even to the practicing teachers and learners. Learners were without any textbooks as they only relied on their teachers. There was no single Technology Education project. Teachers claimed that they did not have resources. They could not think of using the available resources lying around. Student teachers initiated the utilisation of resources in the surroundings and guided their mentors and learners in that regard. Student teachers ended up exchanging roles with their mentors because they seemed to know more than their mentors.

Critical theory underpinned this study so we could explain the phenomenon at hand – teachers’ challenges – guided by the current knowledge (Johnson & Christensen, 2004). Through critical theory teachers could be helped to transform their situation (Giroux, 1988; Fullan, 1993). Our choice of critical theory was motivated by our intent to help emancipate the Technology teachers form their challenges. These teachers could be helped by letting them reflect critically on their challenges and think about creative ways to free themselves from such. The participatory principles of AR would enable us achieve this goal.

We proceed by detailing the research design of the study.

**Research design**

The main research question is stated as: *How can AR interventions be used to strengthen subject knowledge and pedagogy of South African secondary school Technology teachers?*

This research question is addressed through the following sub-questions:

- What are secondary Technology teachers’ challenges to teach the subject?
- How can these teachers be emancipated from these challenges in order to capacitate them to teach Technology?

This study was an AR study. AR was desired as a design to better address teachers’ challenges. Our standpoint was that the optimum way to intervene in the situation of teachers was to engage them actively via AR with a desired goal for them to be ready to implement what they have acquired from the intervention activities. AR was sought in this study since it can convert Technology teachers to serve as change agent (Carson, 1990) and be self-reflective practitioners (Schön, 1983). AR thus became the springboard to address the ‘how’ question above. The sample was drawn from the Capricorn Region at Mankweng Circuit in Mankweng District. The choice of Mankweng Circuit was prompted by the lack of Technology knowledge observed previously by one of us stated above. The aim of
delineating the scope of the study was to implement some intervention strategies to a manageable sample of secondary school Technology teachers teaching Grades 8 and 9. Mankweng Circuit cluster sampled. In this cluster sampling Technology teachers were randomly selected (Gay, 1987) from their schools. Table 1 shows how the sample was done.

**Table 1.** Sampled schools and Technology teachers

<table>
<thead>
<tr>
<th>School Name</th>
<th>Number per school</th>
<th>Grade 8</th>
<th>Grade 9</th>
<th>School milieu</th>
</tr>
</thead>
<tbody>
<tr>
<td>KMK secondary</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td>Rural</td>
</tr>
<tr>
<td>VMV secondary</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>Semi - urban</td>
</tr>
<tr>
<td>RMR secondary</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>Rural</td>
</tr>
<tr>
<td>BMB secondary</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>Rural</td>
</tr>
<tr>
<td>WHW secondary</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>Semi - urban</td>
</tr>
<tr>
<td>Total</td>
<td>18</td>
<td>7</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

Cluster samples were drawn from five secondary schools (see Table 1) at Mankweng Circuit. Cluster sampling is characterised by some degree of homogeneity (Maree & Pietersen, 2010; McMillan & Schumacher, 1989). Though the sampled schools are located in varied milieus (rural and semi-urban), they were all secondary schools who also taught Technology. It should also be noted that our focus was on eighteen Technology teachers sampled from these schools. Pseudo names were assigned to the schools to conceal their true identity.

Permission to conduct research was sought and granted by Department of Basic Education in Limpopo Province. The Circuit Manager was duly informed. The teachers consented to participate in the study. Even though learners were not directly involved in the study, we sent consent letters to their parents with the help of teachers. This was because as part of data gathering to assess the extent of the needed intervention we observed teaching take place in the classrooms and audio-visually recorded lesson presentations. Observation, interviews and questionnaire (Phase 1) and observation and interviews (Phase 2) were the predominant monitoring and data gathering techniques. Those in Phase 2 were supplemented by field notes, logs of plenary and reflection meetings, audio-visual pictures (for class presentations).

In Phase 1 we were interested in observing whether Technology teaching took place in Techno-labs if there were any, or if it took place in ordinary classrooms, whether the classrooms were equipped with Technology materials and equipment like cutting tools, vice benches, Technology kit, etc, whether there were any learner-made artefacts, whether teachers had the Technology curriculum document to plan their lessons from and if they did, whether they understood it, whether there was a support of some sort given to the teachers by schools and circuit/district, etc. While a few of these were also observed in Phase 2, the main focus of Phase 2 observations was on lesson presentations by teachers. The interviews during Phase 1 were aimed at establishing the challenges that teachers faced. The questionnaire sought to elicit biographical information presented in Table 2. It also contained items related to the challenges that teachers faced in their practice – Technology teaching experience, Technology lesson planning, Technology assessment, level of internal and external support for Technology teaching, resources for Technology teaching and learning, interpretation of Technology curriculum policy and implementation, and teacher-learner ratio in a Technology class. The data gathered through this questionnaire helped to identify and confirm challenges that teachers faced regarding the knowledge and pedagogy of Technology, thus addressing
the first research question – What are secondary Technology teachers’ challenges to teach the subject? This question has also been addressed by exploring the challenges that Technology teachers face in the theoretical section above. We always started with observations.

During Phase 2 AR intervention strategies were developed to address the identified challenges. Strategies were planned during plenary meetings by the team (researchers and teachers) whilst ensuring data gathering through the stated monitoring techniques above. The intervention strategies entailed four cycles starting with the second cycle (Phase 1 was viewed as Cycle 1) characterised by plenary and empowerment meetings, implementation of the planned actions from the meetings, which included planned lessons, and reflection meetings/sessions. These cycles are briefly explained:

Cycle 2: Each school was visited to provide feedback emanating from the findings of the reconnaissance and to discuss the way forward to address the identified challenges.

Cycle 3: The third cycle about a plan to address the challenges raised by the participants. A meeting was held to first reflect on the second cycle and chart the way forward. Figures 1 and 2 depict the meetings held where teachers were divided into two grade groups, i.e. Grade 8 and Grade 9 groups.

Figure 1. Grade 8 participants

Figure 2. Grade 9 participants

These photo pictures in figure 1 and 2 are sourced from Mapotse (2012:96)

In this meeting the two groups discussed the design projects and assessment rubric for their learners. Grade 8 group decided to do a project on containerisation and transportation of chemistry test tubes, while Grade 9 group settled for a “Classroom Dustbin” project. In order not to side-track the schools plan, this planning followed the Mankweng Circuit’s work schedule, which at that stage was on the theme “Processing”. Addressing the teachers’ challenges was coupled with unpacking the Technology curriculum policy; contextualizing the teaching of Technology; processing; systems and control and electronics; graphics in Technology; and the interrelationship of the knowledge areas in Learning Outcome 2, i.e. processing, structures and systems and control and electronics (resistor and resistance, Ohm’s Law and colour coding, and logic gates). The two groups went further to design a marking rubric for their grade projects. Teachers specially requested a session on electronics as they perceived it as their most challenging thematic aspect.

Cycle 4: This cycle started with a visit to the schools. Grade 8 and 9 brought along their made projects and portfolios. We noticed and confirmed that this was for the first time teachers let their learners make a Technology project. Then at a common venue, during a meeting, teachers wrote down their reflections on the projects, including the challenges encountered,
gaps identified and their remedies. A revision on graphics and electronics was done. A
teacher from RMR secondary school had a perfect-to-scale grid. She said that she was helped
by a colleague who had a drawing background. This cycle also unpacked the Curriculum and
Assessment Policy Statement for Technology to teachers. The cycle ended with reflections
and the planning for the following two weeks cycle (Cycle 5).

Cycle 5: After lesson presentations the evaluation of the very lesson presentations was done
and feedback given by the peers from the same school and us researchers. The schedule was
jointly drafted during Cycle 4. Each participant was assigned a date for lesson presentation.
Thus, a programme was drawn for this seminar. Based on the responses of the teachers to the
reflection interview questions, which showed a high level of satisfaction about what had been
covered, this was also the closing cycle for the AR.

The audio-visual recording was used for lesson presentations and the made artefacts. Field
notes were taken throughout the investigation, particularly during the meetings (logs of
meetings) and observations. These monitoring methods helped to answer the second question –
How can these teachers be emancipated from these challenges in order to capacitate them to
teach Technology? However, observation and interviews were the key data gathering
techniques. The observation focused on the lesson presentations that were planned by the
team of co-researchers (researchers and teachers) in keeping with the AR principles of co-
researching with participants and co-participation. We thus co-observed with teachers their
colleagues who presented in order to provide multiple perspectives of the observation data.
The interviews were about gathering data from the teachers about any improvement that they
thought took place in their knowledge of Technology and practice post-intervention. The
interview schedule included five important items that teachers were asked to respond to:
What have you learnt from the action research activities planned and implemented this far? What are you taking along to your school from these activities? Indicate the technological
themes that you can now implement with confidence in your lesson, especially those that you
couldn’t before the cycle. What gaps have you identified that still need to be filled regarding your knowledge of Technology? Any other inputs/suggestions/proposals you have for a way forward?

Data analysis followed a thematic and narrative form. Themes that emerged from the analysis
of the three main data types – observations, interviews and questionnaire for Phase 1 and
observations and interviews for Phase 2, guided the presentation of findings. The data were
thus triangulated based on these data types to strengthen the study (Anderson, 1993; Patton,
2002). The use of multiple data gathering techniques/methods is to overcome the weakness or
bias of a single method (Denzin, 1988). Triangulation is a contested idea compared to
crystallisation (Richardson, 2000), the former enabling one to shift from seeing something as
a fixed, rigid, limited dimensional object than seeing a crystal, which allows for an infinite
variety of shapes, substances, transmutations, dimensions and angles of approach. Triangulation helped us to map out and explain the richness and complexity of teaching Technology by studying it from more than one standpoint (Cohen & Manion, 1994; Manion & Morrison, 2000).

**Findings from reconnaissance (Phase 1)**

**Observations**

It was observed that there were technology labs in the five sampled schools. Technology
teaching took place in ordinary classrooms without any Technology materials or equipment.
As stated above, teachers did not have curriculum policy documents in their possession. We
wondered as to what they used to plan lessons and to guide their teaching. How did they approach the assessment of learners in this kind of a situation? We waited to see! Teachers seemed left on an island because there seemed not to be any support of whatever kind coming their way. Except at BMB, high learner number compared to teacher capacity was a cause for concern considering the theory-practice nature of Technology.

Participants’ biographical information

We start by presenting the results from Technology teachers’ biographical information as captured in Table 2. Basically these were findings from the first part of the questionnaire administered to teachers.

Table 2. Grade 8 and 9 technology teachers’ biographical information

<table>
<thead>
<tr>
<th>Gender</th>
<th>Technology teaching experience</th>
<th>Technology qualification</th>
<th>School milieu</th>
<th>Can plan Technology lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>less than 6 yrs</td>
<td>yes</td>
<td>Rural</td>
<td>yes</td>
</tr>
<tr>
<td>F</td>
<td>more than 6 yrs</td>
<td>No</td>
<td>Semi-urban</td>
<td>No</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>7</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>11</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

There were eighteen participants from the five participating secondary schools composed of nine males and nine females. Eleven participants had less than six years of Technology teaching experience while seven had more than six years of Technology teaching experience. Eleven participants did not have any Technology qualification, which explains their incapacity in the knowledge and pedagogical domain of the subject. Seven teachers had some form of Technology qualification. Thirteen of the participants worked in rural areas and five in semi-urban areas. Ten participants could plan a Technology lesson whereas eight still needed some help. This again tends to attest to the non-possession of the curriculum policy documents by these teachers at the time of investigation.

Integrated findings from interviews, field notes, logs of meetings and questionnaire

The findings revealed challenges that teachers faced in their practice of Technology. These challenges included Technology-specific teaching experience, Technology lesson planning, Technology assessment, level of internal and external support for Technology teaching, resources for Technology teaching and learning, Technology curriculum policy interpretation and implementation, and teacher-learner ratio in a Technology class.

Many teachers were asked to volunteer to teach Technology. As a result many do not have any qualifications in Technology. Given this background and findings from the biographical information in Table 2, teachers held reasons for teaching Technology. These reasons ranged from being coerced into teaching Technology to basically having the passion for it. For instance, “It was just allocated to me”. Bearing in mind this situation, most Technology teachers are generally uncomfortable with the pedagogy of Technology as it was observed and revealed from the interviews. Some did not even have any interest in teaching Technology: “It just came along while I am already teaching and I didn’t develop any interest in the subject”.

It can be gathered from the teachers’ responses, that they were not grounded in the learning area of Technology. By implication this suggests that they had challenges planning Technology lessons. It would appear that teachers expected to be supplied with a plan so that
all that they needed to do was get to class and teach without working hard in developing lesson plans: “We want to be supplied with pace-setters, scheme of work and draft lesson plans”. This was confirmed when we requested teachers’ lesson plans so that we could follow their teaching properly during the observations. Many could not produce them. This could only mean two things: either they did not prepare any lessons, or they were uncomfortable to disclose them to us out of anxiety that their planning was not right. Only two out of five schools’ teachers produced their lesson plans.

In the questionnaire teachers were asked to indicate if they planned any Technology design projects or any tasks (capability, case study or resource tasks) for their learners. One teacher blamed lack of support by the school on this matter: “Technology at our school is not taken into consideration because learners are not doing any practical work”. This is quite unfortunate because the teaching of Technology mainly follows a design project approach in the context of problem solving. Only three teachers indicated in the interviews that they had a copy of provincial or national assessment manuals. This confirms their lack of capacity in terms of assessment in Technology.

Technology, being relatively new in the curriculum, may not thrive without a concerted commitment to empowering Technology teachers. Teachers were keen to see support both from within and outside their schools to help them acquire knowledge and teaching of Technology. Teachers expressed this need as follows during the interviews: “The principal should develop interest in Technology Education so that he cannot have a problem in allocating a budget for Technology Education”. Teachers urged their School Management Teams (SMTs) to take Technology seriously and to allocate its budget and teachers per phase accordingly. The responses from the questionnaire indicated that the support received from the district office was very weak compared to the support received from their colleagues and SMTs. Teachers expressed that lack of support from the district was attributed to the fact that the district had not yet appointed a district-based subject advisor for Technology.

Lack of support was also experienced in the provisioning of resources as can be seen from the next set of findings. It was observed that there was a lack of textbooks for both teachers and learners. In some schools there was not even one textbook among the learners. Some teachers in the same schools were sharing a textbook. This state of affairs was confirmed from the interviews findings expressed by one teacher: “I guess it’s a hands-on subject and there are no resources available”. The sampled teachers pleaded for Department of Education in Limpopo Province and their schools to provide the necessary resources – resources for learner group projects, a workshop centre where learners can do Technology hands-on, Technology materials to expose and orientate learners on, and a Technology resource centre.

It was evident from all data sources that there was a great need for teachers to have access to the Technology curriculum document in the first place, and to be helped to interpret and implement it. One teacher remarked in this regard: “I don’t think the challenges I meet as stated would have happened should I have had the technology curriculum policy document as a guide”. We requested the teachers to show us the curriculum policy documents that they used. No one had any across all the sampled schools. This was quite surprising to us since the documents are freely available at the department’s offices and on the internet. We could only suspect teachers’ ignorance about where to find the documents, or they were reluctant to produce them thinking they might be asked questions about them.

Unmanageable learner numbers in a Technology class render the teaching of same ineffective. This was an added constraint Technology teachers were faced with. We observed that teachers’ movement in the classroom during their lessons, and their interaction with
learners was extremely limited due to overcrowding. It was difficult to have a chair or even a space to sit down as a non-participant observer. The teacher-learner ratio ranged from 1:60 to 1:90. One teacher expressed a concern: “The department needs to improve the teacher-learner ratio so that an educator is faced with a manageable class”. From the observation, this had to do with overcrowding.

**Findings from Phase 2**

Findings in this section are presented following the interview questions asked. The responses show teachers’ views about how they were helped to address the challenges that they faced.

*What have you learned from the AR cycle activities?*

The responses to this question appreciated learning together as a team: “I have learnt that cooperative learning is vital”. Having been helped to identify resources in their environments for use in the teaching of Technology, teachers hastened to point out the need for resources, as one stated: “I learnt that Technology cannot be taught theoretically but resources are needed for learners to see what you mean, for example when you teach about different types of systems you need to actually show them those systems and they can relate to the topic by giving their own examples of those systems. I also have learnt that Technology can be so challenging and frustrating when you don’t have resources as a Technology educator”.

We made it our responsibility to obtain the Technology curriculum policy documents and workshop teachers. Having gone through this workshop, two of them remarked: “As a new Technology educator I have learnt a lot about the policy, learning outcomes and assessment standards. I now understand and know how learning outcomes and assessment standards are interrelated from one grade to the other, for example Grade 7, Grade 8 to Grade 9”. “Technology policy was clearly analyzed and that gave me a strong foundation into understanding the learning outcomes and how they relate to their assessment standard”.

One teacher in particular appreciated having learnt the technical content of Technology, i.e. systems and control: “Some sections were clearly expanded which were always a problem to most of the teachers e.g. resistors and their calculations. The OR-gates and AND-gates and their application”.

*What are you taking along to your school from the cycle session?*

One teacher responded that “I managed to develop new knowledge and techniques, for better approaches in the teaching of the subject”. It seemed that systems and control, which gave teachers a tough time, was now made understandable to them, as one teacher stated: “I am taking to my school the ways in which system and control should be taught guided by both the revised curriculum statement and CAPS documents”.

The intervention through this study build some degree of confidence in teachers to teach Technology, as this was expressed through their colleague: “I have been supplied with some handouts and learners will be able to do some projects. From this session I have been guided on how to teach Technology content as well as assessing their work”.

*Indicate the technological themes that you can now implement with confidence in your lesson presentation especially those that you couldn’t before this cycle.*

After the teachers successfully designed and implemented the projects, they felt confident that they could plan projects using the design process – investigate, design, make, evaluate and communicate. One teacher actually stated what he was not able to do before compared to
what he was now able to do: “Before the cycle I could not handle the following themes: binary notation and conversion into the decimal, resistors and resistance, determining and calculating the resistance using the colour code sequences 1st, 2nd 3rd up to the last. I am really thankful for the knowledge gained during this contact session”.

The most challenging theme was also mentioned: “I can now present with confidence the following themes in Technology: structures, system and control and indigenous technology and culture”. A colleague to this teacher elaborated: “We can implement system and control with confidence because we realized that there are more practical things that we do in our daily lives, but we were not aware that Technology is around us. We can also teach design, make, evaluate and communicate, e.g. structure of putting a water tank on”.

What gaps have you identified that still need to be filled regarding your knowledge of technology?
The gaps identified by teachers include systems and control, identifying electronic components, mechanical and pneumatic systems, identifying classes of levers and their performance, and electricity and calculations. Given the uniqueness of Technology Education, its relative newness in the curriculum and the low to no qualification of teachers in the field that surfaced from the biographical data, it does not come as a surprised that teachers still needed some more training in the covered themes and even new themes.

Any other inputs/suggestions/proposal you have for a way forward?
Teachers suggested the following:

- More contact workshop/sessions on regular basis, at least once per month; a more detailed workshop for Technology.
- Let the Department of Education partner with universities to develop curriculum advisors in the field.
- “I suggest that we have more workshops so that we can equip ourselves. I don’t have more knowledge of Technology. I need to learn more; the more the workshops, the more knowledge”.
- Schools should be supplied with relevant and updated materials such as textbooks, study guides and overhead projectors.
- Certificate of attendance at the end of the training to encourage teachers.

These suggestions are very closely related to the identified gaps above. Thus, they raise the same situation like that explained in the gaps above. A way forward was agreed upon with teachers and the Department of Basic Education officials to develop a plan to expand the study and training to other schools in the province. To teachers who participated in this study, some degree of emancipation happened (see Table 3), despite some more areas of need.
### Table 3. Emancipation of Technology teachers

<table>
<thead>
<tr>
<th>Area of emancipation</th>
<th>Before intervention</th>
<th>After intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology lesson planning</td>
<td>Only ten teachers could plan a Technology lesson whereas eight indicated that they needed some help (table 2).</td>
<td>Teachers could plan a Technology lesson after being shown how. They could design projects following the design process and teach their lessons.</td>
</tr>
<tr>
<td>Teaching of Technology</td>
<td>Lack of Technology content knowledge, qualification/ experience to a greater extent (table 2); discomfort with pedagogy of Technology (as observed and revealed from interviews); some teachers had interest in teaching Technology but they encountered challenges during their teaching.</td>
<td>Teachers can now teach Technology with some degree of confidence compared to before intervention (see findings of Phase 2 above). Planning and implementing lessons together helped to build courage and knowledge into teachers; they could for the first time do design projects with their learners.</td>
</tr>
<tr>
<td>Technology assessment</td>
<td>Almost all teachers confined themselves to giving assignments, class work, homework, tests and examinations as it was observed during Phase 1.</td>
<td>After the teachers had engaged their learners in the containerization project some new ways of assessing were evident in addition to the ones that they were accustomed to; a milestone being that they were able to design a rubric for their choice projects.</td>
</tr>
<tr>
<td>Internal and external support</td>
<td>District office-based support by subject advisors was rated lowest compared to school-based support by colleagues and School Management Teams.</td>
<td>Teachers circumvented lack of support by developing self-reliance and engaged self-work-shopping for their own emancipation.</td>
</tr>
<tr>
<td>Resources for Technology teaching and learning</td>
<td>Observation revealed lack of textbooks for teachers and learners; in some cases schools shared a textbook, a very inconveniencing situation to the teachers.</td>
<td>In order to cope with the before-intervention situation, teachers were made aware of the materials available in the surroundings to use to teach Technology, that enabled them to make the artifacts from the designed projects, e.g. plastic, wires, foil material, etc.</td>
</tr>
<tr>
<td>Technology curriculum policy interpretation and implementation</td>
<td>Teachers’ responses to the questionnaire and interviews pointed out that before they did not have the policy document, therefore there was nothing to interpret.</td>
<td>After the policy document was organized and teachers shown how to interpret and implement it they managed to develop an understanding of the learning outcomes, their relationship, and planned lessons successfully.</td>
</tr>
<tr>
<td>Teacher-learner ratio in Technology classes</td>
<td>Teachers’ movement within the classroom and their interaction with learners were profoundly limited due to over-crowdedness (teacher-learner ratio ranged from 1:60 to 1:90).</td>
<td>Helped to manage big classes and through assigning a project to the learners, teachers realized that big classes were not necessarily a hindrance, rather presented an opportunity for teaching Technology.</td>
</tr>
</tbody>
</table>
Recommendations

We recommend the following to the Department of Basic Education from national down to the circuit:

- Build a working relationship with educational partners interested in development of Technology teachers.
- Establish Technology clusters in each circuit with a well-established leadership. This will help make the training easier.
- Identify Technology teachers within a cluster who are good in technological content knowledge and afford them an opportunity to empower their colleagues.
- A needs questionnaire should be administered to Technology teachers to indentify gaps in the Technology curriculum.
- The questionnaire should be analyzed and interpreted together with leaders.
- A four weeks emancipation schedule should be drawn up, which may be interspersed within school holidays.
- Technology subject advisors fulfil the ‘code for quality education’ promise signed by Department of Education and teacher unions during the annual World Teachers’ Day on 5th October 2011. Kliptown Pledges (Hartley, 2011) stresses that the departmental official should:
  - ensure that all schools receive the necessary resources in time for teaching to commence;
  - always be available to assist schools, principals and teachers;
  - visit all schools within the district to offer support on regular basis;
  - respond to requests or concerns of education stakeholders.

Constraints experienced during the study

Certain challenges were experienced in the process of conducting field work:

- The events building up to the hosting of FIFA 2010 Soccer Tournament by South Africa interfered to some extent with the planned AR schedule as some teachers did not report at work.
- Teachers voted for industrial action about salary negotiations after FIFA 2010 Soccer Tournament. When the strike was called off, the teachers started to prepare learners for examinations in 2010.
- One secondary school redeployed 50% of its staff in 2010 which included Technology teachers.
- Some of the teachers had new subject allocation for 2011, which caused them to technically withdraw from participating in the study.
- Two female Technology teachers went on maternity leave, thus terminated their participation.
- The worst was the termination of teaching contracts for some Technology teachers.
- On some days Maths, Science and Technology Education’s Heads of Department were not keen to release Technology teachers due to some school related duties like examinations invigilation.

In the process we gained new teachers in the place of the lost ones. This added to the constraints to some extent because they did not start with us at the beginning.
Conclusions
This study set out to identify the gaps that secondary school Technology teachers at Mankweng Circuit of Limpopo Province faced regarding their knowledge and teaching of Technology. A reconnaissance study as part of AR was employed to identify those challenges. There was a definite problem regarding the interpretation of the technology curriculum policy document, learning programme, work schedule and lesson planning, lack of taking advantage of the available resources, among other things. Phase 2 was rolled out with participants to address the challenges they faced. Thus, we employed AR to intervene in the challenges that teachers faced. This intervention yielded findings about the emancipation of teachers and pertinent recommendations were made subsequently.

References


Teachers and Learners Level of Computer Literacy to Use Educational Technologies at Some Secondary Schools in Attridgeville Township

Olika Moila & Moses Makgato
Department of Educational Studies, Tshwane University of Technology, Soshanguve North Campus, South Africa
olikamoila@gmail.com; makgatom@tut.ac.za

Introduction

The White Paper on e-Education, DoE, (2004) states that Information and Communication Technologies (ICTs) are the combination of networks, hardware and software as well as the means of communication, collaboration and engagement that enable the processing, management and exchange of data, information and knowledge.

The rapid development in ICTs have made tremendous changes in the twenty-first century, as well as affected the demands of modern societies. Recognizing the impact of new technologies on the workplace and everyday life, today’s educational institutions try to restructure their educational programs and classroom facilities, in order to minimize the teaching and learning technology gap between developed and the developing countries. This restructuring process involves effective integration of ICTs into the curricula in order to provide learners with knowledge of specific subject areas in an attempt to enhance meaningful learning. However, this is not the case at the six public secondary schools in Atteridgeville at the TSD 4. The challenges that hinder effective ICT integration to enhance teaching and learning at the schools prompted this study. To this end, Roblyer, (2004), argues that one of the things that make teaching so challenging is that it goes on in an environment that mirrors and sometimes magnifies some of societies’ most profound and problematic issues and adding ICTs to this mix makes the situation even more complex, yet to integrate them successfully into their teaching, educators need to adapt to the effects that ICTs have on education.

It has also been noted that schools have been slow to adopt ICT as an integral tool across the curricula. Teachers and learners lack the necessary knowledge and skills to use educational technologies effectively for teaching and learning purposes, i.e. to access information from internet, write and send emails, using PowerPoint, using MS programmes (Word, Excel,) to produce content documents of their own, etc (DoE, White Paper 2004). It is to this regard that the study embarked on investigating the level of knowledge and skills to use ICT at six schools in the Attridgeville area around Pretoria. The study sought to respond to the following research questions:

- What is the level of teachers’ computer literacy at the six Atteridgeville secondary schools?
- What is the level of learners’ computer literacy at the six Atteridgeville secondary schools?

Computer literacy and the use of ICT for teaching and learning

The use of ICT for learning is currently associated with computers and internet to facilitate teaching and learning (Mbah, 2010). Using ICTs for teaching and learning includes the technologies used in conveying and storage of data, emailing browsing the internet looking
for information (googling), emailing, twittering. According to Mbah, (2010), educational technologies provide an array of powerful tools to transform the present teacher-centered and text-bound classrooms into rich, student-centered, interactive knowledge environments. It is important for higher education institutions and schools should embrace the new technologies and ICT tools for learning. Most of universities have started with compulsory computer literacy courses. Computer literacy has evolved over time as the use of technologies improved and society became more dependent on computers (Nawaz, & Kundi, 2010; Hammed, 2007). With changes in technologies, the contents of computers literacy are constantly changing to include the latest technological developments (Martin & Dunsword, 2007). The most common educational technology used at schools includes computer hardware, software, video-player and the internet. According to Iyamu and Adawu (2008), these forms of technology provide teachers and students with vast quantities of information in an easily accessible way that can be used as a teaching tool.

The global demand for computer literacy emanates from the dominance of information and communication technologies in different aspects of our modern life (Oliver, 2002). Computer literacy has evolved overtime as technology improved and society became more dependent on computers. In a modern technological society, basic computer literacy is compulsory and is emphasised in every educational institution (Ezziane, 2007). Aviram and Eshet-Alkalai (2006) argue that Information and Communication Technology (ICT) is important for ‘mindful learning’ in the information technology society. Students, teachers and employees acquire their computer or technology-literacy formally by means of formal courses or informally at home, from friends (Ezziane, 2007). According to Vrana, (2007); Macleod, (2005); Wims and Lawler, (2007) the use of ICT was found to be helpful in reducing the problems of ‘isolation’ and empowering the developing countries and marginalised groups. The use of ICT are proving to be powerful tools for ‘poverty-alleviation’ and ‘economic-development’ in developing countries (Nawaz, and Kundi, 2010; Hammed, 2007). In South Africa a concerted efforts have been made by various stakeholders (NGOs, businesses) to implement the policy on the use of ICT in education, but many schools are still not using educational technology in the classrooms. A substantial body of research asserts that teachers have difficulty in integrating ICT with teaching and learning due to several challenges or barriers, Ertner, 1999; Pelgrum, 2001 and Schoepp, 2005).

Theoretical framework

This study is informed by the diffusion of innovations theory of learning postulated by Rogers (2003). Diffusion is defined as the process by which an innovation is communicated through certain channels over time among the members of a social system Rogers (2003, 5). Furthermore, Rogers (2003, 12) describes an innovation as the idea, practice, or project that is perceived as new by an individual or other unit of adoption. According to Rogers, people’s attitude towards a new technology is a key element in its diffusion. ICT integration is a technological innovation because it is considered to be a new idea by potential adopters. Hence, to establish the extent to which ICT integration is being implemented at the schools in this case study, the four-stage process in Rogers’ diffusion of innovation theory guided my research. His theory is the most suitable framework for this study as the terms innovation and technology are used interchangeably Rogers (2003, 12). The four-stage process comprise: knowledge; persuasion; decision; implementation; which are each briefly explained.

The Knowledge Stage

Rogers (2003, 21) explains that the innovation starts with the knowledge stage, whereby an individual learns about the existence of the innovation and seeks to get answers to questions
such as ‘what’, ‘why’ and ‘how’ about the innovation. Rogers asserts that the questions comprise three types of knowledge, which are (i) awareness-knowledge, which represents the knowledge of the existence of the innovation. Individuals can be motivated by this type of knowledge, to learn more about the innovation and eventually adopt it; (ii) how-to-knowledge, which entails information and skills to use an innovation correctly. This type of knowledge influences the extent to which innovations are adopted or rejected; (iii) principles-knowledge, which describes how and why an innovation works. The lack of ‘how’ and ‘why’ to integrate ICTs with teaching and learning may be a barrier which leads to misuse or even resistance to the implementation of the innovation. Like many other schools in the country, the six schools at the TSD 4, they have been included in the provincial’s government initiatives to implement ICT as an innovation.

The Persuasion Stage

This stage follows the knowledge stage and occurs when an individual moulds their attitude, after they know about the innovation. Rogers (2003, 176) asserts that the formation of a favourable or unfavourable attitude toward an innovation does not always lead to an adoption or rejection. Rogers further maintains that while the knowledge stage is more cognitive(- or knowing-) centered, the persuasion stage is more effective- or (feeling-) centered. It is at this stage that an individual is sensitively involved with the innovation. The teachers and learners at the six TSD 4 schools would be more motivated to implement these tools if the relevant resources as well as professional development were adequate.

The Decision Stage

It is at this stage that the individual chooses to adopt or reject the innovation. Rogers (2003, 177) describes adoption as referring to full use of an innovation and rejection as not adopting an innovation. ICTs are being adopted as an innovation at the six TSD 4 schools through the government’s efforts in collaboration with the GoL, and other NGO initiatives, such as Vodacom. However, full use of this innovation has not been realized as a result of the ICT challenges that the schools face. These challenges include lack of skill; lack of support; as well as insufficient time to access the resources.

The Implementation Stage

The innovation is put into practice at this stage; and this may continue for a long period of time until the innovation eventually loses its distinctive and noticeable quality as a new idea. Rogers (2003, 180) also asserts that reinvention is an important part of this stage and describes it as the degree to which an innovation is changed or modified by a user in the process of its adoption and implementation. However, Rogers (2003, 6) argues that some degree of uncertainty is involved in diffusion. The individual may be uncertain about the outcomes of the innovation, hence the need for support from experts in order to reduce the degree of uncertainty and build up confidence of the implementer. The implementation of ICT integration to enhance teaching and learning at the six TSD 4 schools is still at its early stages due to the existing ICT challenges that they face. Rogers’ diffusion of innovation theory was suitable for the study of the integration of ICTs with teaching and learning in these schools as its five stages guided the researcher to investigate how teachers, learners and school management teams (SMTs) interacted with ICTs to enhance teaching and learning. The study investigated the degree of each school’s exposure to ICTs and their understanding of how to use these powerful tools (knowledge stage). The attitudes of teachers and the school personnel were also investigated as these have a direct impact on the integration of ICTs with their practices (persuasion stage). Issues relating to ICT policies in the schools
were also investigated as these would verify the schools’ degree of readiness to adopt ICTs (decision stage). The extent of ICT integration with teaching and learning was also explored (implementation stage). The levels and quality of professional development were investigated as well because the findings would indicate the measures that had been taken to reinforce ICT skills in order for teachers to effectively use ICTs to enhance teaching and learning (confirmation stage).

Methodology

The study used both quantitative and qualitative approach in its methodology, by using questionnaires and interviews to collect data from both teachers and learners, which is a form of triangulation. Cohen & Manion (2000) define triangulation as an attempt to map out, or explain more fully, the richness and complexity of human behaviour by studying it from more than one standpoint. In this view, Altrichter, Fieldman, Posch & Somekh (2008) contend that triangulation gives a more detailed and balanced picture of the situation.

Terre Blanche, Durrhein & Painter (2006) describe population as any clearly defined group of people, events or things that are of interest to and under investigation by the researcher and from which sampling elements are drawn. Although initially 8 public secondary schools with 363 teachers and 7656 learners were targeted, only six schools were part of the case study since the other two did not respond favourably to the application to do research. The schools were selected using non-probability technique of convenience. At each school the participants were school principals, ICT coordinators, teachers and learners. Initially 10% of the learners and teaching staff had been sampled, but due to withdrawal of participants for various reasons, the participants in the six schools were reduced to 24 teachers and 670 learners. Data from questionnaires were analysed by means of frequencies, tables, means and standard deviations. The qualitative data from the interviews with 6 teachers and 6 School Management Team members were analyzed by reading repeatedly the transcripts in order to identify the patterns, categories and themes using relevant codes. These themes were presented and discussed in a narrative way together with some verbatim.

Data from the six schools were collected using questionnaires, interviews. Data was simultaneously collected from the six schools during February and March 2013.

Results and discussion

Questionnaire results

Teachers and learners were asked to indicate whether or not they had the following skills and access to ICT as well related training. Table 1 provide the findings on biographical data of teachers at the six schools.
Table 1. Teacher’s biographical data

<table>
<thead>
<tr>
<th>Gender</th>
<th>Frequency</th>
<th>Percentage</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>11</td>
<td>46</td>
<td>24</td>
</tr>
<tr>
<td>F</td>
<td>13</td>
<td>54</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teaching Years of experience</th>
<th>Frequency</th>
<th>Percentage</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 2</td>
<td>2</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>2 – 3</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4 – 5</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>6 – 10</td>
<td>4</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>10 and above</td>
<td>18</td>
<td>75</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 shows most (54%) of the participants were female, while 46% were male. Most of participants had 10 and above years of teaching experience. Several studies have reported relationships between demographic characteristics of teachers and their reported use of technologies; age, gender, race, education level, socio-economic status of students taught, years of teaching, years of technology use, (Becker, 1994; Ely, 1999; Hadley & Sheingold, 1993; Jaber & Moore, 1999). In a study done by Kobak, & Taşkın, (2013), there was no significant difference between males and females in using technology as well as their perceptions on using technology. However, this study did not conduct statistical tests on correlation between gender and usage of technology.

Table 2. Teachers’ knowledge and skills of using educational technology

<table>
<thead>
<tr>
<th>No</th>
<th>Item</th>
<th>Yes</th>
<th>Frequency</th>
<th>Percentage</th>
<th>No</th>
<th>Frequency</th>
<th>Percentage</th>
<th>Total</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Use MS-Excel Spreadsheet</td>
<td></td>
<td>21</td>
<td>88</td>
<td>3</td>
<td>12</td>
<td>46</td>
<td>24</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Use PowerPoint</td>
<td></td>
<td>20</td>
<td>83</td>
<td>4</td>
<td>17</td>
<td>34</td>
<td>24</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Store data on a flash drive</td>
<td></td>
<td>21</td>
<td>88</td>
<td>3</td>
<td>12</td>
<td>42</td>
<td>24</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Use a printer</td>
<td></td>
<td>20</td>
<td>83</td>
<td>4</td>
<td>17</td>
<td>34</td>
<td>24</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Use data projector</td>
<td></td>
<td>18</td>
<td>75</td>
<td>6</td>
<td>25</td>
<td>41</td>
<td>24</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Professional development on ICT</td>
<td></td>
<td>5</td>
<td>21</td>
<td>19</td>
<td>79</td>
<td>100</td>
<td>24</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Table 2 indicate the responses of participants on knowledge and skills they possess in using ICT (computer). Majority (88%, 83%, 88%, 83%, 75%) of participants indicated that they have skills and knowledge of using computer (MS Excel, PowerPoint, data storage, printer). These items listed form the basic indicators of computer literacy which can enable teachers to use ICT for teaching and learning. Another key factor affecting the integration of computers in classrooms is continuous professional development teachers (Chin & Hortin, 1993, 1994; Dupagne & Krendl, 1992). Technology-related professional development plays a crucial role in developing teacher’s competency with computer applications (Gilmore, 1995) as well as influencing teachers’ attitudes towards computers (Becker, Ravitz, & Wong, 1999). Majority of participants indicated that they don’t receive related training in the use of ICT, which tend to create a barrier in using ICT in the classroom.

However, it interesting to note that, most of participants indicated that they had basic knowledge and skills in using ICT.

Table 3. Learners’ biographical data

<table>
<thead>
<tr>
<th>Gender</th>
<th>Frequency</th>
<th>Percentage</th>
<th>Total Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>309</td>
<td>46%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>361</td>
<td>54%</td>
<td>670</td>
<td>100%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grades</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>141</td>
<td>21%</td>
</tr>
<tr>
<td>9</td>
<td>159</td>
<td>24%</td>
</tr>
<tr>
<td>10</td>
<td>194</td>
<td>29%</td>
</tr>
<tr>
<td>11</td>
<td>99</td>
<td>15%</td>
</tr>
<tr>
<td>12</td>
<td>77</td>
<td>11%</td>
</tr>
</tbody>
</table>

Table 3 shows that most (54%) of the participants were female learners at the six schools. Learners were spread across grades 8, 9, 10, 11 and 12, whereby more than quarter (29%) of them were from grade 10. The least participants (11%) were from grade 12, 15% from grade 11, about of them (24%) were from grade 9, while 21% of them were from grade 8.

Table 4. Learners’ knowledge and skills of using educational technology

<table>
<thead>
<tr>
<th>No</th>
<th>Item</th>
<th>Yes</th>
<th>No</th>
<th>Non Responses</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Freq.</td>
<td>%</td>
<td>Freq.</td>
<td>%</td>
</tr>
<tr>
<td>1.</td>
<td>Use of e-mail</td>
<td>242</td>
<td>36</td>
<td>414</td>
<td>62</td>
</tr>
<tr>
<td>2.</td>
<td>MS Word processing skills</td>
<td>407</td>
<td>60</td>
<td>247</td>
<td>38</td>
</tr>
<tr>
<td>3.</td>
<td>Use of power point</td>
<td>226</td>
<td>34</td>
<td>420</td>
<td>63</td>
</tr>
<tr>
<td>4.</td>
<td>Use of Excel spreadsheet</td>
<td>407</td>
<td>61</td>
<td>247</td>
<td>37</td>
</tr>
<tr>
<td>5.</td>
<td>Use of internet</td>
<td>497</td>
<td>74</td>
<td>162</td>
<td>24</td>
</tr>
<tr>
<td>6.</td>
<td>Storing data in flash drive</td>
<td>416</td>
<td>62</td>
<td>242</td>
<td>36</td>
</tr>
</tbody>
</table>
From table 4, most learners (74%) responded that they know how to use internet more than any other items mentioned in the computer technology. The ability to use internet will enable learners to search information related to their content knowledge of their subjects, thereby enhancing knowledge acquisition from the World Wide Web (www). Storing data from flash was the second item which learners (62%) indicate that they have ability to use. Students should know how to use ICTs effectively across the curriculum, with specific requirements to investigate with ICTs, create with ICTs, communicate with ICTs, manage and operate ICTs, and use ICTs in socially and ethically appropriate ways (Pelgrum, Oakley & Faulkner, 2013).

Interview results

- ICT policies and their implementation

Tondeur et al (2007,963) argued that ICT policy is desirable as it assists ICT integration in the teaching and learning environment by building the willingness of teachers and school principals’ professional attitudes. With this regard it was imperative to explore through semi-structured interviews, the extent of ICT policy awareness and implementation in the six schools. During these interviews some of the teachers’ perceptions that were given below. The designation of the respondent is indicated in brackets.

School A [Teacher]:
No…there are not any ICT policies that I am aware of in the school.

School B [Teacher]:
I have no information on this.

School C [Teacher]:
We are working on this.

- Computer Literacy and its Impact on Education

The DoE (2003) stated that every African manager, teacher and learner in the general and further education and training bands will be ICT capable that is, use ICTs confidently and creatively to help develop the skills and knowledge they need as lifelong learners to achieve personal goals and to be full participants in the global community. In this view, this theme focused on the question of computer literacy levels at the six schools.

It was noted from the interviews that although the teachers acknowledge the positive impact that ICTs have on teaching and learning activities; their eagerness to use these tools, the levels of computer levels are still very low at these schools:

School A [SMT]:
Level of literacy is low due to limited access to equipment.

School D [Teacher]:
The low levels of ICT literacy on the part of teachers hinders the enhancement of effective learning on students who apparently seem to be better equipped to integrate ICT with their learning activities. Our learners have smart phones which they use to access information whereas most of us teachers just use these cell phones for receiving, making calls and sending as well as receiving messages.
School E [ICT Coordinator]:

There are problems that the teachers encounter in trying to access ICTs for teaching in the classrooms. These include lack of time and inadequate equipment. However, most teachers use computers to prepare worksheets, print out hand-outs and process learner mark sheets.

School F [SMT]:

There hasn’t been enough training on how to use ICTs to integrate these tools with all the subjects in the curricula, the ICT literacy levels are still very low.

These findings were also reflected in UNESCO (2002) where it is stated that simply providing the technology for learners and teachers is not enough. The type and level of access is also important. ICTs will improve learning very little if teachers and students have rare and occasional access to the tools for learning.

- **ICT integration with Teaching and Learning**

The responses that were given under this theme gave a clear indication of the present teachers’ pedagogical practices in relation to ICT usage, in terms of moving away from teacher-cantered classroom practices to learner-cantered ones. Although the teachers are aware of the need to keep abreast with the rest of the information society, they indicated their interests to use ICTs in their teaching but with an emphasis on obstacles hindering this integration:

School C [Teacher]:

Individual teachers have to find their way to get access to ICTs, which is a difficult process as the computer lab is locked always, some ICT equipment are not in good condition.

School F [SMT]:

Integration is almost impossible due to a very small number of computers which are usually offline; being available for large numbers of learners and teachers.

- **ICT support systems**

ICT integration with teaching and learning is not feasible without adequate support systems as indicated in Balanskat et al (2006) that amongst other things, policy makers should motivate and reward teachers to use ICT and integrate the ICT strategy into the school’s overall strategies. It was therefore of utmost importance that this research elucidate the teachers’ perspectives on this issue through the semi-structured interviews. Although the teachers reported that they were receiving ICT support from various organizations, it was quite insignificant. The teachers also indicated the need for the DoE to provide sufficient ICT equipment and relevant professional developmental programs:

School A [Teacher]:

The Department of Education has not offered enough training on how to use ICT in the teaching of various subjects. Only basic computer skills like typing and using the internet have been provided by GoL.

School B [Teacher]:

We have not heard anything from the District office. Vodacom is giving us ICT support but this is barely enough to meet the needs of the whole school. They donated only two computers and a limited amount of data bundles to be accessed by a staff of more than 40 teachers.
School C [ICT Coordinator]:

Only 5 teachers received training from Vodacom and 3 from Microsoft (SA). We received 3 computers from Vodacom for 40 teachers; check the ratio 3:40, (bad); 0 computers from the government, this shows very poor support systems. For example, if you want state doctors to supply good diagnosis of diseases, you buy CT-scans and surgical equipment for theatres. Teachers must buy their own laptops with abusive salaries. The support system is virtually non-existent.

These responses are supported by Dawes (2001) who argues that problems arise when teachers are expected to implement changes in what may be adverse circumstances. It implies that if the teachers are expected to effectively integrate ICTs in their practice, they must receive appropriate, thorough training and continuous development on ICT usage in the classroom. Balanskat et al. (2006) also stress that many teachers still choose not to use ICT skills in teaching situations because of their lack of ICT skills rather than for pedagogical or didactical reasons. Despite teachers’ ICT training, there is still a lack of follow-up on the utilization of newly acquired skills. Unsuitable teacher training programmes fail to engage teachers in using ICTs both during their lessons and also in the preparation of lessons. It is also argued that the most commonly-mentioned cause of this lack is that the training courses focus mainly on the development of ICT skills and not on the pedagogical aspects of ICTs.

**Conclusion**

A comprehensive picture of how teachers and learners at the six schools that were involved in the study were derived from the given responses. It is quite clear that the ICT statuses at these schools are similar. The findings revealed that the teachers and learners are equipped with basic ICT skills which still need to be improved on if effective integration of these powerful tools with teaching and learning should take place.

With regard to this integration, the respondents expressed overwhelming excitement and eagerness to implement ICTs in education; however, these attitudes are dampened by various challenges that they face in their schools. These obstacles to effective ICT integration with teaching and learning included those mentioned by Pelgrum (2001) who stated that these included the lack of ICT knowledge/skills; difficult to integrate ICT into instruction; scheduling computer time; insufficient peripherals; not enough copies of software; insufficient teacher time; not enough simultaneous access; not enough supervision staff.; lack of assistance; not enough training opportunities; lack of information about software; not enough connections; weak infrastructure; slow network performance; lack of interest of teachers and lack of support.

This research is a case study bounded on six public secondary schools at the Atteridgeville Township of the Tshwane South District 4 in the Gauteng Province of the Republic of South Africa. The findings in this study cannot be generalized to any other public secondary schools in the district. With regard to this aspect; the researcher henceforth suggests that broader research be done on more schools in the Tshwane South District. From a wider research scale, the findings from the teachers’ and learners’ perceptions of ICT integration with teaching and learning can then be generalized and applied to more schools.

The lack of adequate resources and infrastructure; lack of pedagogical skills to use ICTs; and the lack of support on ICT implementation are some of the important challenges that were indicated in the findings of this research. It is therefore imperative for the researcher to recommend further in-depth research at a wider scale, to investigate and explore how ICT integration can be effectively implemented to enhance teaching and learning in public
schools. It is only then, that the findings from such research may be generalized and their relevant recommendations are adapted by the schools in generally similar situations.

References


Oliver, O. 2002. The role of ICT in higher education for the 21st century: ICT as a change agent for education.
Pre-service Technology Teachers’ Misconceptions about the Concept of a Lever

S. M. Ramaligela, A. Mji & Ugorji I. Ogbonnaya
Department of Mathematics, Science and Technology Education, Tshwane University of Technology, South Africa
RamaligelaSM@tut.ac.za; MjiA@tut.ac.za; ogbonnayaUO@tut.ac.za

Levers are among the simplest mechanical devices. The concept of the lever is taught in Technology as a subject at school level. The aim of this study was to investigate pre-service teachers’ misconceptions about the principle of the lever. A case study of two pre-service teachers at the University of Technology in South Africa was explored. The data were collected through classroom observations on their regular teaching routines during teaching practice. A video recorder was used to capture the students’ practice in the classroom. Misconceptions of these pre-service teachers were analysed and arranged according to Swartz and Parks’ (1994) lesson plan concepts and design. The results revealed that both participants held conceptual misunderstandings about the principle of the lever. The study first identified misinterpretations when presenting the concepts. Secondly, it detected misrepresentation of the concepts when demonstrating. Lastly, it identified misconceptions in the students’ explanations of the aspects of the lever. This study therefore recommend intensive dialogue on Technology courses to help students to identify their misconception as well as their peers and identify a way on how to evade and correct misconceptions.

This article addresses how these inaccuracies can occur, what historic missteps may contribute, and which strategies teachers can use to help students move toward conceptual

Introduction

Misconceptions play a critical role in teaching and learning (Alkhalifa, 2006:124). A misconception, in a scientific context, is an idea or explanation that differs from the accepted scientific concept (Keeley, 2012). Gooding and Metz (2011) point out that teachers have a significant influence on their students’ learning; teachers’ misconceptions can heighten students’ misconceptions. Yip (2007) indicates that many students’ erroneous concepts or inaccurate views of concepts have been propagated during classroom instruction by either the teacher or textbooks. In order for teachers to change students’ misconceptions, they need to possess a very high level of content knowledge and an acute awareness of students’ thinking. Shuttleworth (2009) points out that teachers who have inadequate or inaccurate knowledge of concepts may well pass on such knowledge to their students. They may also fail to challenge students' misconceptions and will use texts uncritically, or modify them inappropriately. Alkhalifa (2006) adds that teachers' conceptions of knowledge shape their classroom practice, the type of question they ask, the ideas they present and reinforce and the type of task they design.

Gören (2008) conducted a study on student teachers’ misconceptions of scientifically accepted principles of mass and gravity. The study found that student teachers had difficulty in distinguishing between some concepts; student teachers of Science could not articulate the important role of gravitation and inertia concepts, even if they had studied these concepts from secondary school to university. Identifying misconceptions in Technology Education in a South African context is an important step towards developing better understanding of technology concepts, and essential for designing and constructing effective instructional
strategies that aim to prevent or rectify misconceptions (Yip, 2007). The aim of this study is to identify pre-service Technology teachers’ misconceptions on teaching the concept of the lever. The research question addressed is “What types of misconception do pre-service Technology teachers hold about the principle of the lever?”

**Misconception**

This paper uses the term 'misconception' to denote any ideas held by Technology pre-service teachers that are inconsistent or in conflict with those generally accepted by engineers and scientists. Misconceptions arise from poor science, oversimplification of truths or belief in myths that everybody accepts as true (Shuttleworth, 2009). Keeley (2012) indicates that misconceptions are all ideas that students bring to their learning that are not completely correct. Scientifically inaccurate ideas have also been categorised in a variety of ways, including preconceived notions; non-scientific belief; conceptual misunderstanding; vernacular misconception; and factual misconception (NRC, 1997). A preconceived notion is defined as popular conceptions rooted in everyday experience. Non-scientific beliefs are views learned by students from sources other than scientific education, such as religious or mythical teachings. Conceptual misunderstanding is defined as students’ models that combine correctly taught scientific information with their own preconceived notions and non-scientific beliefs. Vernacular misconception is defined as confusion created when words have both an everyday meaning and a scientific one. Factual misconception is defined as false concepts learned at an earlier age and retained unchallenged into adulthood.

Gooding and Metz (2011) in their study *From misconception to conceptual change* also identify five types of misconceptions: preconceptions, non-scientific beliefs, conceptual misunderstanding, vernacular misunderstanding and factual misconception. Preconceptions are based on incomplete observations or previous experience. A non-scientific belief is, again, based on views learned from sources other than scientific education, which include religious or mythical teachings. Conceptual misunderstanding is based on misapplying a general principle or example. Vernacular misunderstanding is based on the meaning of words. Factual misconception is based on misinformation. Yip (2007) conducted a study to identify misconceptions of novice Biology subject teachers. Yip identified three categories of causes of misconceptions: personal experience, factual misinformation, and the teacher’s subject-matter knowledge. Personal experience referred to life experiences and indiscriminate use of everyday language. Inaccurate factual information is due to improper or distorted prior knowledge of the student that is necessary for the construction of a new concept. Subject-matter knowledge refers to teachers’ level of competence in content knowledge. Mitchell (2007) identified students’ misconceptions and misinterpretation of terminology on stress management.

In terms of teachers’ own misconceptions, many studies have been well documented in the field: in Science (e.g Chambers & Andre, 1997; Hammer, 1996; Gönen, 2008), Mathematics (Gooding & Metz, 2011; Muzangwa & Chifamba, 2004) and Life Science (Sanders 1993; Shuttleworth, 2009), but those relating to the field of Technology are relatively unexplored especially pre-service teachers. One possible reason for this lack of research is the fact that Technology Education is an emerging school subject in many countries, especially in South Africa. There were a few studies that focus on the implementation of Technology Education (Potgieter, 2004; Khumalo, 2004), evaluation studies in Technology (Mouton, Tapp, Luthuli & Rogen 1999), Design and Technology (Higgins, 2002), Evaluation of Textbook (Ramaligela, Gaigher & Hattingh, 2014) but there was a lack of research on identifying pre-
service teachers misconception. Therefore, to identify misconceptions of pre-service Technology teachers is critical.

Lever

A lever is a bar that moves around a fixed point (Backer et al., 2011). The fixed point, or pivot, on which the bar turns is called a fulcrum. Makgato, Ramaligela & Khoza (2014) define a lever as one of the simple mechanical devices which consist of stick or rod, though the lever cannot be effective without a pivot or fulcrum. McKenna and Agogino (1998) define a lever basically as a long stick that one pushes or pulls against a fulcrum to move something. According to Kurtus (2011), a lever is a simple machine that allows you to gain a mechanical advantage when moving an object or applying a force to an object. Kurtus explains that a lever consists of a fulcrum, load and applied force. This means that a lever involves a fulcrum, a load and effort. The concept is illustrated in Figure 1.

![Man lifting a stone using a lever](image)

**Figure 1.** Man lifting a stone using a lever (Riteshmaurya, 2013)

In order to lift the stone the man uses a lever (the pole) balanced on a small stone, which is called a fulcrum or fixed pivot point. According to Kurtus (2011), a lever consists of a fulcrum, a load (the large stone) and applied force (here, the force the man exerts to lift the load). Levers can be classified in three categories: first-, second- and third-class levers. These classes can be defined according to the positioning of the fulcrum relative to effort and load.

First Class lever

Figure 2 shows an example of a seesaw, which uses the principle of the lever. This object is classified as a first-class lever. Backer et al. (2011: 94) explain: “when you push one end of the bar down with force, the other end of the bar goes up, lifting the load”.

In a first-class lever, effort and load are at the end of either side of the lever, while the fulcrum is in the middle.
Second class lever

Figure 3 shows a wheelbarrow, which uses a lever mechanism. This object is classified under second-class levers. In order to use a wheelbarrow a person must lift the handles and push it, which means the person can push a heavier weight load by exerting less force. (Makgato et al. 2014).

Third class lever

Figure 4 shows a fishing rod which uses a lever mechanism. This object is classified as a third-class lever. As McKenna and Agogino (1998) point out, the fishing rod consists of a reel, a rod and fishing line. The fisherman throws the fishing line forward by hand without letting go of the fishing pole. He then manually uses a mechanism on the reel that allows him to set the line in place and reset before each cast to allow the line to fly loose. Once the bait or the lure has been cast into the water, the fisherman must wait until the fish bite. When the tugs on the line show that a fish has taken the bait, the fisherman can then begin to drag the fish to the surface using the reel.
In this third-class lever, the load is at the front, the force in the middle and the fulcrum, or fixed pivot, at the back, next to the person who is holding the rod.

Methodology

The participants in this study were two final-year B. Ed pre-service teachers taking a degree in Technology Education. The data were collected through lesson observation during their teaching practice, during the period of the participants’ final-year teaching practice. During their final year of study, students spend two months (August–September) on teaching practice. During this teaching practice one lesson was observed for each pre-service teacher. The data were collected through classroom observations on their regular teaching routines during teaching practice. A video recorder was used to capture the students’ practice in the classroom. Misconceptions of these pre-service teachers were analysed and arranged according to Swartz and Parks’ (1994) lesson plan concepts and design. Lesson plan concepts include general procedures; lesson outcomes; introductory procedure; instructional procedure; concluding procedure; and evaluation. This study focuses only on introductory, instructional and concluding procedures.

Background of participants

Tefu and Tinny were both young female pre-service teachers aged 19 and 23. During the data-collection process, both students were doing fourth-year teaching practice to meet the last criterion to fulfil their qualification requirement. Tefu and Tinny completed their teaching practice in one of the secondary schools in Pretoria, South Africa. One of their major subjects was Technology.

Introductory procedure

Introductory procedure is the way in which the topic is introduced, which includes diagnosing learners’ pre-knowledge and linking the topic to a real-life situation that will attract learners’ interest and attention.

Tefu started the lesson by asking:
The learners defined a lever correctly, but when Tefu continued she revealed several misconceptions. Firstly she indicated that “we are working with levers that contain or surround a fixed point”, which is a misinterpretation of the concept. She seemed to think that a lever is an object that contains something, rather than a mechanism used to make work easier. Secondly, she said “there are three important things”, implying that a lever has three important things. This indicates that Tefu did not know enough to describe load, effort and fulcrum as characteristics, but referred to “things”. This can be seen as a misrepresentation of the characteristics. Lastly, she thought that these three characteristics are found in a lever, whereas a lever has characteristics based on the object used. The principle can also be confusing if it is not made clear that there are three classes of levers, which need understanding of the subject on the part of the teacher.

In the introductory procedure, firstly both Tinny and Tefu showed misconceptions about what a lever is. Even though they could define “a lever”, it was clear that they had misunderstandings about it as the lesson progressed. Secondly, they both were confused about whether the three characteristics were “found in” a lever rather than identified when using a lever as a mechanism. Lastly, they both misinterpreted the concepts of load, effort and fulcrum by referring to them as “things”.

Instructional procedure

Instructional procedure involves introduction of new knowledge, using different methods and techniques, such as substantiating concepts with relevant examples, asking learners questions to keep their attention and carrying the discussion forward.

During exposition of new knowledge of content, Tefu said:
Tefu did not know where the fulcrum is found when a person is pushing a wheelbarrow. Firstly, she said “where the movement takes place”, which could be interpreted as where the load is, because the wheelbarrow is moving. Even though her words could be interpreted incorrectly, she also stated: “my elbow bends, thus my fulcrum”. Tefu misrepresented the fulcrum on her demonstration. Figure 3 shows that in fact the fulcrum is at the wheel of the wheelbarrow.

In order to present new knowledge, Tinny said that:

“Today we are starting with classes of levers.” She asked, “what is an effort?” Learners raised hands to answer and the teacher pointed on one learner, who answered that “It is the energy that we put on something”. She asked another question: “What is a fulcrum?” and one learner answered that “It is the point where a lever turns” (the student was referring to the textbook). She also asked them, “what is a load?” One student answered wrongly so she pointed to another learner, who said “It is a weight.” Then the teacher added that “It is a weight that we put on something.” The student teacher then said “We are having three levers. We are having three levers: scissor, nail clipper and wheelbarrow” (she had these out for demonstration).

If we look at Tinny’s continuation from the introduction, the explanation was correct but she continued by saying “We are having three levers: scissor, nail clipper and wheelbarrow”. Firstly, this statement reveals that even though Tinny could explain those three characteristics correctly, she did not understand the concept of the lever. Lastly, she used confusing language when indicating that scissor, nail and wheelbarrow are levers themselves, whereas they are objects that use the mechanism of the lever.

The instructional procedure used by Tinny and Tefu’s illustrates the argument by Zook and Maier (1994) that most educators use irrelevant information to represent the concepts of the lever and fulcrum. According to Zook and Maier (1994), conceptual misrepresentation is the selection of irrelevant information to demonstrate a certain concept. In addition, Tinny revealed a misconception about the concept of the lever, and Tefu misinterpreted the concept of fulcrum.
Concluding procedure

Concluding procedure refers to concluding the lesson, which includes wrapping up or summarising of the new content knowledge. In concluding the lesson, Tefu said:

“Effort is the energy you apply in order to get the outcome. The effort is gonna be applied by the right hand side, and we know that at the end there is where we are going to catch the fish, whereby the load will take place and the fulcrum, my hand (showing the left hand) that will allow me to do this movement is going to be my fulcrum. We said the fulcrum is a point that allows the movement”.

Tefu explained effort correctly. However, when demonstrating how the fishing rod works, she said that where there is a movement is where the fulcrum is. This was a misrepresentation of the positioning of the fulcrum. Figure 4 shows that the fulcrum is positioned on the right hand that supports the fishing rod, but the left hand makes the movement.

To conclude the lesson Tinny said:

“We use levers in our daily life,” and learners said “Yes”. “Give me an example of levers?” Learners did not respond to this question and the student teacher then said, “When we go to buy groceries we use a trolley: that is a lever”.

Tinny was trying to relate the concept to a real-life context, though this was supposed to be done at the beginning of the lesson. In the above statement she used more inaccurate language, in that a trolley is not a lever; it is not even an object that uses a lever mechanism because there is no fulcrum.

Surprisingly, in terms of concluding procedure both students has conceptual misrepresentation. However, misrepresentation were identified in different characteristics. For example, Tinny misrepresented the concept of the lever, while Tefu misrepresented the position of the fulcrum when demonstrating the three characteristics of the lever in the fishing rod.

Conclusion and recommendation

Understanding is sometimes incomplete at every level, and the generation of misconceptions is a natural and probably unavoidable part of the learning process (Gönen, 2008). Therefore, there is a need to identify misconceptions on the part of future teachers, especially in Technology, as a new subject in the South African context. The findings of this study indicate that the participants held three misunderstandings of the concept of the lever. They both misinterpreted and misrepresented the concept and those of the load, effort and fulcrum. The findings of this study add to the evidence found by other researchers like Gönen (2008:79), indicating that regardless of students’ level of schooling, misconceptions are prevalent and resistant. Technology teachers have a vital role to play in developing Technology education,
because they have to educate and introduce our younger generation to the technology age. For this reason, teacher-training programmes need to assess students’ scientific concepts and the effectiveness of their subject content knowledge in order to avoid misconceptions. It is critical to weigh the long-term consequences of having Technology teacher graduates who hold misconceptions that will be further propagated to the learners. More research needs to be done in order to be able to identify and understand pre-service teachers’ misconceptions with regard to Technology Education content knowledge.

References
Sara. (2013). (bv8science.wikispaces.com)


Examining the Impact of Dialogical Argumentation on Grade 9 Learners’ Beliefs about Weather and Indigenous Knowledge

Alvin Daniel Riffel

Department of Education, University of the Western Cape, South Africa.

alvinriffel@yahoo.com

This paper looks at those aspects of Indigenous Knowledge (IK) that are socially and culturally relevant in South Africa for teaching meteorological science concepts in a grade 9 geography class room using dialogical argumentation as an instructional model (DAIM). Focussing on the Western Cape Province, and using a quasi-experimental research design model, the study employed both quantitative and qualitative (mixed methods) to collect data in a public secondary school in Cape Town, in the Western Cape Province.

A survey questionnaire on their attitudes towards, and perceptions of, high school, as well as their understandings and conceptions of weather, was administered to Grade 9 learners before conducting the main study in order to provide the researcher with baseline information and to develop pilot instruments to use in the main study. The study employed a dialogical instructional model (DAIM) with an experimental group of learners exposed to the DAIM intervention, and recorded differences like: responses to the DAIM method of teaching/learning; learner performance (scores in the post MLT test); depth of learners’ understanding about weather/meteorological concepts; their perceptions/attitudes towards Geography - between this group and a control group which had no intervention. Learners from the two groups were exposed to a meteorological literacy test (MLT) evaluation before and after the DAIM intervention. The results from the two groups were then compared and analysed according to the two theoretical frameworks that underpin the study namely: Toulmin’s Argumentation Pattern - TAP (Toulmin, 1958) and Contiguity Argumentation Theory - CAT (Ogunniyi, 1997).

Introduction

My reasons for focussing on indigenous knowledge systems (IKS) in this study were twofold: the latest Curriculum Assessment Policy Statement (CAPS) (2011, p. 39) document for Natural Science from the Department of Basic Education indicates a change in the curriculum plan which incorporates “indigenous knowledge”, which was effective from January 2011. I was also concerned about the vague explanation provided at the time by the Department of Basic Education concerning plans to follow and sketchy indications of assessment strategies to be employed by teachers in using IKS in the classroom in Specific Outcome 3.2 of the Natural Sciences Learning Area:

3.2 RELATIONSHIP OF INDIGENOUS KNOWLEDGE TO NATURAL SCIENCES

Examples that are selected (and that should, as far as possible, reflect different South African cultural groupings) will also link directly to specific areas in the Natural Sciences subject content. (CAPS, 2011, p. 39)

In this paper I argue that implementing aims such as this is likely to have, and has already had, direct implications for the Natural Science curriculum in the majority of South African schools, which do not have the space, framework or resources to develop IKS properly, or in
adequate fulfillment of such Specific Outcomes as the one quoted above. In this context the introduction of IKS into various Learning Areas in the National Curriculum has opened up a debate amongst IKS-researchers, curriculum developers, academics and education specialists, based on the premise that IKS is an integral part of any culture or society and cannot be ignored simply in the interests of creating a smoother assessment curriculum policy statement (Riffel, 2011). I would argue that IKS forms the basis of every learning area, whether Natural Science, Life and Living Skills, or Social Sciences, and should be regarded at school level by education researchers, policy makers and practitioners as a point of departure for learning and for both acquiring and creating new knowledge (according to the Constructivist perspective underpinning the curriculum), as well as generating healthy argumentation in the classroom and amongst learners of all ages.

**Background of study**

This study investigates the perceptions of high school learners’ in their particular social context as well as their cultural beliefs about, and their attitudes towards, indigenous knowledge systems (IKS), and the relationship of these systems to meteorological knowledge and education, including weather predictions and their cultural-religious associations and values. This study also focuses on the perceptions of high school learners and the views that they hold concerning cultural knowledge, world/Western scientific knowledge and the Nature of Science (NOS) in relation to meteorological concepts. A further aim of this study is to investigate and establish the nature and extent and the kinds of views high school learners’ have of indigenous knowledge systems as constituting and informing instruction in the classroom. The authors of the newly drafted CAPS (2011) document from the Department of Basic Education in South Africa have included IKS in the curriculum and in the assessment policies and practices as one of the ways of enhancing learners’ understanding of the Nature of Science (NOS) and developing their ability to make a positive connection with those aspects of cultural knowledge that support IKS.

**Problem statement**

Both the Revised National Curriculum Statement (RNCS) (DOE, 2002) and the Curriculum and Assessment Policy Statement (CAPS) (DoBE, 2011) suggest the need to relate knowledge in the Natural and Social Sciences to IK as a way of closing the gap between the knowledge learners acquire and develop at home and what they learn at school, as well as a way to develop a holistic understanding of their bio-physical and socio-cultural environment. In this regard the expectations inscribed in the Revised Curriculum Statement and the CAPS (under ‘Critical Outcomes’) are for learners to be able to develop certain process skills that they need to solve the problems they encounter in their daily lives. These process skills call for the mobilization of critical thinking skills in classroom discourse (Riffel, 2013). However this presupposes that learners have the freedom and the opportunity for self-expression and for questioning in the classroom. This form of classroom interaction where learners are able to express their views without feeling intimidated or lacking in knowledge comes under the general umbrella of ‘argumentation instruction’, a teaching and learning approach that has been receiving increasing attention in both local and international literature since the beginning of the 21st century. In a study Stears and Malcolm (2005) conducted in the Cape Flats region of the Western Cape, South Africa they found that “relevance and participation go together: relevance encourages learners to participate in classroom processes more deeply, learning in their own ways and bringing together ideas, interests and experiences” (Stears & Malcolm, 2005 p.22). Stears and Malcolm (2005) refer to previous studies which have identified a number of dimensions of relevance. In studies done in Australia Linkson (1999)

**Argumentation**

Argumentation is a form of conversation involving two people or members of a group with the sole purpose of reaching some consensus. In a classroom context argumentation instruction is a method of teaching and learning in the course of which the teacher creates opportunities in class for learners to argue about and discuss freely a particular topic in a situation where learners possess a variety of viewpoints or worldviews (Ogunniyi, 2004). Meteorological concepts in theory provide ample opportunities for learners to argue from various scientific, religious or cultural perspectives. For example, learners from various cultural groups may hold different views, or carry cultural ‘myths’ or subscribe to a cosmology concerning the causes of natural phenomena (e.g. lightning, thunder, changes in the seasons, floods, or cold fronts as a major source of rainfall, etc.)

As a way of pointing a way forward for achieving the outcomes of the new curriculum, the study adopted the dialogical argumentation method of teaching and learning as an intervention in one Geography classroom. Numerous studies (e.g. Amosun et al., 2013; Qhobel, & Moru, 2011; Jegede, & Aikenhead, 1999; Stears, & Malcom, 2005; Erduran et al., 2004) have demonstrated the value and effectiveness of dialogic argumentation as an instrument for discovering teachers’ and learners’ conceptual understandings, as well as making them aware of the tentative and material-discursive nature of the natural sciences (Ogunniyi & Hewson, 2008). According to this argument, the use of dialogical argumentation in the Geography and/or Social or Natural Sciences class room should help teachers and learners in other learning areas to understand the processes involved in ‘border crossing’ into the cultural spaces of hidden indigenous knowledge (Ogunniyi, 2002). This process, I would argue, not only allows the teacher the time to build on both school and every day/home knowledge of learners, but to become acquainted with, and rediscover, the indigenous knowledge that has been lost to many in indigenous communities or cultures due to urbanisation and to the pervading impact of ‘school science’ based on Western science and empirical knowledge.

**Purpose of the study**

The main aim of the present study is to investigate what kind of IK in South Africa, in the Western Cape specifically, is socially and culturally relevant in teaching meteorological science concepts to a group of Grade 9 learners in a specific socio-economic and cultural context using a dialogical argumentation-based instructional model (DIAM) in a CAPS related classroom.

In the course of the study emphasis is placed on how this group of learners relate socially and culturally to selected meteorological concepts and how they interpret them within the context of the current school curriculum, CAPS. The study also aims to create awareness among these learners, and their teachers, about the importance of certain weather concepts, and their interpretation of such concepts.

In particular, the study focused on the following questions:
1. What indigenous knowledge system (IKS) concepts do Grade 9 learners currently hold?

2. What are Grade 9 learners’ ideas about, and attitudes towards, integrating indigenous knowledge (IKS) with school/Western meteorological science?

3. What are the effects of dialogical argumentation instruction on Grade 9 learners’ indigenous knowledge (IK) conceptions, understandings, and attitudes towards Geography?

**Literature Review**

The literature review focuses on theoretical and practical issues regarding grade 9 learners’ conceptions and understandings of selected meteorological concepts. In this context theoretical issues are those theory related issues raised in the literature and based on the views expressed by recognised scholars in the area, while the practical issues deal with actual studies done in the area. This study investigates grade 9 high school learners’ understandings (social and cultural) of meteorological concepts and phenomena. Such conceptions include, or subsume, both (Western) scientifically valid ideas and what is understood in this study as indigenous knowledge systems (IKS) regarding a group of learners’ conceptions about weather conditions/phenomena and/or environmental factors.

**Dialogical Argumentation**

According to Ogunniyi and Hewson (2008), dialogical argumentation occurs when different perspectives of a subject are expressed by opposing groups or individuals with the hope of their ultimately reaching consensus. The purpose of argumentation is to persuade others of the validity of the claim made by the individual or group through well-reasoned or well-grounded arguments. Through dialogical argumentation learners articulate their “reasons for supporting a particular claim and then strive to persuade or convince” others about the truthfulness of such a claim (Ogunniyi & Hewson, 2008, p. 161). Dialogical argumentation provides the critical “environment for learners to externalize their doubts, clear their misgivings or misconceptions, reflect on their own ideas and those of their peers in order to arrive at clearer and more robust understanding of a given topic than would have otherwise been the case” (Ogunniyi & Hewson, 2008, p.161).

Besides direct instruction and direct questions posed by the teacher, classroom talk can take the form of argumentation. Argumentation in a classroom setting would be mostly between students, if a viewpoint is tabled and justified, or if others demand a justification (Qhobela & Moru, 2011). This means learners in a science classroom setting need to use the available data to both justify and make sense of a specific claim (Riffel, 2011). To complete the process, warrants and backings are given by the arguers to support the original claim, and rebuttals are given as counter-claims in showing non-compliances to the original claim. The use of argumentation as a way of learning science derives its strength from the nature of (Western) science and the nature of scientific enquiry. Qhobela and Moru (2011) see ‘scientific knowledge’ as a product of intense and robust discussion within the community of scientists. A scientist must convince other members of this community that a finding amounts to a new, acceptable and important contribution to knowledge (Ford & Forman, 2006).

Beets and Le Grange (2005, p. 1201) argue that in Africa “schools are the sites where most learners first experience the interaction between African and Western worldviews”. They argue and advocate for the need for teachers working in these contexts to be alert, especially in a South African science teaching context, to “this type of interaction and understand the
way it could complicate the learning process” (2005, p. 1201). Much literature has been produced over the years (see Ogawa, 1986; 1988; Fraser, 1990) about the experiences of the majority of African and South African learners with learning and understanding science in the classroom. Despite the volume of literature, and the “fact that indigenous knowledge systems reside among the majority of South Africans, the topic has not been given the attention in educational curriculum development policies it deserves, resulting in a lack of attention to indigenous knowledge in the discursive terrains of all learning areas/subjects” (Le Grange, 2007, p. 581).

**Theoretical Framework**

The study is underpinned by an Argumentation Framework based on Toulmin’s (1958) Argumentation Pattern (TAP) and Oggunniyi’s (1997) Contiguity Argumentation Theory (CAT). The two theories accord with Vygotsky’s notion of constructivism (Vygotsky, 1978) whereby an individual learns or acquires new experiences from his/her interactions with his/her physical or socio-cultural environment. The TAP construes learning as a “product of self- or cross-conversation and reflection” (Riffel, 2014). This study explores the application of both TAP and CAT in the context of classroom discourse dealing with selected meteorological concepts.

The two theoretical frameworks TAP and CAT were chosen because of their amenability to the investigation of classroom discourse dealing with phenomena about which learners might be holding conflicting worldviews.

**Toulmin’s Argumentation Pattern (TAP)**

In order to participate in a scientific community, students and novices need to know “how to construct substantive arguments to support their” position (Toulmin, 1958 p.98). Toulmin (1958) developed the Toulmin’s Argumentation Pattern, a theoretical model that can be used “as a basis for characterizing argumentations in science lessons” (Pedemonte, 2005 p.29). Toulmin (1958) also suggested that a substantive argument requires providing supporting data to a claim. In the current research study this model was used to compare and analyse the cognitive understandings of grade 9 learners in terms of their conceptions of selected meteorological concepts.

**The Contiguity Argumentation Theory (CAT)**

I would argue that as useful as the Toulmin Argumentation Pattern is in assessing the quality of arguments, subsequent research (see Oggunniyi & Hewson, 2008 ; Qhobel at el, 2011) found that it does not address metaphysical IK-rated beliefs that impinge on, or could enrich, learners’ understanding of diverse phenomena. It was because of this limitation that Oggunniyi (2004) proposed the Contiguity Argumentation Theory. The effectiveness of argumentation instruction in enhancing learners’ understanding of school science is well supported by a plethora of studies (e.g. Erduran et al., 2004; Oggunniyi, 2004, 2005, 2007a & b, 2011; Osborne et al., 2004a). In an attempt to mediate between (Western/school) science and IK Oggunniyi (2007a) proposed the Contiguity Argumentation Theory (CAT) in order to capture learners’ experiences which exist beyond the scope of school/Western science. The basis of CAT is that “two distinct co-existing thought systems”, such as, in the context of this study, science and IK, “tend to readily couple with, or recall each other to create an optimum cognitive state” Unlike TAP, which only deals with logical substantive arguments (Toulmin, 1958), CAT “deals with both logical or scientifically valid arguments as well as non-logical metaphysical discourses embraced by IKS” (Oggunniyi & Hewson, 2008, p.161).
Teaching methods that use or facilitate argumentation through the use of appropriate activities and teaching strategies in the Social Science class (in Grade 9 Geography in the South African curriculum context) can provide a means of promoting and fulfilling a wider range of educational goals, including social skills, reasoning skills and the skills required to construct argument and knowledge using evidence (Osborne, Erduran & Simon, 2004a), skills are included under ‘Critical Outcomes’ in the introduction to the NCS and CAPS. Thus, in advocating that teachers need argumentation during the teaching of Social Science, it needs to be emphasised that they need to adapt their teaching styles from direct instruction/transmission approaches to more dialogic approaches (Alexander, 2005) that involve the participation of students in class and lesson discussions, and the use of those teaching methods which include interacting with students in a process of deliberate fostering of students’ argumentation skills.

According to Ogunniyi (2011), the CAT assumes that when different ideas coming from different students interact, they tend (through a sort of dialogical process) to find areas of commonality i.e. areas where their subsumed elements are compatible and may ultimately result in, or metamorphose into, a higher form of meaning than was previously possible (Ogunniyi in Book2 SIKSP, 2009). Furthermore Learning Outcome 3 of the Revised National Curriculum Statement Grades R-9 (2002) Science curriculum carries the expectation of learners being able “to demonstrate an understanding of the interrelationship between science and technology, society and the environment” (Department of Education, 2002, p. 10). The same for CAPS (2011) where the Science curriculum “serves a dual purpose: it must enable learners to make sense of the world in scientific terms and prepare learners for continuing with a science(s) into the Further Education and Training (FET) phase … and beyond” (Caps, 2011 p.57). This outcome serves as an appropriate departure point from which to consider the central concern of the study, namely, grade 9 learners’ conceptions of selected meteorological concepts, and their understanding and use of these.

Research Design

The study is based on a case study design with two main components namely, a quasi-experimental design component and a qualitative research design component. The purpose of experimental research is to describe “the consequences of a direct intervention into the status quo” (Ogunniyi, 1992, p. 81).

Research Setting

This research was conducted in a secondary school in what is called a ‘previously disadvantaged’ community in Cape Town, Western Cape, South Africa. This area formerly hosted the oldest informal community in Western Cape which, in the interests of anonymity, will be referred to as ‘Blikkiesdorp’ (pseudonym) with a 67 year history dating back to 1948 under the apartheid government in South Africa. With its tin roof and shack-like informal housing structures, it gave permanent residence to more than 750 families in the southern suburbs of the Western Cape. The first primary school of the area was erected as a temporary structure (wooden walls on concrete slabs) adjacent to Blikkiesdorp in the early 1950s. The area and residents are characterised by poverty, and low income and education levels.

Sampling

This included two intact school class groups from a public high school (school “X” for purposes of anonymity) in the Western Cape. The school is situated in a distinct category of socio-cultural and economic area of ward 801* (pseudonym) of the City of Cape Town. The sampling involved two grade 9 classes from the same school. Participants ranged from 14-
17 years of age. Classes were positively selected on the basis of comparability with respect to:

D. Classes doing the same subject and taught by the same teacher
E. Formal class test and reports
F. Learners within comparable socio-cultural and economic backgrounds

Quantitative and Qualitative Research Methods

For the purpose of this study both quantitative and qualitative “mixed methods” were used to complete the data collection process. The qualitative data were derived from learners’ written responses in the Conceptions of Weather (CoW), Meteorological Literacy Test (MLT), as well as Classroom Observation Schedules (COS). The quantitative data were derived from learners’ performance scores in the Meteorological Literacy Test (MLT).

A teacher on the staff of the school undertook the Comparison (C) group intervention, while I as the researcher took charge of the Experimental (E) group Dialogical Argumentation Instructional Model (DAIM) intervention. The teacher who handled the (C) group used traditional teaching methods, discussions and resources to provide the control component of, and to complete, the research process. The teacher was also asked to use her ‘normal’, accustomed teaching methods according to those specified in the National Curriculum Statement (NCS) (DoE, 2002) and the Curriculum Assessment Policy Statement (CAPS) (DoBE, 2011) that encourage the implementation of IKS into a school Science curriculum as s/he understood these while I used the Dialogical Argumentation Instructional Model (DAIM) with the experimental (E) group. Both comparison (C) group and experimental (E) groups were exposed to the same Meteorological Literacy Test (MLT) assessment before and after the teaching that happened over a period of eight weeks.

The quantitative aspect of the study was in the form of quasi-experimental (pre-test), case study control group design. Only the results from the survey and pre-test were to be evaluated to interpret the learners’ conceptions, and/or the effectiveness/depth of their weather conceptions, as shown below in Figure 1:

Quasi-experimental Control Group Design

This research design can be both quantitative and qualitative in nature e.g.

\[
\begin{align*}
O_1 & \times O_2 \ (\text{Experimental group}) = (E) \\
\hline
O_3 & \quad O_4 \ (\text{Control or Comparison Group}) = (C) \\
\end{align*}
\]

\[
\begin{align*}
O_1 & \times O_2 = \text{Pre-observations} \\
O_3 & \quad O_4 = \text{Post-observations} \\
\end{align*}
\]

Figure 1. Quasi-experimental control group design

The Intervention

The intervention referred to as (X) focused on the Experimental (E) group only. The intervention was in the form of a CoW (Conceptions of Weather) questionnaire, and based on local indigenous knowledge. The Dialogical Argumentation Instructional Model (DAIM)
class room model was used to apply the teaching part of the intervention to the E group. The Control (C) group was used to compare pre-and post test results from the two groups in order to establish whether the intervention applied had been successful.

During the two-month intervention stage the experimental group (E-group) formed four randomized groups. Each group was asked to choose a group leader who would present the outcomes of its discursive arguments on the various tasks to the whole class. It is worth noting that in each session the group had to choose another leader and rotate their individual roles such as reader of the worksheets containing the instructions, recorder, and manipulator of apparatuses or materials. This process was also intended to develop skills other than the understanding and use of meteorological concepts using appropriate resources. For reasons of space limitation further details of the procedures followed in the study which have already been published elsewhere are not repeated here (e.g. Ogunniyi, 2004, 2007a & b; Siseho, 2013; Siseho & Ogunniyi, 2010 & 2011a & b).

Figure 2. A Pedagogical Scheme for Implementing Dialogical Argumentation Instruction

The author is a member of SIKP (Science Indigenous Knowledge Project) from the School of Science and Mathematics Education at the University of the Western Cape where the above diagram (Figure 2) was generated in order to illustrate the process and trajectory of the DAIM. The ‘pedagogical schema’ (Siseho, 2013) for enacting a dialogical argumentation-based discourse (as shown in Figure 2 above) is a descriptive model arising out of the series of SKIP workshops and has been piloted successfully based on empirical evidence (e.g. Ogunniyi, 2007a & b). The result of using the pedagogical schema was the attainment of some level of cognitive synchronization on the part of the participants based on convincing evidence and warranting what was recorded during the intervention stage.
Results of CoW

The report of results is to be seen in the context of the original questions and combines the descriptive data with the survey data. Where survey and descriptive data (pre-test) were noted, the CAT was applied to inform and describe the types of findings.

This questionnaire was administered two weeks after the survey questionnaire was completed. The same group that completed the survey completed the CoW questionnaire.

The CoW questionnaire indicated that the learners had little knowledge about IKS and the nature of the impact they have on the local and global community. It appeared that very little indigenous knowledge as such was made use of at home by parents and other family members. If any IKS knowledge was used in daily traditions or practices it was never noted as such by those elders and others as belonging to a cultural system of IK. From this one could assume that it was very difficult or even impossible for learners to be acquainted with the nature and content of cultural knowledge or of IKS when answering the CoW questionnaire.

Much of the traditional/cultural knowledge is derived from elders in our communities. Some of the traditional knowledge has made its way into present day situations through stories and folk-lore passed on through traditional dances, song, rituals and other cultural engagements such as community festivals, weddings, prayer meetings and seminars (Riffel, 2011).

A total of 25 subjects from the E and C group completed the CoW questionnaire.

<table>
<thead>
<tr>
<th>Nominal scale</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Like-it Scale</td>
<td>Strongly disagree</td>
<td>Agree</td>
<td>Disagree</td>
<td>Strongly disagree</td>
<td></td>
</tr>
<tr>
<td>Boys Beliefs (%)</td>
<td>15.71</td>
<td>40.71</td>
<td>34.29</td>
<td>9.29</td>
<td>= 100 %</td>
</tr>
<tr>
<td>Girls Beliefs (%)</td>
<td>28.57</td>
<td>42.15</td>
<td>17.14</td>
<td>12.14</td>
<td>= 100 %</td>
</tr>
<tr>
<td>Total %</td>
<td>44.28</td>
<td>82.86</td>
<td>51.43</td>
<td>21.43</td>
<td>(200) / 2 = 100%</td>
</tr>
</tbody>
</table>

In examining the overall results of the conceptions of weather (CoW) questionnaire in Table 1 (above) one could conclude that the answers from learners display a more positive and inspired view on the nominal scale 1= Strongly Agree (44.28%) and nominal scale 2 = Agree (82.86%) then disagree, nominal scale 3 = (51.43%) and nominal scale 4 = (21.43%) respectively on the original questions that were administered to them in the CoW questionnaire. On item 1 ‘Learning geography through school science is interesting.’ the collective response count of boys and girls was surprisingly low. Only two boys (20%) and two girls (20%) agreed with the statement that they find ‘geography interesting’. This claim can be supported by reasons given by learners’ as to why they chose to agree or disagree in the following excerpts:

On item 1: some boys (2) responded (direct transcription)

L1: Because there is a lot of stuff to do and to learn about.
L3: Because I don’t like it and I don’t know what to do when the teacher give the class to me.

Learner L1 is saying that the geography subject has too many complicated sections to deal with, and he shows an awareness of the difficulty of dealing with subject learning material that you do not understand. He also finds it complicated to handle all the various concepts within the geography subject. For L3 being, or having no choice but to be, in the geography classroom and in the presence of other learners, besides his subject teacher, is in itself just too much for him to deal with because he does not like geography as a school subject at all.

The excerpts below are representative of the learner’s responses (direct transcription) to certain items in the CoW questionnaire:

On item 1: some girls (2) responded (direct transcription)

L10: The teacher doesn’t make it interesting but I make it myself sometimes he makes it interesting.

L12: The children in class makes it not easy to understand the teacher.

No connection between IKS and weather related geography was indicated, although 80% (16) agree that using IKS to learn geography can help understand the weather better. Out of the 20 participants only 6, three girls and three boys, responded to their choice of answers.

On Item 2: To ‘Using my indigenous knowledge to learn geography helps me to understand weather’, some girls (3) responded:

L8: I don’t always know what the weather would be.

L9: To know everything about the weather is better than to understand that.

L10: Sometimes I’m wrong but it help a little with geography.

On Item 2, some boys responded:

L15: No, if you don’t know geography how can you understand the weather.

L16: Yes, it show all the countries and show the weather.

L18: To know what is going on with the weather.

On Item 4 the girls (2) responses were:

L10: I don’t believe in both.

On Item 4 boys (3) responses were:

L16: It shows on the news what is the weather going to be.

L18: Their knowledge is better than ours.

L20: Because indigenous knowledge will not show you about the weather.

Findings on CoW

Among other responses, the excerpts above suggest that most learners have never been exposed to IKS in the field of Geography, and that for them there is no connection or relation between IKS and their understandings of weather phenomena.
Not all learners completed the response section, and this resulted in less than half of the total questionnaires that were administered being fully completed. This also demonstrated that the learners did not show interest in reading and making time to answer questions properly. The lack of interest shown in many of the partially completed questionnaires leads to the impression that most of the learners are either not interested in Geography as a subject, or the teaching of the subject is poor or uninteresting or ineffective, particularly in terms of the impact it has on their daily lives.

Summary

From the responses of the Grade 9 learners to the CoW questionnaire one can conclude that Geography is not a favourite subject at their school, possibly mainly due to the lack of resources both at their school and from the DoBE (Department of Basic Education). Most of the learners’ current knowledge on weather and the environment seemed to be derived from school/Western knowledge, with very little knowledge gleaned from an indigenous knowledge perspective. In fact some learners had never heard of IKS. Some even mentioned that they had never encountered the term IKS. From this I would argue that teaching strategies should be initiated, and resources provided or acquired, that showcase the importance of tapping into indigenous knowledge for easier and more relevant acquisition and use of geography concepts by learners, and concepts which are of value to them both in the classroom and in their everyday lives.

DAIM Findings

DAIM findings which emerged from analyzing both the quantitative and qualitative data and are as follows:

- E group learners being exposed to argumentation in the classroom during the DAIM intervention process seemed to have enhanced their awareness of the Nature of Science (NOS) and the Nature of IKS more than those in the C group who were not exposed to this learning strategy.
- E and C groups held relatively good conceptions of weather processes. Their attitudes as revealed in the questionnaire indicated that both groups possessed valid scientific conceptions about weather and meteorological science.
- Both groups’ pre-test responses to the attitudes questionnaire based on the framework of the Contiguity Argumentation Theory (CAT) revealed that they held largely equipollent views (i.e. Scientific and IKS-based) of weather (Geography). This suggests that they held both the Western scientific and the IKS-based views of weather knowledge in a co-existing manner.
- Learners that were exposed to the DAIM intervention developed a better understanding and attitude towards a science worldview, and value the use of IK embedded science lessons.

The Experimental (E) group learners appeared to have developed a positive attitude and better understanding towards science and an enhanced awareness of the Nature of Science (NOS) and the Nature of IKS (NOIKS), when compared to the Control (C) group

Both groups were exposed to the conceptions of weather science and IKS content test, the Meteorological Literacy Test (MLT), and the average scores were almost the same for both groups in the pre-test. The E group had an average of 18.88 (37.76%) and the C group had an average of 14.33 (28.67%), as indicated in Table 2 below. The mean results of the E group
had significantly improved from an 18.88 mean to a 29.87 mean, with an overall 10.87 mean value improvement after the IKS and DAIM intervention. On the other hand the C group scored a 14.33 mean value in the pre-test and a 15.45 mean value in the post-test, with a mere 1.36 overall mean value improvement with no intervention exposure. Thus a significant improvement of 10.87 in the overall mean results in the E group as opposed to the 1.36 mean values in the C group can be noted. I argue that this difference in mean value score improvements between the two groups can be attributed to the DAIM intervention that was applied only to the E group and not to the C group. Both groups completed the MTL in the pre-post test.

**Table 2. EXPERIMENTAL & CONTROL GROUP – Pre-post –test scores compared**

<table>
<thead>
<tr>
<th></th>
<th>E GROUP</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PRE-TEST</td>
<td>POST-TEST</td>
<td>PRE &amp; POST DIFF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MEAN</td>
<td>18.88</td>
<td>29.87</td>
<td>10.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>3.31</td>
<td>4.28</td>
<td>3.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**t-TEST: N = 15, Critical t-value = 2.131, Alpha = 0.05**

<table>
<thead>
<tr>
<th></th>
<th>C GROUP</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PRE-TEST</td>
<td>POST-TEST</td>
<td>PRE &amp; POST DIFF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MEAN</td>
<td>14.33</td>
<td>15.45</td>
<td>1.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>4.39</td>
<td>3.72</td>
<td>6.34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**t-TEST: N = 10, Critical t-value = 2.201, Alpha = 0.05**

**Benefits of DAIM**

I would argue that the Dialogical Argumentation Instructional Model (DAIM) provides an alternative lens for explaining many of the empirical findings in science/IKS education. DAIM also addresses the shortcomings of alternative frameworks for scientific reasoning. DAIM can also be used and taught as a model for the reasoning processes of science.

**Conclusion**

At the school serving as the research site, Geography was clearly not a favourite subject of learners, I would argue mainly because of the lack of resources, but possibly also because of poor teaching strategies. I would of course see these as interrelated, or a combined cause of learners’ lack of interest in the subject. Most of the learners’ current knowledge about weather and the environment is derived from school knowledge, and very little geographical or meteorological knowledge seemed to have been gleaned by them from an indigenous knowledge perspective. In fact, some learners were completely unfamiliar with the concept of IKS. Some had never had any encounters with the term IKS. Thus, as I have mentioned, and re-emphasise, this paper advocates for the introduction of teaching strategies and resources that showcase the importance of tapping into indigenous knowledge for easier and more relevant acquisition and use of Geography concepts by learners, and concepts which are of value to them both in the classroom and in their everyday lives.

**References**


Ogunniyi, M.B. (1997). *Curriculum 2005: A Panacea or a Pandora's Box?* (Vol. Seminar series 1). Bellville: Published by the School of Science and Mathematics Education at the University of the Western Cape.

Ogunniyi, M.B. (2011a). Exploring science educators'cosmological worldviews through the


Riffel, A. (2011). What effect will dialogical argumentation have on a grade 9 learners conceptual understanding of selected meteorological concepts when used as an instructional method in a science classroom in the Western Cape, South Africa. *Proceedings of the 19th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education* (pp. 121-128). Mafikeng, South Africa: North-West University.


**Acknowledgement**

This paper is in memory of my late father Izak Daniel Riffel (17:01:1930 – 08:12:2013)
Adapted Mathematical Knowledge for Teaching Measures: Reliable, But Still Challenging

Reidar Mosvold 1, Janne Fauskanger 2, & Arne Jakobsen 3
1Department of Education and Sports Science, University of Stavanger, Norway. 2Department of Education and Sports Science, University of Stavanger, Norway. 3Department of Education and Sports Science, University of Stavanger, Norway.
1 reidar.mosvold@uis.no; 2 janne.fauskanger@uis.no; 3 arne.jakobsen@uis.no

In this paper, we discuss why researchers need to be critical when using measures, although – from a measurement perspective – the measures seem to work well. As a starting point for our discussion, we investigate the inter-coder reliability and internal consistency of mathematical knowledge for teaching measures that were translated and adapted for use in Norway. We analyse the responses of 142 teachers to a set of 61 items. The overall conclusion is that we can report on high inter-coder reliability and high internal consistency of the test overall. Despite these positive results, the complexity of measuring teachers’ knowledge still makes it challenging.

Introduction

In the last decades, there has been a strong focus on the mathematical knowledge needed for teaching (e.g., Davis & Simmt, 2008). Attempts have been made to model or describe (e.g., Petrou & Goulding, 2011), and also measure the knowledge a teacher needs to teach mathematics (e.g., Hill, Sleep, Lewis, & Ball, 2007). In this paper, we focus in particular on the theories and measures developed by Deborah Ball and her colleagues at the University of Michigan (e.g., Ball, Thames, & Phelps, 2008). In the Learning Mathematics for Teaching (LMT) project, these researchers developed sets of multiple-choice items to measure teachers’ mathematical knowledge for teaching (MKT), and this development of items has been regarded as indispensable to the development of a conceptual understanding of MKT. Researchers have described MKT as a practice-based theory – defined as the “mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students” (Ball et al., 2008, p. 399).

Although the framework relates to knowledge needed for performing all tasks of teaching mathematics, the researchers in the LMT project have only developed items to measure certain aspects of MKT so far. Ball and colleagues have been particularly interested in the content knowledge needed for teaching mathematics, and they have focused on the distinction between common content knowledge (CCK) and specialized content knowledge (SCK); SCK has been particularly emphasized (Ball et al., 2008). An example of a problem that requires CCK is to divide 1¼ by ½ (see Figure 1). This problem is not particularly related to the work of teaching mathematics, but it is a problem that can be solved by people who use mathematics in other professions than teaching. CCK includes knowledge of number facts and algorithms, and the focus is on calculating an answer or solving a mathematical problem correctly; it can also include knowledge of concepts and connections (e.g., “why” an algorithm for computing fraction division would work). A mathematics teacher, however, needs more than the knowledge to solve such a problem, (s)he also should be able to understand a variety of algorithms for division by fractions and assess whether or not these algorithms could be used to divide any two fractions. This is an example of specialized content knowledge (SCK) – mathematical knowledge not typically needed in other contexts than teaching. In order to determine which of the story problems presented in Figure 1 that could be used to illustrate 1¼ divided by ½, teachers also need SCK.
The theory of MKT has evolved from empirical studies, and factor analysis suggests that MKT is multidimensional (Ball et al., 2008; Schilling, Blunk, & Hill, 2007). The existence of SCK and CCK as two different constructs within the subject matter knowledge needed for teaching is, however, only partly documented in the U.S. (e.g., Schilling et al., 2007). In studies where the MKT instrument has been used outside the U.S., the results are mixed. In the Norwegian context, Drageset (2009) reported good reliability of the measures, and he concluded that the empirical evidence indicated the existence of CCK and SCK as two distinct dimensions of MKT. In a study of Indonesian teachers’ MKT, on the other hand, Ng (2012) found that the adapted measures had a relatively low reliability. In light of these mixed results it seems relevant to analyse the reliability of measures carefully, and this could be particularly relevant to investigate and discuss critically when using measures in different cultures.

An aspect of reliability that is particularly relevant in this context is that of internal consistency; this is connected with the inter-relatedness of the items within a test and describes the extent to which the items in a test measure the same construct. Tavakol and Cennick (2011) suggests that internal consistency should be determined before a test can be employed for research or examination purposes to ensure validity, and Cronbach’s alpha – a commonly employed index of internal reliability – is important in the evaluation of assessments.

Cronbach’s alpha is calculated from the squared correlation between observed scores and true scores; this interpretation of reliability is the correlation of the test with itself. A reliable test should minimize the measurement error so that the error is not highly correlated with the true score. The relationship between true score and observed score – which can be examined with Cronbach’s alpha (Ary, Jacobs, & Razavieh, 1996) – should be strong. It is expressed as a number between 0 (denoting no internal reliability) and 1 (denoting perfect internal reliability) (Bryman, 2004). In general, reliabilities of 0.80 or above are often considered adequate (Bryman, 2004); some researchers accept lower reliabilities – such as 0.70. A high alpha indicates that the test consistently measures an underlying construct, but additional analyses must be performed to be able to assure that the measure is unidimensional (e.g., exploratory and/or confirmatory factor analysis).

In this paper, we aim at going into a discussion of why researchers need to go beyond a technical discussion of reliability measures when using LMT measures, and we approach the following research question:

**Figure 1.** Testlet item from the public released LMT items (Ball & Hill, 2008, p. 8).
What are some possible reasons why researchers should look beyond reliability measures when applying measures of mathematical knowledge for teaching?

We investigate the inter-coder reliability and internal consistency of mathematical knowledge for teaching (LMT) measures that were translated and adapted for use in Norway, and we discuss possible explanations for the results.

Methods

As a part of a research project focusing on adapting the LMT measures for use in a Norwegian context, a complete form (Elementary form A, MSP_A04) of items was translated and adapted (e.g., Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012) for use among Norwegian teachers. In total, 61 items were included; 26 in a sub-scale on number concepts and operations, 19 in a sub-scale on geometry, and 16 in a sub-scale on patterns, functions and algebra. We selected these sets of items because these three areas are emphasized in the Norwegian curriculum.

In our project, 142 in-service teachers answered this individual test. The participants were selected from a convenience sample of 17 schools – all of which were connected to our university as practice schools. The teachers had the same formal qualifications although they taught different grade levels.

Initially, the test items were not developed to distinguish between sub-domains of MKT; the theory of MKT – as presented by Ball and colleagues (2008) – was only published after the test had been adapted. As a result of this, the items included in the test had to be categorized using the sub-domains of MKT in retrospect. The following definitions were used as codes:

- CCK: Calculating an answer or solving a mathematical problem correctly.
- SCK: Mathematical knowledge not typically needed in other contexts than teaching. Many of the tasks of teaching are distinctive to this special work.

In order to assure inter-coder reliability, the three authors coded the items independently and reconciled.

The analysis of internal consistency – measured by Cronbach’s alpha – was calculated by the use of IBM SPSS Statistics version 21.

Results

Reliability is important when assessing the precision of a test. In previous publications, we have reported and discussed item response theory (IRT) reliability estimates of the adapted measures (Fauskanger et al., 2012; Jakobsen, Fauskanger, Mosvold, & Bjuland, 2012). The IRT reliability estimates are good for all three scales in the measures: number concepts and operations, (0.838) geometry (0.799), and patterns, functions and algebra (0.861). The point of maximum information is below the mean, indicating that the test as a whole provides optimal measurement of teachers with an IRT score below average (from 0.75 to 0.875 standard deviation below the mean score). In this paper, we report on two additional aspects of reliability: inter-coder reliability and internal consistency – using classical test theory (CTT). We start with internal consistency, which is normally measured by Cronbach’s alpha.

Cronbach’s alpha is not robust to missing data, and includes only data from teachers who answered all the items (listwise deletion). For the test as a whole, the reliability was 0.886 for the 61 items – which is good. All items appeared to be worthy of retention; the greatest increase in alpha would come from deleting item 1c, but removal of this item would increase alpha only by 0.003.

Another finding is that all items do not correlate well with the total scale. Some of the r-values are less than 0.3, indicating that these particular items do not correlate well with the scale overall. One of the items, 17c, even had a negative correlation. This is reported and discussed elsewhere (Fauskanger et al., 2012).
The LMT items were developed to measure content knowledge for teaching. With high alpha value the items seem to measure that construct consistently across the entire test. The next step in our analysis was to look closer into the sub-domains of MKT included in the test: SCK and CCK. The items were not originally categorized, so we had to make a categorization in retrospect – according to the definitions found in the literature (e.g., Ball et al., 2008). All three coders placed the items in the same category in 90% of the cases, and the inter-coder reliability is thus regarded as good. For the last 10% of the cases, the items were discussed and codes reconciled.

A total of 44 items were categorized as CCK items. Cronbach’s alpha was 0.853 for these 44 items, and the CCK sub-scale thus seemed to have good internal consistency. All the CCK items appeared to be worthy of retention, and the greatest increase in alpha would come from deleting item 1c. In this item the teachers are challenged to assess students’ work on grouping a three-digit number, some of them non-standard. By removing this item, however, the alpha would only increase by 0.004.

Out of the 61 items in the test, 17 items were categorized as measuring SCK. Cronbach’s alpha was 0.634 for these 17 items. The greatest increase in alpha would come from deleting item 30a (an algebra item where teachers should decide whether or not a text problem could be linked with a given equation), but removing this item would only increase alpha by 0.009 – still not close enough to the recommended alpha of 0.80.

**Discussion**

When assessing the internal reliability of measures, it is relevant to consider 1) the overall consistency of the instrument as a whole, 2) the consistency of the different parts of the instrument (e.g., the CCK and the SCK sub-scales), 3) the internal consistence if an item is deleted and 4) the correlation between each item and the total score (Pallant, 2013). In our study, the internal reliability of the instrument as a whole was good (0.886), and the instrument thus seems to consistently measure the same construct: content knowledge for teaching mathematics. When comparing with the results from other studies where adapted LMT measures have been used, and reliability discussed, our results were slightly better than those of Drageset (2009) and much better than those of Ng (2012). From those studies we do not, however, get information about missing data.

When investigating the sub-scales, our results were more mixed. Cronbach’s alpha for the CCK sub-scale was good. Our findings in this respect were more promising than those of Drageset (2009). In a similar study – also in the Norwegian context – he got an alpha of 0.72 for the CCK sub-scale; we got an alpha of 0.853. The alpha value found for our SCK sub-scale, on the other hand, was not so good. A Cronbach’s alpha coefficient of 0.634 is below what is normally accepted. When comparing with Drageset again, he got 0.77. The two forms used in these two studies did not, however, include the exact same items, and the reliabilities cannot be compared directly. Although the overall result seems promising, it is necessary to discuss the results of the SCK sub-scale in particular. Hill and colleagues (2008) suggest that a low Cronbach’s alpha can result from different issues: 1) it might indicate that multiple constructs are present in the scale, 2) it might indicate that some items are not sensitive to discriminate among participants, and 3) it might indicate that there is a mismatch of items when compared with the ability level of the target population (see also Ng, 2012). In our discussion here, we want to pay particular attention to the first possibility – although other explanations are also possible.

The items used in our study were not originally separated in CCK and SCK sub-scales. We made such a categorisation in retrospect. We reached high inter-coder reliability in this process, but some items were problematic to categorize. With two of the items, we discussed whether they should be coded as SCK items or knowledge of content and students (KCS) items. The decision depends a lot on how the actual item is presented. Items that asked teachers to evaluate different student-responses were coded as SCK items, but one might argue that people with a high CCK might be able to answer some of these items. Ball and colleagues (2008) discussed that a strict distinction between sub-categories of MKT is not always easy; one teacher might use SCK when responding to an item, whereas another teacher might draw upon KCS. We found the same tendency in our analysis of the connection...
between teachers’ responses to the multiple-choice items and their corresponding written reflections (Fauskanger & Mosvold, 2012); teachers seemed to draw upon different aspects of MKT when responding to items. A possible explanation for the low Cronbach’s alpha in our SCK sub-scale might then be that multiple constructs are present.

Other researchers suggest additional possible factors that might affect reliability coefficients (e.g., Ary et al., 1996) – thus extending the list that was suggested by Hill and colleagues (2008). One such factor is related to the length of the test. In general, longer tests will increase reliability (Ary et al., 1996). Our adapted measures included 61 items, and this is more than both Drageset (2009) and Ng (2012); Drageset included 46 items and Ng included 35. Another factor is group heterogeneity. The reliability coefficient increases as the heterogeneity of subjects who take the test increases. In our study, the teachers were teaching in different grade levels, and the heterogeneity was thus high.

Conclusion

Studying the validity of adapted MKT measures is highlighted in the literature (e.g., Shilling et al., 2007). Reliability is a necessary condition for valid interpretation. Tests should measure MKT and its sub-domains consistently, and, although reliable test may or may not be valid, an unreliable test can never be valid. This means that reliability is the upper limit of validity and important to study.

In previous publications, we have investigated and critically discussed various aspects related to validity – as well as the IRT reliability – of a set of MKT items that we have translated and adapted for use in Norway. We have scrutinized issues regarding translation, adaptation (e.g., Fauskanger et al., 2012), issues with the multiple-choice format, the pitfalls of including “I’m not sure” as a suggested solution (Fauskanger & Mosvold, in press), the connection between item responses and written reflections (e.g., Fauskanger & Mosvold, 2012) and more. When analysing the results from a measurement perspective, however, the overall conclusion is that the adapted measures seem to work well in a Norwegian context (Fauskanger et al., 2012; Jakobsen et al., 2012). In this paper, we can report on high inter-coder reliability and high internal consistency of the test overall. In light of this, a natural question to ask might be: When all seems well, why worry so much about all the possible perils and pitfalls? This quandary – and an outside observer might indeed suggest that there is a quandary when we have a reliable test that seems to work well on the one hand and ongoing critical discussions on the other hand – can be approached in different ways. On the one hand, it can be seen as part of our overall approach to ensure that the measures we use are as high-quality and precise as possible – and we believe this is an iterative rather than linear process (Fauskanger et al., 2012). On the other hand, it can be seen as an attempt to scrutinize the measures from different perspectives to increase the overall quality. We have used a combination of quantitative and qualitative approaches in this process, and we have been conscious about not only following the herd in our efforts.

As a conclusion, we could simply point to the results and claim that the measure is reliable and all is well. This would be a too simple conclusion. Despite the high reliability, there are still issues. First, reliability resides in the interaction between a particular test and a particular population of test-takers. It is thus important to note that the test, even though it demonstrates high reliability in this situation may show lower reliability in another situation. Another issue is that some items – an example is item 17c, which related to the number of sides for polygons – seem to behave in strange ways (Fauskanger et al., 2012), and the distinction between the sub-domains of MKT is not always straightforward. The internal consistency of the CCK items was high in our study, but the Cronbach’s alpha for the SCK items was too low. One possible explanation is that more constructs could be involved. In our coding, we discussed whether two particular items should be categorized as SCK or knowledge of content and students (KCS). In previous publications, we have also suggested that teachers seem to draw upon other aspects of MKT when replying to particular items (Fauskanger & Mosvold, 2012); Ball and colleagues (2008) also indicate that some teachers might draw upon different parts of their knowledge to answer particular items. This indicates that even further studies might be needed to investigate the distinction between different aspects of MKT. A possible result of such studies could be that measures are reliable and seem to work well, but the complexity of “reality” still makes it challenging. From a
research perspective, this complexity is relevant to investigate further; when attempting to adapt and use measures in other cultures this is a particularly relevant topic of investigation.

References


